Credit Scoring and Competitive Pricing of Default Risk[1]

Satyajit Chatterjee
Federal Reserve Bank of Philadelphia

Dean Corbae
University of Texas at Austin

José-Víctor Ríos-Rull
University of Pennsylvania and CAERP

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Abstract

When people cannot commit to pay back their loans and there is limited information concerning their willingness to do so how do lending institutions draw inferences about the likelihood of default? How does this inference impact the ability of people to consumption smooth? We study an environment populated with two types of people who differ with respect to their rates of time preference. Lenders cannot directly observe a person's type but make probabilistic assessments of it based on the person's financial history. The probability assigned to a person being of a given type is a person's *type score*. In a recursive competitive equilibrium, a person's actions in the asset market (how much to borrow or lend and whether to default on a previous loan), along with the person's beginning-of-period type score and beginning-of-period assets, determine the person's updated type score. The terms of credit depend only on the person's updated type score, the amount of credit requested and the risk-free rate. If in equilibrium one type of person is always more likely to default than the other, the type score can be interpreted as a *credit score*. In this case the model delivers an integrated theory of terms of credit and credit scoring that seems broadly consistent with the data.

1 Introduction

This paper is an attempt to further our understanding of how consumption smoothing - via saving and borrowing - works when households cannot commit to payback their loans and financial intermediaries who lend to these households cannot commit to long-term contracts.

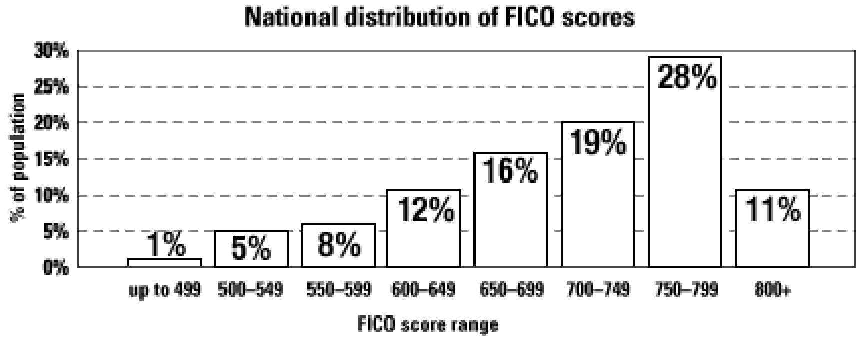
The institutional arrangement of the U.S. unsecured consumer credit market is consistent with this two-sided lack of commitment. On the borrowing side, glossing over some details, U.S. law (more precisely, the U.S. Constitution) gives people the right to declare bankruptcy and walk away from their debt. Since this right is inalienable (cannot be sold by a person in exchange for credit, say), the household cannot commit to not use the option to file for bankruptcy. On the lending side, the standard credit card contract gives the card issuer the right to the change the terms of the credit contract at any time.

The inability of market participants to commit does not prevent people from borrowing. Currently, the level of unsecured consumer credit in the U.S. is between 10 to 15 percent of annual aggregate consumption *and* there is 1 bankruptcy filing per year for every 75 U.S. households. Evidently, the current market arrangement works. The questions are: *how* (exactly) does it work and could it work *better*?

Given the inability of people to commit, it's important for a lender to assess the probability that a borrower will fail to pay back – that is, assess the risk of default. In the U.S., lenders use *credit scores* as an index of the risk of default. The credit scores most commonly used are produced by a single company, the Fair Isaac and Company, and are known as FICO scores.[1] These scores range between 300 and 850, where a higher score signals a lower probability of default. The national distribution of FICO scores are given in Figure 1.

[1]Over 75% of mortgage lenders and 80% of the largest financial institutions use FICO scores in their evaluation and approvals process for credit applications.

Figure 1



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Source: http://www.myfico.com/myfico/Credit Central/ScoringWorks.asp

A FICO score takes into account a person's payment history (most particularly the presence of adverse public records such as bankruptcy and delinquency) and current amounts owed.[2] These scores appear to affect the extension of consumer credit in three primary ways.

1. Credit terms appear to improve with a person's credit score.
2. The presence of adverse public records lower the score substantially.
3. Taking on more debt (paying off debt) tends to lower (raise) credit scores.

