Optimal Fiscal and Monetary Policy with Heterogeneous Agents and Nonlinear Income Taxation

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Abstract

Previous papers consider optimal fiscal policy in an economy without involving money, or discuss optimal monetary policy with homogeneous agents in reduced-form approaches, putting money in the utility function, or imposing cash in advance. Different from them, this paper studies the optimal fiscal and monetary policy with heterogeneous agents in an environment where explicit frictions give rise to valued money, making money essential in the sense that it expands the set of feasible trades. In the spirit of Mirrlees’s private information framework and based on the search-theoretical environment, the paper first solves the households’ problem in the centralized and decentralized market, and finds out the optimal conditions. Then, the paper describes the problem that social planner faces by involving uncertainty and agents whose types are continuous. By comparing the optimal conditions in this generous setting, I show that the Friedman rule is no longer optimal when combined with nonlinear taxation of income. In addition, the capital income taxation is not zero.

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1 Introduction

Friedman (1969) states that in order to achieve the first-best, the nominal interest rate should be set to zero since positive nominal interest rates represent a distorting tax on real money balances. This prescription, known as the Friedman rule, is a critical conclusion in monetary economics. Chari, Christiano and Kehoe (1996) show that in three different representative agent models of money demand: the money-in-utility function model of Sidrauski (1967), the cash-credit model of Lucas and Stokey (1987), and the shopping-time model of Kimbrough (1986), when preferences are homotheticity and weakly separability in labor, the Friedman rule holds. However, if we consider the heterogeneity, other than representative agents in the economy, what is the optimal monetary policy, and whether the Friedman rule still holds?

Since a disproportionate amount of money holding, monetary policy would have different effects across agents. For example, for those families with lower income, the amount of cash in hand has always occupied a higher proportion of their total wealth. Eroso and Ventura (2002), based on US data, show that low income households use cash for a greater fraction of their total purchases relative to those households with high income. Wen (2010) uses the cash-in-advance model with heterogeneous money demand and shows that under an annual inflation rate of 10% and sufficiently large inequality in cash holdings to match that in the U.S. economy, households are willing to reduce up to 15% of average consumption each year in exchange for the Friedman rule inflation rate, which is significantly bigger than that of with considering homogeneous agents (Lucas (2000)). Therefore, the heterogeneity could lead to dramatically different results of monetary policies from those under the representative-agent assumption.

In recent years, some studies have explored the optimal monetary policy by involving the heterogeneity in the economy, and achieved different results. Costa and Werning (2008) study an economy where agents differ in labor productivities and government sets nonlinear labor income taxes and the nominal interest rate. They use the money-in-utility model and allow that individual labor ability are private information, by following Mirrlees (1971). The paper gains the conclusion that the Friedman rule is still optimal if money and labor effort
in utility are gross complements. However, as Albanesi (2008) points out the empirical works is inconsistent with their assumption which would lead to a cross-sectional distribution of money holdings. Moreover, their model considers the money in a reduced-form model and has not supplied the micro-foundation of money in the economy.

Bhattacharya, Haslag and Martin (2005) discuss several alternative monetary economies in which agents have heterogeneous money holding, and find that the Friedman rule does not maximize ex post steady-state welfare with considering the redistribution effects. However, their results are not robust as they ignore the asset market in their economy. If the asset market is incomplete, as noted by Aiyagari (1994, 1995), agents would have strong incentives to self-insure against idiosyncratic shocks through precautionary savings, which motivate agents to hold excessive amount of cash in hand to avoid liquidity constraints. Therefore, in this condition, money serves not solely as a medium of exchange, but also as a store of value.

On the other hand, normally, there exists two different approaches to consider the optimal fiscal policy, based on Ramsey (1927) or Mirrlees (1971). In the Ramsey approach, the starting point is postulating tax instruments. Usually, it is assumed that only linear taxes are allowed and lump sum taxation is prohibited. The crucial assumption is that all activities of agents are observable. The main lessons of Ramsey taxation are uniform commodity taxation (Atkinson and Stiglitz (1972)) and zero capital tax in the long run (Chamley (1986) and Judd (1985)). Conversely, the Mirrlees approach to optimal taxation is built on a different foundation, which assumed that an informational friction endogenously restricted the set of taxes that implement the optimal allocation. While the information friction posed is unobservability of agents’ skill, the objective of the Mirrlees approach is to find the optimal incentive-insurance trade-off (Golosov and Tsyvinski (2006)). By extending the static models into dynamic, Golosov, Kocherlakota, and Tsyvinski (2003) and Werning (2001) have the conclusion that any optimal allocation includes a positive intertemporal wedge. However, when they are considering about the optimal taxation issues ether in Ramsey approach or in Mirrlees approach, they have not involved the money in the economy or provide the micro-foundation about holding money.