Table 1

FICO Score	Auto Loan	Mortgage
720-850	4.94%	5.55%
700-719	5.67%	5.68%
675-699	7.91%	6.21%
620-674	10.84%	7.36%
560-619	15.14%	8.53%
500-559	18.60%	9.29%

Source: http://www.myfico.com/myfico/Credit Central/LoanRates.asp

[2]The score also takes into account the length of a person's credit history, the kinds of credit accounts (retail credit, installment credit etc.) and the borrowing capacity (or line of credit) on each account. It's also worth noting the kinds of information that are not used in credit scores. By law, credit scores cannot use information on race, color, national origin, sex, and marital status. Further, FICO scores do not use age, assets, salary, occupation, and employment history.

Table 1 provides information on the relationship between FICO scores and the average interest rate on a new 60-month auto loan or a new 30-year fixed mortgage consistent with item 1. Item 2 is consistent with evidence provided in Musto [6]. Musto studied the impact of striking an individual's bankruptcy record from his or her credit history after 10 years (as required by the Fair Credit and Reporting Act). He found "[a] bankruptcy filing significantly constrains unsecured financing for the entire period when potential creditors can observe it on credit files. Among filers, the better credits acquire substantial new access to unsecured credit when the bankruptcy finally leaves their files, and the concurrent dynamics of credit scores indicate that this reflects a big upward boost in estimated creditworthiness". In conjunction with Table 1, item 2 suggests that an individual who fails to pay back an unsecured loan will experience an adverse change in the terms of (unsecured) credit. Thus, a failure to pay back a loan adversely impacts the terms of credit and may result in outright denial of credit. Item 3 is consistent with the advice given by FICO for improving one's credit score.[3] Additionally, item 3 in conjunction with Table 1 indicates that even absent default, the terms of credit on unsecured credit worsen as an individual gets further into debt – people face a rising marginal cost of funds.

These facts suggest the following characterization of the workings of the unsecured consumer credit market. Given the inability of borrowers to commit to pay back, lenders condition the terms of credit (including whether they lend at all) on an individual's credit history. This history is somehow encapsulated by a credit score. Individuals with higher scores are viewed by lenders as less likely to default and receive credit on more attractive terms. The failure to pay back a loan (default) leads to a drop in the individual's credit score. Consequently, post-default access to credit is available on worse terms and may not be available at all. Even absent default, greater indebtedness leads to a lower credit score and worse terms of credit. Finally, there is considerable amount of unsecured credit extended under these circumstances and a non-trivial fraction of borrowers default.

There is now a fairly substantial literature on how (and to what extent) borrowing can

[3]To improve a score, FICO advises to "Keep balances low on credit card and 'other revolving credit"' and "[p]ay off debt rather than moving it around". Source:www.myfico.com/CreditEducation/ImproveYour Score

occur when agents cannot commit to pay back (the key papers will be noted below). The challenge, as we see it, is to use the insights of this literature to specify a structure that can make quantitative sense of the characterization of the unsecured consumer credit market offered in the previous paragraph.

This paper takes some tentative steps toward meeting this challenge. We consider an environment with a continuum of infinitely-lived people who may be one of two types. The two types could differ in terms of their risk aversion (equivalently, the reciprocal of their inter-temporal elasticity of substitution) and their discount factors. Importantly, a person's type is private information.

These people interact with a competitive financial intermediation sector that can borrow in the international credit market at some fixed risk-free rate and make one-period loans to individuals at an interest rate that reflects that person's risk of default. Because differences in preference bear on the willingness of each type of person to pay back a loan, intermediaries must form some assessment of a person's type. We model this as a (recursive) Bayesian inference problem: intermediaries use a person's current actions in the credit market to update their prior probability of the person being of a given type and then charge an interest rate that is appropriate for the posterior probability.