This paper reconsider the optimal fiscal and monetary policy problem with search-theoretic
framework. Different from the classical search-theoretical papers such as Lagos and Wright (2005) and Williamson and Wright (2010), an important assumption of this article is that individuals also need to accumulate physical capital to produce in the economy. Just as Kocherlakota (2005) states, we need to answer the following questions: what are the micro-foundations for the difference in returns between money and other assets? How are these micro-foundations likely to affect the nature of optimal monetary policy? Since in the paper, each period is divided into two subperiods, and there exists two kinds of markets, opening separately in the economy which are centralized market and decentralized market. In the centralized market, economy is operating as in the classical growth models. Households supply their labors and capitals, receive their payment and adjust their asset levels in the frictionless market. While in the decentralized market, some parts of households could trade with some one by anonymous matching, which supplies the microfoundations of monetary theory.

While the households in the economy are heterogeneity in skills which are private information and always changing over time, the paper develops a full characterization of the socially optimal allocation in this environment. In order to classify different kinds of agents, we need to consider a large number of incentive constraints, which would lead us to reconsider the optimal monetary and fiscal policy under these settings. The paper finds out that Friedman rule is no longer optimal when combined with nonlinear taxation of income. In addition, the capital income taxation is not zero and marginal labor income taxation is negative correlated with skills.

By investigating different kinds of assets and heterogeneity in the standard search-theoretical model, the main goal of this paper is to find out the solutions of the above questions. Section 2 introduces the model and the basic problems for households. Section 3 derives the optimal conditions for social planner’s problem and find out the optimal monetary policy. Section 4 presents the conclusions.
2 The Model

The model extends the framework in Lagos and Wright (2005), and is similar to the baseline model in Aruoba and Chugh (2010), Aruoba, Waller and Wright (2011) and Williamson and Wright (2010), by allowing capital accumulation. The main difference is that in this economy, there exists a continuum of infinitely lived individuals with differences in productivity other than homogeneous agents. The purpose of this assumption is to introduce the heterogeneity, in spirit of Mirrlees’s (1971) private information approach, in a tractable way with considering the micro-foundation of monetary economics.

The article considers an infinite-horizon economy where time is discrete $t = 0, 1, \ldots$. The economy is populated by a continuum unit measure of agents, indexed by $i \in [0, 1]$ with identical preferences, whose skills differ across households and over time. Let $\Theta = [\theta, \theta]$ be the set of ex-ante types and the distribution is stable over time by applying the law of large number. At the beginning of period $t$, an agent $i$ privately knows his history $\theta^t_i = (\theta^t_{i0}, \ldots, \theta^t_{it}) \in \Theta^t$ of current and past skill vectors but not his future skill vectors which are governed by Markov process. At each period $t$, the probability measure of type $\theta^t$ is denoted by $\mu(\theta^t) \geq 0$, with $\int_{\theta^t \in \Theta^t} d\mu(\theta^t) = 1$. This implies that the individual’s choices in period $t$ can only be a function of his history. Moreover, the skill processes are independent across households and there is no aggregate uncertainty in the environment, as argued in Aruoba and Chugh (2010), this is without loss of generality since we always gain the conclusions in the deterministic steady states.

As in the typical search model, each period is divided into two subperiods, say day and night. A frictionless centralized market (CM) opens during the day, and a decentralized market (DM) opens at night. During the CM, households rent their previously accumulated capital and supply labor in the competitive market, and they also choose their consumption level in the goods market. Meanwhile, they also adjust their holdings of money, capital and

\footnote{The heterogeneity in productivity may have the same effect, in some sense, as the preference shocks in the model, as in Waller (2008) and Wen (2010).}
one-period nominal government bond.\(^2\)

When entering the DM, each household receives an idiosyncratic shock that governs his trading status, which is independent with households’ skills. A given household is a buyer in the DM with fixed probability \(\sigma\), a seller with fixed probability \(\sigma\), neither a buyer nor a seller with probability \(1 - 2\sigma\) which means that with probability with \(1 - 2\sigma\), a household will not trade at all in the DM, here \(\sigma \in [0, 1/2]\).\(^3\) As there is no credit or record keeping, households interact with anonymous bilateral matching. The seller produces goods for the buyer by using his own labor and capital, and receives money as a payoff from the buyer, while the terms of trade is determined through bargaining just like the standard search models.\(^4\)

In what follows, I provide the information about production, household behavior and government in more details, and define the monetary equilibrium.