We model the pricing of unsecured consumer loans in the same fashion as in our predecessor paper Chatterjee, *et.al.* [2]. As in that paper, all one-period loans are viewed as discount bonds and the price of these bonds depend on the size of the bond. This is necessary because the probability of default (for any type) will depend on the size of the bond (i.e., on the person's liability). In addition, and this is a feature that is new to this paper, the price of the bond also depends on the posterior probability of a person being of a given type, *conditional* on selling that particular sized bond. This is necessary because the two types will not have the same probability of default for any given sized bond and a person's asset choice is potentially informative about the person's type. With this asset market structure, competition implies that the expected rate of return on each type of bond is equal to the (exogenous) risk-free rate.

This is, possibly, the simplest environment one could imagine that could make sense of the observed connection between credit history and the terms of credit. Suppose it turns out that, in equilibrium, one type of person, say type a, always has a lower probability of default. Then, under competition, the price of a discount bond (of any size) could be expected to be positively related to the probability of a person being of type a. Further, default will lower the *posterior* probability of being of type a because type a people default less frequently. If we interpret a person's credit score as (some positive transform of) the probability of a person being of type a, we would explain Table 1 and item 1. We caution the reader, however, that although this sounds intuitive the statement that a person of type a is *always* less likely to default is a very strong restriction on equilibrium behavior and may require correspondingly strong assumptions on fundamentals.

There are two strands of existing literature to which our paper is closely related. One strand relates to the banking literature where Diamond's [4] well-known paper on acquisition of reputation in debt markets is a key reference.[4] Diamond considers a situation where there are two types of infinitely-lived risk-neutral entrepreneurs who interact with a competitive financial intermediation sector. Financial intermediaries make one-period loans without directly observing the entrepreneur's type. One type of entrepreneur always chooses the safe project but the other type chooses between a safe project and risky project and an entrepreneur defaults if the project fails. Since an entrepreneur's loss is bounded below, the second type has an incentive to choose the risky project. In this environment, an entrepreneur's payment history (did the project ever fail?) reveals something about an entrepreneur's type. Consequently, the terms of credit offered to an entrepreneur will depend on the entrepreneur's payment history. Diamond's set-up clearly has parallels to our own. The main difference is that for us the decision to default is the key decision (in Diamond this happens only when the project fails) and we don't permit any choice with regard to the riskiness of the income stream.

The second strand of literature to which our paper is related is the literature on sovereign debt. This literature shares with us the concern about the inability of the borrower to commit to pay back. The inability of the sovereign to commit stems from the fact that the sovereign

[4] Phelan [7] studies reputation aquisition by a government in a related framework.

does not (by definition) answer to a higher authority. In this strand, the paper that is most closely related to ours is Cole, Dow and English [3]. They focus on an interesting aspect of sovereign defaults, namely, that a sovereign who defaults is shut out of international credit market until such time as the sovereign makes a payment on the defaulted debt. In our case, the inability to commit stems from a right to bankruptcy granted to an individual by the legal system. Consequently, a bankruptcy results in a *discharge* of existing debt and individuals do not have the option of making payment on discharged debt in the future.[5]

2 Model Economy

We begin by describing the market arrangement in our model economy. This is followed by a recursive formulation of the individual's decision problem and a description of profit maximizing behavior of firms serving the unsecured credit industry.

2.1 Default Option and Market Arrangement

We model the default option to resemble, in procedure, a Chapter 7 bankruptcy filing. If an individual files for bankruptcy, the individual's beginning of period liabilities are set to zero (i.e., the individual's debt is discharged) and the individual is not permitted to save in the filing period.

There is a competitive credit industry that accepts deposits and makes loans to individuals. An individual can borrow at an interest rate that depends on the size of the loan and on the market's belief about the individual's type. We will assume that there are only two types of people denoted type a and b. As noted earlier, belief about an individual's type is important because an individual cannot commit to repay and the probability of repayment

[5] A bankruptcy filing that results in the discharge of debt is called a Chapter 7 filing. Individuals also have the option to file under a different chapter, Chapter 13, wherein their debt is not discharged and they agree to a repayment schedule that typically involves a partial default. Given the choice, individuals would chose a Chapter 13 filing only if they wished to keep assets they would lose under a Chapter 7 filing. Since borrowers in our model have negative net worth (there is only one asset), Chapter 7 is always the preferred Chapter to file for bankruptcy.

can vary across types. An individual can also save via deposits and all deposits fetch a constant risk-free rate.

Let $\ell \in L \subset R$ be an individual's asset holding, where $\ell < 0$ denotes debt and $\ell \geq 0$ denotes deposits. The set L is taken to be finite. Motivated by the three items mentioned in the introduction, we assume an asset market structure where the price of a loan of size ℓ made to an individual whose probability of being type a is σ is given by $q(\ell, \sigma) \geq 0$.[6] We will refer to σ, the market's assessment of an individual's probability of being of type a, as an individual's type score.

An individual's type score is affected by the individual's actions in the asset market. An individual with a loan of size ℓ and a current type score s will have a type score s' at the start of next period, where

$$s' = \Psi\left(\ell', d, \ell, s\right).$$

Here ℓ' is the individual's choice of asset holdings and d is an indicator variable that takes on the value of 1 if the individual defaults on an existing loan ℓ and zero otherwise (in the event of default ℓ' is constrained to be 0).[7]

The functions $\{q, \Psi\}$ are equilibrium objects.

2.2 People

Time is discrete and indexed by t. There is a unit measure of people. As already noted, people can be one of two types, indexed by $i \in \{a, b\}$.

Within a period, the timing of events is as follows. At the start of a period, each person learns his type and this type is drawn in an i.i.d. fashion from a Markov process. In particular, if an agent was of type i in the previous period, he will remain type i in the current period with probability $1 - \delta_i$ and changes type with probability δ_i. Next, each individual receives a random endowment of goods and this endowment is an i.i.d. draw with measure μ on a compact support $E \subset R_{++}$. After observing his type and endowment, an

[6]In principle, the price of a loan could depend on the entire financial history of an individual.

[7]Note that Ψ depends on the actual ℓ and not partitions of asset holdings. If this assumption is violated, then the entire credit history may be needed to determine s'.

individual chooses whether to default. Finally, the individual chooses his asset position and consumes. While an individual's type and endowment are private information, his default decisions and asset position are observable.

Given our asset market structure, it is natural to adopt a recursive formulation of an individual's decision problem with state variables given by (i, e, ℓ, s). The value function of an agent of type i, denoted by $v_i(e, \ell, s)$, solves the following functional equation:[8]

Case 1: When $\ell < 0$

$$v_i(e, \ell, s) = \max_{d \in \{0,1\}} v_i^d(e, \ell, s) \tag{1}$$

where the value function when the agent decides not to default ($d = 0$) is given by

$$\begin{aligned} v_i^0(e, \ell, s) = & \max_{(c,\ell') \in B(e,\ell;q,\Psi) \neq \varnothing} u_i(c) + \\ & \beta_i \int_E [(1 - \delta_i) v_i(e', \ell', \Psi(\ell', 0, \ell, s)) + \delta_i v_{-i}(e', \ell', \Psi(\ell', 0, \ell, s))] \; \mu(de') \end{aligned} \tag{2}$$

where

$$B(e, \ell; q, \Psi) = \{c \geq 0, \ell' \in L \mid c + q(\ell', \Psi(\ell', 0, \ell, s)) \cdot \ell' \leq e + \ell\}$$

and the value function when the agent chooses to default ($d = 1$) or $B(e, \ell; q, \Psi) = \varnothing$ (in which case default is the only option) is given by

$$v_i^1(e, \ell, s) = u_i(e) + \beta_i \int_E [(1 - \delta_i) v_i(e', 0, \Psi(0, 1, \ell, s)) + \delta_i v_{-i}(e', 0, \Psi(0, 1, \ell, s))] \; \mu(de').$$

Here, $u_i(c)$ is the utility that an individual of type i receives from consuming c units of the good and β_i is the discount factor of individual of type i. The value function under default, $v_i^1(e, \ell, s)$, assumes that default wipes out all debt and that a defaulting individual cannot accumulate any asset in the period of default.