### 2.1 Production

Both in the CM and DM, the inputs of production are capital and effective labor. A household with type \(\theta_i\) produces effective labor \(h_i\) according to the function \(h_i = \theta_i l_i\), where \(l_i\) is the household’s labor input. Effective labor \(h_i\) is observable, but actual labor \(l_i\) and skill \(\theta_i\) are not.

In the CM, as in standard growth theory, there is a general good that can be used for consumption or investment, produced by the constant-returns technology \(Y_t = F(K_t, H_t)\), here, the function \(F(\cdot)\) is strictly increasing and strictly concave, \(K_t = \int k(\theta^t)d\mu(\theta^t)\) denotes the aggregate capital, \(H_t = \int h(\theta^t)d\mu(\theta^t)\) denotes aggregate effective labor. In competitive markets, firms hire labor and capital from household, and profit maximization implies the real wage \(w_t = F_H(K_t, H_t)\) and real rental rate \(r_t = F_K(K_t, H_t)\).

\(^2\)Different from Svensson (1985), the asset market opens first before individuals face the cash-in-advance constraint, which is consistent with Lucas and Stockey (1983).

\(^3\)This structure is similar to Trejos and Wright (1995).

\(^4\)They also could trade by directed search and price posting, such as Mortensen and Wright (2002), other than random matching and bargaining, but it will increase the complexity of the model considerably with the heterogeneous agents. If not having additional constraints about production function, the agents with highest ability may monopoly the DM, which will violate the original intention of considering the behaviors in DM.
In the DM, although firms do not operate, the sellers’ own effective labor $e$ and capital $k$, carried from the CM in this period, can be used to produce with the technology $q = f(e, k)$, which implies that the cost of production is $e = c(q, k)$ and disutility is $c(q, k)/\theta$. Since the production function $f(.)$ is strictly increasing and strictly concave, it is easy to show that $c_q, c_{qq}, c_{kk} > 0, c_k, c_{qk} < 0$.

2.2 Government

As in Aruba and Chugh (2010), government consumption takes place in the CM while the expenditure could be financed by taxes, money creation and debt issuance. As for the form of taxation, the model is based on the dynamic Mirrlees literature that differ from those of Ramsey’s. At period $t$, the taxation is a nonlinear function of agent’s effective labor supply and capital stock in that period, which could be denoted as $T(h_t, k_t)$. The budget constraint for government in period $t$ is

$$M_t + B_t + P_t \int T(h_t, k_t)d\mu(\theta^t) = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}$$

The government also face the standard no-Ponzi constraint, which is

$$\sum_{t=0}^{\infty} \left[ \frac{P_t}{P_0} \prod_{s=0}^{t-1} \frac{1}{R_{s-1}} \left( G_t - \frac{M_t - M_{t-1}}{P_t} - \int T(h_t, k_t)d\mu(\theta^t) \right) \right] \leq 0$$

2.3 Households

At period $t$, CM opens first and I consider the household’s problem in CM and DM, respectively.

2.3.1 Household’s Problem in CM

In the beginning of period $t$, a household enters the CM with money holdings $m_{t-1}$, nominal government bond $b_{t-1}$ and capital $k_t$, and knows his own skill type at time $t$ denoted by $\theta_t$, which means that the history of his skill $\theta^t = (\theta_0, ..., \theta_t)$. Instantaneous utility for everyone

\footnote{Since the agents face the skill shock over time and there is no record keeping, which is similar to the basic environment in Albanesi and Sleet (2006), the taxation just depends on agents’ behaviors in each period, other than Kocherlakota (2005) which depends on the whole history of agents’ labor supplies.}
in the period $t$ CM is $U(x_t) - l_t$, where $x_t$ is consumption level and $l_t$ is labor. Assume that $U$ is twice continuously differentiable with $U' > 0, U'(0) = \infty, U'' < 0$ and there exist $x^* \in (0, \infty)$ such that $U'(x^*) = 1$ with $U(x^*) > x^*$. Let $W(\theta^t; m_{t-1}, b_{t-1}, k_t)$ be the value function for an agent with type $\theta^t$ at the beginning of CM, and $V(\theta^t; m_t, b_t, k_{t+1})$ be the value function in the DM. Agent discount between the DM and CM at rate $\beta \in (0, 1)$ but not between the CM and DM. Then, the household’s problem in CM is

$$W(\theta^t; m_{t-1}, b_{t-1}, k_t) = \max_{x_t, l_t, m_t, b_t, k_{t+1}} U(x_t) - l_t + V(\theta^t; m_t, b_t, k_{t+1})$$

subject to

$$P_t x_t + P_t (k_{t+1} - (1 - \delta)k_t) + m_t + b_t = P_t w_t h_t + m_{t-1} + P_t r_t k_t + R_{t-1} b_{t-1} - P_t T(h_t, k_t)$$