Case 2: When $\ell \geq 0$,

$$v_i(e, \ell, s) = v_i^0(e, \ell, s). \tag{3}$$

In what follows we denote the set of earnings for which an individual of type i and score s defaults on a loan of size ℓ by $D_i(\ell, s; q, \Psi) = \{e \mid d_i(e, \ell, s) = 1\} \subseteq E$. We will also denote

[8] We omit the proof of existence and uniqueness of the value function. To apply the Contraction Mapping Theorem we need the following assumptions: (i) u is bounded and continuous; (ii) μ has no atoms; (iii) $q(\ell, \sigma)$ is continuous in σ for all ℓ; and (iv) $\Psi(\ell', d, \ell, s)$ is continuous in s for all admissible ℓ', d, ℓ.

by $E_i(\ell', \ell, s; q, \Psi) = \{e \mid \ell_i'(e, \ell, s) = \ell'\} \subseteq E$ as the set of earnings for which an individual of type i, score s and a loan or deposit of size ℓ chooses ℓ'.

3 The Credit Industry

Financial intermediaries have access to an international credit market where they can borrow or lend at the risk-free interest rate $r \geq 0$. The profit on a loan of of size $\ell < 0$ made to an individual with score σ is the present discounted value of inflows less the current value of outflows and the profit on deposit of size $\ell > 0$ made to an individual with score σ is the current value of the inflows less the present discounted value of outflows. Then the profit on single contract of type (ℓ, σ), denoted $\pi(\ell, \sigma; q, \Psi)$, is:

$$\pi(\ell, \sigma; q, \Psi) = \begin{cases} (1+r)^{-1}[1 - p(\ell, \sigma)](-\ell) - q(\ell, \sigma)(-\ell) & \text{if } \ell < 0 \\ q(\ell, \sigma) - (1+r)^{-1}\ell & \text{if } \ell \geq 0 \end{cases} \tag{4}$$

where $p(\ell, \sigma)$ is the fraction of individuals with score σ expected to default on a loan of size ℓ. If $a(\ell, \sigma)$ is the measure of type (ℓ, σ) contracts sold, the decision problem of an intermediary is to maximize $\sum_{\ell,\sigma} \pi(\ell, \sigma; q, \Psi) \cdot a(\ell, \sigma)$ subject to the constraint that $a(\ell, \sigma) \geq 0$.

4 Equilibrium

An equilibrium is (q^*, Ψ^*) which satisfies the following three sets of conditions. The first set are the optimization conditions of individuals. That is, given q and Ψ, $D_i(\ell, s; q, \Psi)$ and $E_i(\ell', \ell, s; q, \Psi)$ must be consistent with (1) and (3).

The second set are zero profit conditions for loans and deposits

$$q(\ell, \sigma) = \begin{cases} (1+r)^{-1} \left\{[1 - \mu(D_a(\ell, \sigma; q, \Psi))] \cdot \sigma + [1 - \mu(D_b(\ell, \sigma; q, \Psi))] \cdot (1 - \sigma)\right\} & \ell < 0 \\ (1+r)^{-1} & \ell \geq 0 \end{cases} \tag{5}$$

where we have substituted the objective default probabilities for $p(\ell, \sigma)$.

The third and most important set is the formula for updating an individual's type score. We require that the formula must be consistent with Bayes' rule whenever applicable.[9] Recall that according to Bayes' rule, the probability that event A is true given that event B is true is $P(A \cap B)/P(B) = P(A)P(B \mid A)/P(B) = P(A \mid B)$, provided $P(B) > 0$. In Bayesian terminology, $P(A)$ is the prior probability that A is true and $P(A \mid B)$ is the posterior probability that A is true given that B is observed.

Let event A be the event "an individual with score s is of type a" By the definition of s, $P(A) = s$. The event B can be one of two mutually exclusive events. In the first case, B is the event "an individual with score s defaults on loan ℓ." Then, $P(B \mid A) = [\mu(D_a(\ell, s; q, \Psi))]$, $P(B) = [\mu(D_a(\ell, s; q, \Psi))] \cdot s + [\mu(D_b(\ell, s; q, \Psi))] \cdot (1 - s)$, and the posterior probability that an individual with score s who defaults on a loan of size ℓ is of type a is given by:

$$\varphi(0, 1, \ell, s) = \frac{[\mu(D_a(\ell, s; q, \Psi))] \cdot s}{[\mu(D_a(\ell, s; q, \Psi))] \cdot s + [\mu(D_b(\ell, s; q, \Psi))] \cdot (1 - s)} \tag{6}$$

In the second case, B is the event "an individual with score s and loan or deposit ℓ chooses ℓ'." Then, $P(B \mid A) = [\mu(E_a(\ell', \ell, s; q, \Psi))]$, $P(B) = [\mu(E_a(\ell', \ell, s; q, \Psi))] \cdot s + [\mu(E_b(\ell', \ell, s; q, \Psi))] \cdot (1 - s)$, and the posterior probability that an individual with score s and loan or deposit of size ℓ is of type a is given by

$$\varphi(\ell', 0, \ell, s) = \frac{[\mu(E_a(\ell', \ell, s; q, \Psi))] \cdot s}{[\mu(E_a(\ell', \ell, s; q, \Psi))] \cdot s + [\mu(E_b(\ell', \ell, s; q, \Psi))] \cdot (1 - s)} \tag{7}$$

Then, given that type can change at the beginning of the next period,

$$\Psi'(\ell', d, \ell, s) = (1 - \delta_a)\varphi(\ell', d, \ell, s) + \delta_b \left[1 - \varphi(\ell', d, \ell, s)\right]. \tag{8}$$

Since Bayes' rule is applicable only if the conditioning event has positive probability, we may also need to assign values to $\Psi'(\ell', d, \ell, s)$ in some fashion when the conditioning set is empty.[10] Theory does not restrict the assignment but there may be existence and computa-

[9] This notion of assigning beliefs "whenever possible" as to individual types on the basis of Bayes Rule is similar to what is assumed as part of a definition of Perfect Bayesian Equilibrium (see Fudenberg and Tirole (2000), p. 331-333).

[10] One restriction on off-equilibrium beliefs is that Ψ' function be continuous in s in order to satisfy assumption (iv) in footnote 7.

tional issues involved in the choice. An equilibrium requires $\Psi'(\ell', d, \ell, s) = \Psi(\ell', d, \ell, s)$.

It is worth pointing out that the equilibrium conditions do not involve the *distribution* of people over the state variables (i, e, ℓ, s). This "independence" is the result of two assumptions. First, there is no cross subsidization across loans of different types. Therefore, the number of loans of each type is not relevant for determining the probability of repayment on any given loan. Second, the risk-free rate is exogenous and therefore there is no feedback from the distribution of people over (i, e, ℓ, s) to the rate of return on deposits. In a closed economy this feedback will be present and the distribution will matter.

5 When is a Type Score a Credit Score?

We will set aside for the moment the question of whether an equilibrium exists – we will provide several examples of equilibria below. Here, we are interested in knowing when a type score will have the three properties of a credit score noted in the introduction. For instance, under what conditions on $u_i(.)$ and β_i is $D_a(\ell, s; q, \Psi) \subseteq D_b(\ell, s; q, \Psi)$ for any s and ℓ? Such a ranking would give content to the statement that, from the perspective of lenders, type a is the good type and type b is the bad type and, therefore, give some basis for identifying the *type score* (the probability that a person is of type a) with a *credit score*. However, the potentially complex dependence of a person's decision rule on the q and Ψ functions makes it challenging to provide such a ranking – unless very strong assumptions are made on preferences and choice sets.

To make progress, we will specialize the model to a case that is simple enough so that with a combination of reasoning and numerical simulation we can develop some intuition on the basic economics of the situation. With this mind, we will make the following assumptions:

A1. $\beta_b = 0$ and $0 < \beta_a$.

A2. $\delta_i \in (0, 1)$ and $1 - \delta_a > \delta_b$.

A3. $L = \{-x, 0, x\}$.

A4. If $\mu(D_i(\ell, s; q, \Psi)) = 0$ for all i, $\Psi(0, 1, \ell, s) = (1-\delta_a)s+\delta_b(1-s)$ and if $\mu(E_i(\ell', \ell, s; q, \Psi)) = 0$ for all i, $\Psi(\ell', 0, \ell, s) = (1-\delta_a)s + \delta_b(1-s)$.

Under the strong assumption A1 about myopia of type b agents, we can characterize their decision rule independent of their type score s.