Here, $P_t$ denotes the nominal price of the consumption goods in the CM, $\delta$ is the discount rate of capital, $r_t$ is the capital’s real rent rate, $R_{t-1}$ is the gross nominal return on the one-period government bond purchased in the previous period. While the effective labor $h_t = \theta^t l_t$, replace it in the above equation and we could find out the first-order conditions of this problem are

$$U'(x_t) = \frac{1}{\theta^t_t(w_t - T_h(h_t, k_t))}$$

(1)

$$W_m(\theta^t; m_{t-1}, b_{t-1}, k_t) = V_m(\theta^t; m_t, b_t, k_{t+1}) = \frac{1}{P_t \theta^t (w_t - T_h(h_t, k_t))}$$

(2)

$$W_b(\theta^t; m_{t-1}, b_{t-1}, k_t) = R_{t-1} V_b(\theta^t; m_t, b_t, k_{t+1}) = \frac{R_{t-1}^t}{P_t \theta^t (w_t - T_h(h_t, k_t))}$$

(3)

$$W_k(\theta^t; m_{t-1}, b_{t-1}, k_t) = (1 + r_t - \delta - T_k(h_t, k_t)) V_k(\theta^t; m_t, b_t, k_{t+1}) = \frac{1 + r_t - \delta - T_k(h_t, k_t)}{\theta^t_t(w_t - T_h(h_t, k_t))}$$

(4)

With the quasi-linear utility, we could derive many results analytically, while with general preferences, the model requires numerical methods, just like Chiu and Molico (2010). Rocheteau, Shell and Wright (2008) show how to get the same simplification with general preferences.
Define \( \lambda_{\theta_t} = \frac{1}{T_t \theta_t (w_t - T^t_h h_t, k_t)} \) as the agents with skill \( \theta_t \)' marginal value of entering period \( t \) with one extra unit of money. From equation (1), we know that the consumption good demand \( x_t \) in the CM is an increasing function of agent’s skill \( \theta_t \). The first-order conditions (2)-(4) tell us that the value functions \( W(\cdot) \) and \( V(\cdot) \) are linear, and a given household’s marginal utility of wealth is independent of its trading status in the previous DM, but depends on the skill type. Different from Lagos and Wright (2005), the holdings of money, bond and capital are not identical any more.

2.3.2 Household’s Problem in DM

After the CM closes, agents draw the shocks determining whether they are buyer or seller with the same probability \( \sigma \). When the DM opens, households know their types and could match bilaterally. While the capital cannot be used for DM payment, as Williamson and Wright (2010) state, the reason is that capital is fixed in place physically and consumers have to travel without their capital to producers’ locations to trade.\(^7\) Therefore, producers could use their capital as an input in the DM but consumers could not use their holding of capital as payment. The household’s problem in DM is

\[
V(\theta^t; m_t, b_t, k_{t+1}) = \sigma[u(q^b_t) + \beta E_t(W(\theta^{t+1}; m_t - d^b_t, b_t, k_{t+1}))] \\
+ \sigma[-c(q^s_t, k_{t+1})/\theta_t + \beta E_t(W(\theta^{t+1}; m_t + d^s_t, b_t, k_{t+1}))] \\
+(1 - 2\sigma)\beta E_t(W(\theta^{t+1}; m_t, b_t, k_{t+1}))
\]  

(5)

Here, \( E_t(\cdot) = E(\cdot|\theta^t) \) means the conditional expectation based on the information until period \( t \), \((q^b_t, d^b_t)\) and \((q^s_t, d^s_t)\) denote the quantity of goods and dollars exchanged when the agent is buyer and seller respectively. The first line of equation (5) represents the expected payoff if the household is a buyer, the second line describes the expected payoff if the household is a seller, and the last line represents the expected payoff if the household does not participate in the DM.