Proposition 1. (i) $E_b(x, \ell, s; q^*, \Psi^*) = \varnothing$, (ii) $D_a(-x, s; q, \Psi) \subseteq D_b(-x, s; q, \Psi) = E$, and (iii) for $\ell \in \{0, x\}$, $E_a(-x, \ell, s; q^*, \Psi^*) \subseteq E_b(-x, \ell, s; q^*, \Psi^*) \in \{\varnothing, E\}$.

These results follow because a type b person cares about an action *only* to the extent it affects current consumption – what any action might entail about the person's future type-score is not relevant because the person does not care about the future at all. This is true even though the type b agent may switch to being type a at the start of the next period simply because that switches happen in the *future* and a type b person does not give *any* weight to the future. Therefore, if choosing $\ell' = x$ is feasible it is strictly dominated by choosing $\ell' = 0$ (– the latter is a feasible choice if the former is feasible) and part (i) follows. To see part (ii), observe that paying the debt back and not borrowing (i.e., choosing $(d, \ell') = (0, 0)$) results in a reduction of current consumption and is strictly dominated by choosing $(d, \ell') = (1, 0)$. Paying the debt back and borrowing also results in a drop in current consumption since current consumption under this action is $-(1 - q^*(-x, \Psi^*(-x, 0, -x, s)) < 0$ by virtue of the fact that in equilibrium the $q^*(-x, \sigma) \leq 1/(1+r) < 1$ for any σ. Therefore, for a type b person with debt, the optimal decision is to default independent of his earnings. To see part (iii), consider the following two cases. First, if $q(-x, \Psi^*(-x, 0, \ell, s)) > 0$, the optimal decision for a type b agent is to borrow since this maximizes current period consumption and that's all the person cares about. Therefore, $E_b(-x, \ell, s; q^*, \Psi^*) = E$. Second, if $q(-x, \Psi^*(-x, 0, \ell, s)) = 0$, then agents are borrowing constrained and neither type can choose $\ell'_i = -x$. Hence $E_a(-x, \ell, s; q^*, \Psi^*) = E_b(-x, \ell, s; q^*, \Psi^*) = \varnothing$.

Given that type b people behave in this way, we can now partially characterize the equilibrium updating function Ψ^*. We have:

Proposition 2. If $\ell \in \{0, x\}$ and $\mu(E_a(x, \ell, s; q^*, \Psi^*)) > 0$, Bayesian updating implies

$\Psi^*(x, 0, \ell, s) = 1 - \delta_a$.

If the person saves, then by Proposition 1(i), he is not of type b. Provided in equilibrium there is some e for which type a agent with ℓ, s chooses to save (i.e. $\mu(E_a(x, \ell, s; q^*, \Psi^*)) > 0$, a requirement that is necessary to apply Bayes' formula), then by (7) $\varphi(x, 0, \ell, s) = 1$.[11] The Proposition follows from (8).

There is also a version of Proposition 2 which applies when a person chooses $\ell' = 0$. We know by Proposition 1(iii) that in equilibrium $E_b(-x, \ell, s; q^*, \Psi^*) \in \{\varnothing, E\}$. If all type b borrow, then lenders can correctly infer, provided $\mu(E_a(0, \ell, s; q^*, \Psi^*)) > 0$, that an agent who chooses $\ell' = 0$ is of type a. So, we have:

Proposition 3. If $\ell \in \{0, x\}$, $\mu(E_a(0, \ell, s; q^*, \Psi^*)) > 0$, and $E_b(-x, \ell, s; q^*, \Psi^*) = E$, Bayesian updating implies $\Psi^*(0, 0, \ell, s) = 1 - \delta_a$.

The next two propositions address issues that are at the heart of this project. These propositions establish that in equilibrium the type score s has properties that resemble the properties of credit scores, namely, that credit scores decline with default (consistent with item 2 in the introduction) and decline (improve) with increasing (decreasing) indebtedness (consistent with item 3).

Proposition 4. Bayesian updating implies $\varphi(0, 1, -x, s) \leq s$.