\(^7\)By introducing the financial intermediation in the economy, we could consider the capital and credit as the payment method just as Williamson (2009) and Woodford (2010).
Since the value function \( W(\cdot) \) is linearity, we could rewrite the equation (5) as follow

\[
V(\theta^t; m_t, b_t, k_{t+1}) = \sigma[u(q_t^b) - c(q_t^s, k_{t+1})/\theta_t - \beta E_t(\lambda_{\theta_t+1}(d_t^b - d_t^s))] + \beta E_t(W(\theta^{t+1}; m_t, b_t, k_{t+1}))
\]

(6)

Therefore, all we have to do is to characterize the solution to the household’s problem is to compute the partial derivatives of \( V(\cdot) \), which means that we need to find out how \( q_t, d_t \) is determined under the pricing schemes.

2.4 Bargaining

Under the search-theoretical framework, the DM terms of trade are most commonly determined by bargaining. Let \( \phi \in [0, 1] \) denote the bargaining power of the buyer, while \( \phi = 1 \) means the buyer makes a take-it-or-leave-it offer to the seller, and \( \phi = 0 \) means the power reverses. Suppose individual \( i \) with skill type \( \theta^t_i \) is a buyer in DM, who will meet the seller with skill type \( \theta^t_j \). The generalized Nash bargaining problem is

\[
\max_{q_{it}, d_{it}} [u(q_{it}) + \beta E_t(W(\theta^{t+1}_i; m_{it} - d_{it}, b_{it}, k_{i,t+1}) - \beta E_t(W(\theta^{t+1}_j; m_{jt} + d_{jt}, b_{jt}, k_{j,t+1}))^\phi
\]

\[
*[-c(q_{it}, k_{j,t+1})/\theta_{jt} + \beta E_t(W(\theta^{t+1}_j; m_{jt} + d_{jt}, b_{jt}, k_{j,t+1}) - \beta E_t(W(\theta^{t+1}_j; m_{jt}, b_{jt}, k_{j,t+1}))^{1-\phi}
\]

subject to

\[
d_{it} \leq m_{it}
\]

(7)

Here, \( d_{it} \) denotes the amount of cash that the buyer \( i \) gives to the seller in exchange of consumption goods \( q_{it} \). The constraint (8) is the feasibility condition for the buyer, which is just like the cash-in-advance constraint in Stokey and Lucas (1983, 1987). A question naturally arises: Whether the constraint (8) always binds as in the representative-agent models? As Walsh (2003) states, even though in the next period, the CM opens first, the constraint may rarely bind because the household can almost fully self-insure itself against random liquidity-demand shocks by working harder and accumulating more cash in hand in the CM, especially the households with high skill type in period \( t \). On the other hand, as households ability is uncertainty in the period \( t + 1 \), the household would like to reduce the consumption in DM and take the money to the next period to smooth the future consumption.
Using the linearity of $W(.)$, rewrite the bargaining problem as

$$\max_{q_{it}, d_{it}} [u(q_{it}) - \beta E_t(\lambda_{\theta_{i,t+1}}d_{it})]^{\phi}[-c(q_{it},k_{j,t+1})/\theta_{jt} + \beta E_t(\lambda_{\theta_{j,t+1}}d_{it})]^{1-\phi}$$

subject to (8). Let $\chi_{it}$ denotes the Lagrange multiplier of the constraint. The first-order conditions are

$$\phi u'(q_{it})[-c(q_{it},k_{j,t+1})/\theta_{jt} + \beta E_t(\lambda_{\theta_{j,t+1}}d_{it})] - (1 - \phi) \frac{c_q(q_{it},k_{j,t+1})}{\theta_{jt}} [u(q_{it}) - \beta E_t(\lambda_{\theta_{i,t+1}}d_{it})] = 0$$

(9)

$$\phi \beta E_t\lambda_{\theta_{i,t+1}}[-c(q_{it},k_{j,t+1})/\theta_{jt} + \beta E_t(\lambda_{\theta_{j,t+1}}d_{it})] - (1 - \phi) \beta E_t\lambda_{\theta_{j,t+1}} [u(q_{it}) - \beta E_t(\lambda_{\theta_{i,t+1}}d_{it})] = -\chi_{it} [u(q_{it}) - \beta E_t(\lambda_{\theta_{i,t+1}}d_{it})]^{1-\phi}[-c(q_{it},k_{j,t+1})/\theta_{jt} + \beta E_t(\lambda_{\theta_{j,t+1}}d_{it})]^{\phi}$$

(10)

$$\chi_{it}(m_{it} - d_{it}) = 0, \chi_{it} \geq 0$$

(11)

From the equation (9), the relation between $d_{it}$ and $q_{it}$ is

$$d_{it} = \frac{\phi u'(q_{it}) c(q_{it},k_{j,t+1}) + (1 - \phi) c_q(q_{it},k_{j,t+1})u(q_{it})}{\phi \theta_{jt} u'(q_{it}) \beta E_t\lambda_{\theta_{j,t+1}} + (1 - \phi) c_q(q_{it},k_{j,t+1})\beta E_t\lambda_{\theta_{i,t+1}}}$$

(12)

Define the left hand side of (12)=$z(\theta_{it}, \theta_{jt}, q_{it}, k_{j,t+1})$, and here, $z_q > 0$ and $z_k < 0$. Different from the previous work with identical individuals, the bargaining game not only depends on buyer and seller’s asset holding, but also related with their skill types. By using the Jensen’s Inequality, the amount $d_{it}$ is lower than the amount under the situation without uncertainty. As the uncertainty in the future, the buyer would like to provide less money to exchange the goods in the DM.

Let’s go back to the previous households’ problem in the DM, and find out the first-order conditions as follows

$$V_m(\theta_t; m_t, b_t, k_{t+1}) = \sigma u_q \frac{\partial q_t^h}{\partial m_t} - \beta E_t(\lambda_{\theta_{i+1}}) \frac{\partial q_t^h}{\partial m_t} + \beta E_t(\lambda_{\theta_{t+1}})$$

(13)

$$V_k(\theta^t; m_t, b_t, k_{t+1}) = \sigma \left[ \frac{c_k}{\theta_t} z_k - \frac{c_k}{\theta_t} + \beta E_t(\lambda_{\theta_{t+1}}) z_k + \beta E_t(W_k(\theta^{t+1}; m_t, b_t, k_{t+1})) \right]$$

(14)
Define \(\gamma(\theta_t, q_t, k_{t+1}) = \frac{c_q}{\theta_t} z_k - \frac{c_k}{q_t} + \beta E_t(\lambda_{t+1}) z_k\), we have intertemporal optimal condition:

\[
U'(x_t) = \beta E_t[U'(x_{t+1})(1 + F(K_{t+1}, H_{t+1})) - \delta - T_k(k_{t+1}, h_{t+1})] + \sigma\gamma(\theta_t, q_t, k_{t+1})
\]

(15)

Except for the last term on the right hand side, (15) is a standard intertemporal Euler equation for capital investment. Since physical capital is used not only in the CM but also in the DM with probability \(\sigma\), optimal investment decisions take this into account. The additional term \(\sigma\gamma(.)\) captures the return to capital in the DM, which reflects the fact that, all else equal, the cost of producing a given quantity of DM output is lower if the seller has accumulated more capital.

### 2.5 Monetary Equilibrium

**Definition 1** Given policy processes \(\{T(.), R_t\}\), the government spending process \(\{G_t\}\), and the initial conditions \(\{M_0, B_0, K_0\}\), an monetary equilibrium is a collection of \(\{x_t, q_t, k_t, l_t, P_t, m_t, M_t, b_t, B_t, r_t, w_t\}\) such that

i. Households optimize \(\{x_t, l_t, k_t, m_t, q_t\}\) to maximize their utility, subject to the budget constraint, taking the price \(\{P_t, r_t, w_t\}\) and the policy processes as given;

ii. Government budget constraints hold every period;

iii. Market clear:

\[
\int m_t d\mu(\theta_t) = M_t; \quad \int b_t d\mu(\theta_t) = B_t; \\
\int k_t d\mu(\theta_t) = K_t; \quad \int h_t d\mu(\theta_t) = H_t
\]

iv. Resource constraint:

\[
\int x_t d\mu(\theta_t) + G_t + K_{t+1} = F(K_t, H_t) + (1 - \delta)K_t.
\]

### 3 Optimal Policy

Different from the traditional public finance approach to macroeconomic policy since Ramsey (1927), the households are heterogeneous and have the private information about their skills,
and I use the Mirrlees approach to formulate the problem of a benevolent planner that chooses allocations subject to their decentralization as a monetary equilibrium (Golosov et al. (2003, 2006), Costa and Werning (2008)).

Since the individual’s skill history $\theta_t$ is unobservable, the objective of the government act as the social planner is to find the optimal incentive-insurance trade-off $^8$, which means the allocation must respect incentive-compatibility conditions. A reporting strategy $\xi$ is a mapping from $\Theta^t$ into $\Theta^t$. Let

$$\tilde{W}(\xi; x, h, q, e) = E_t \left( \sum_{i=0}^{\infty} \beta^i \{ U(x_{t+i}(\xi)) - \frac{h_{t+i}(\xi)}{\theta_{t+i}} + \sigma(u(q_{t+i}(\xi)) - \frac{e_{t+i}(\xi)}{\theta_{t+i}}) \} \right)$$

which is the payoff from reporting strategy $\xi$ for agent with skill type $\theta^t$.

**Definition 2** An allocation $(x, h, K, q, e)$ is incentive-compatible if

$$\tilde{W}(\xi^*; x, h, q, e) \geq \tilde{W}(\xi; x, h, q, e)$$

for any $\xi \in \Theta^t$, while $\xi^*(\theta^t) = \theta^t$ for all $\theta^t$ is the truth-telling strategy.

Besides the incentive-compatible constraint, the series of allocations $(x, h, K, q, e)$ also need to face the resource constraints which are the feasible conditions.

**Definition 3** Define an allocation $(x, h, K, q, e)$ to be feasible if

$$\int x_t d\mu(\theta^t) + G_t + K_{t+1} \leq F(K_t, \int h_t d\mu(\theta^t)) + (1 - \delta)K_t$$

$$\int q_t d\mu(\theta^t) \leq f(\int e_t d\mu(\theta^t), K_{t+1})$$

for all $t$

I restrict attention to direct mechanisms. By the revelation principle, the households’ consumption and output depend only on his own announcements. This is without loss of

$^8$The government have the full commitment power such that we could abandon the issues brought by the time inconsistent.
generality because there is a continuum of agents with independent shock processes. The
government or social planner’s problem $SP(K_t)$ is

$$TV(K_t) = \max_{x,h,K,q,e} \sum_{i=0}^{\infty} \beta^i \int U(x_{t+i}) - \frac{h_{t+i}}{\theta_{t+i}} + \sigma(u(q_{t+i}) - \frac{e_{t+i}}{\theta_{t+i}}) d\mu(\theta^t)$$

subject to two feasible conditions (17) (18) and incentive-compatible condition (16).

In the economy, the planner maximizes the social welfare by given the initially $K^*_t$ units
of capital. Different from Acemoglu et al.(2010), the ex ante objective weights agents are the
same as the distribution of the skill levels.

**Proposition 1** Suppose that any $(x^*, h^*, K^*, q^*, e^*)$ that solves the social planner’s problem
$SP(\cdot)$, then the function $TV(\cdot)$ is strictly increasing, which means $TV(K_t) < TV(K^*_t)$ for
all $K_t < K^*_t$.

*Proof. In Appendix.*

The sketch of the proof is as follows. If the planner has not used up all the given capital level.
Then, the planner could add $\epsilon/U'(x^*_t(\theta^t_t))$ for all $\theta^t_t$, which guarantees that all households’
utility level increase by $\epsilon$. Since all types’ utility are going up with the same value, the change
will not affect incentive-compatibility. Meanwhile, $\epsilon$ is small enough to ensure the feasible
condition will not be violated. Then, we could have the conclusion that the aggregate value
function is strictly increasing.

**Proposition 2** There exists \( \{x^*, h^*, K^*, q^*, e^*\} \) solve the social planner’s problem, which sat-

fies,

$$1 - \delta + \frac{E_t(K^*_{t+1}, \int h^*_{t+1} d\mu(\theta^t)) + \sigma f_K(\int e^*_{t} d\mu(\theta^t), K^*_{t+1})}{U'(x^*_t)} = E_t(\sigma u'(q^*_t) + \beta U'(x^*_t))$$

*Proof. In Appendix*

Here is the basic idea of the proof. I define a new consumption allocation $x'$ to be the same
$x^*$ except that for the agents with skill history $\bar{\theta}^t$ such that

$$x'_t(\bar{\theta}^t) = x^*_t(\bar{\theta}^t) - \epsilon/U'(x^*_t(\bar{\theta}^t))$$

(20)
The change is designed to reduce the instantaneous utility in period $t$ by $\epsilon$ while increasing the utility in the DM and the next period with the same amount. By making the change for all households, the incentive-compatible constraint still holds, which means the truth-telling strategy is still the dominant strategy for all skill types. If save the extra consumption in period $t$, the social planner could have extra resources. By applying the monotonicity of the aggregate value function, we could have the above proposition.

Actually, in Golosov et al. (2006), the above intertemporal optimal condition is a typical Inverse Euler Equation. However, if there is no uncertainty in the economy, we could rewrite the condition as

$$U'(x_t) = \sigma u'(q_t) f_K(t) + \beta U'(x_{t+1})(1 + F_K(t + 1) - \delta)$$

Which is just the Euler Equation.

**Proposition 3** The optimal nominal interest rate is greater than 1, which means the Friedman rule is not optimal. If the probability of being seller is small, the optimal taxation on capital income is positive.

Note: Use the Jensen’s Inequality, then compare the result with the intertemporal optimal conditions, such as equation (15), we could get the results.

4 Conclusion

This paper studies the optimal fiscal and monetary policy with heterogeneous agents in an environment where explicit frictions give rise to valued money, supplying the microfoundation of holding money and making money essential in the sense that it expands the set of feasible trades. In the spirit of Mirrlees’s private information framework, I find that the Friedman...
rule is no longer optimal when combined with nonlinear taxation of income. In addition, the capital income taxation is not zero.

Woodford (2010) points out the financial intermediation play a important role in the economy, especially during the financial crisis. Since in the model, money is an exchange instrument. If we consider other payment methods, such as credit and collateral in the economy, we need to add the financial intermediation, which will help us explain the the phenomenons in the asset market during the financial crisis. I will leave this as the further job.

Appendix

Proof of Proposition 1

Suppose there exists some \( K_t < K_t^* \) such that \( TV(K_t) = TV(K_t^*) \).

Let \((x^*, h^*, K^*, q^*, e^*)\) be the solution of social planner’s problem \( SP(K_t) \), which means that it satisfies the feasible conditions and the incentive-compatible condition. Since \( K_t < K_t^* \) and \( TV(K_t) = TV(K_t^*) \), \((x^*, h^*, K^*, q^*, e^*)\) also satisfies the constraints of social planner’s problem \( SP(K_t^*) \), then it is also the solution of \( SP(K_t^*) \).

While \((x^*, h^*, K^*, q^*, e^*)\) satisfies the feasible conditions, then for all \( \theta^t \in \Theta^t \), we have

\[
\int x_t^* d\mu(\theta^t) + G_t^* + K_{t+1}^* \leq F(K_t, \int h_t^* d\mu(\theta^t)) + (1 - \delta)K_t < F(K_t^*, \int h_t^* d\mu(\theta^t)) + (1 - \delta)K_t^*
\]

For any \( \epsilon > 0 \), let \( x_t' = x_t^* + \frac{\epsilon}{U(x_t^*)} \), we have

\[
\int x_t' d\mu(\theta^t) + G_t + K_{t+1}^* < F(K_t^*, \int h_t^* d\mu(\theta^t)) + (1 - \delta)K_t^*
\]

which means that the feasible condition will always hold as long as \( \epsilon \) is small enough. Therefore, \((x', h^*, K^*, q^*, e^*)\) satisfies the feasible conditions of \( SP(K_t^*) \).

Meanwhile, for all \( \theta^t \in \Theta^t \), we have

\[
U(x_t) - \frac{h_t^*}{\theta_t^*} + \sigma(u(q_t^* - c_t^* \theta_t^* = U(x_t^*) + \epsilon - \frac{h_t^*}{\theta_t^*} + \sigma(u(q_t^* - c_t^* \theta_t^*)) > U(x_t^*) - \frac{h_t^*}{\theta_t^*} + \sigma(u(q_t^* - c_t^* \theta_t^*))
\]

and

\[
\tilde{W}(\xi^*(\theta^t); x', h^*, q^*, e^*) > \tilde{W}(\xi^*(\theta^t); x^*, h^*, q^*, e^*)
\]
While \((x^*, h^*, K^*, q^*, e^*)\) satisfies the incentive-compatible condition of \(SP(K_t^*)\), then 
\[
\tilde{W}(\xi^*(\theta^t); x^*, h^*, q^*, e^*) \geq \tilde{W}(\xi; x^*, h^*, q^*, e^*)
\]
for any \(\xi \in \Theta^t\), hence, \((\hat{x}, h^*, K^*, q^*, e^*)\) also satisfies the incentive-compatible condition of \(SP(K_t^*)\).

Thus, \((x^*, h^*, K^*, q^*, e^*)\) could not be the solution of social planner’s problem \(SP(K_t^*)\), which means that the assumption could not be true. So, \(TV(K_t) < TV(K_t^*)\) for all \(K_t < K_t^*\)
References