To see why the proposition is true, observe first that given the assumptions on δ_i and the definition in (8), it follows that $\Psi^*(\ell', d, \ell, s) \in [\delta_b, 1 - \delta_a]$ so we need only consider $s \in (0, 1)$. Next, by Proposition 1(ii), $\mu(D_b(-x, s; q^*, \Psi^*)) = 1$. Therefore, by (6) we have:

$$\varphi(0, 1, \ell, s) - s = \frac{(1-s)[\mu(D_a(\ell, s; q^*, \Psi^*)) \cdot s - s]}{[\mu(D_a(\ell, s; q^*, \Psi^*))] \cdot s + (1-s)}.$$

If $\mu(D_a(\ell, s; q^*, \Psi^*)) < 1$, then $\varphi(0, 1, \ell, s) - s < 0$; that is, if some type a persons do not default, default increases the probability that a person is of type b. If $\mu(D_a(\ell, s; q^*, \Psi^*)) = 1$, default does not provide any information about type and $\varphi(0, 1, \ell, s) = s$.

[11] If this requirement is not met, then $\varphi = \frac{0}{0}$.

Proposition 5. Suppose $E_b(-x, \ell, s; q^*, \Psi^*) = E$. (i) If $\ell \in \{0, x\}$, Bayesian updating implies $\varphi(-x, 0, \ell, s) \leq s$. (ii) If $\ell' \in \{0, x\}$ and $\mu(E_a(\ell', -x, s; q^*, \Psi^*)) > 0$, Bayesian updating implies $\varphi(\ell', 0, -x, s) = 1$.

Part (i) of the proof is analogous to that of Proposition 4. Intuitively, if type b people can borrow and some type a do not borrow then taking on debt strictly increases the likelihood that the person is of type b (otherwise the score does not change). Since there is only a single level of debt in this model, this property is the model analog of taking on debt in item 3. Again, since all type b are borrowing, part (ii) follows since paying down debt signals the agent is of type a.

It is worth pointing out that Proposition 5 does not hold for people who have debt and choose to continue to be in debt. The next result follows from Proposition 1(ii) and (7).

Proposition 6. If $\mu(E_a(-x, -x, s; q^*, \Psi^*)) > 0$, Bayesian updating implies $\varphi(-x, 0, -x, s) = 1$.

It is important to recognize that Propositions 4 to 6 refer to the function φ.The impact of a person's action on s' will depend not only on how φ is affected, but also on the possibility that the person may change type by the following period. This induces "mean-reversion" in the Ψ function. Namely,

$$\varphi < \frac{\delta_b}{\delta_a + \delta_b} \Longrightarrow \Psi > \varphi \text{ and } \varphi > \frac{\delta_b}{\delta_a + \delta_b} \Longrightarrow \Psi < \varphi.$$

This feature makes it important to distinguish between φ and Ψ in discussing the impact of current actions on a person's type score. In particular, if the person's current period score is low it is possible for his next period score to *rise* after default. For example, consider a person with $s = \delta_b$. If this person defaults his φ will be less than δ_b but positive (provided $\mu(D_a(\ell, s; q^*, \Psi^*)) > 0$). Since φ is positive, it follows from the definition of the Ψ function that $\Psi(0, 1, s, -x) > \delta_b = s$! Basically, when a person's score is low the mean reverting force can end up being the dominant one and can raise a person's score in the period following default.[12]

[12] This need not be a problematic implication. Musto documents that credit scores tend to rise over time following default.

Propositions 1-6 exhaust what we can say analytically about the nature of the equilibrium. Notably, these propositions do not say anything about item 1 in the introduction. By Proposition 1(ii), the probability of default on a loan made to a person with (updated) score σ is

$$p(-x, \sigma; q^*, \Psi^*) = \mu(D_a(-x, \sigma; q^*, \Psi^*)) \cdot \sigma + (1 - \sigma).$$

If $\mu(D_a) < 1$ then, holding fixed $\mu(D_a)$, it is clear that a higher σ is associated with a lower probability of default. However, $\mu(D_a)$ is not in general independent of σ. Therefore, unless we can characterize the behavior of type a people we cannot prove that item 1 is true in this model. But the behavior of type a people is hard to characterize because unlike type b, their decisions are affected by (q^*, Ψ^*) which itself is determined by their actions. Thus, we turn to exploring the behavior of type a people numerically.

At this stage, we have not calibrated the model. Here we simply take $\beta^a = 0.75$, $r = (0.8/0.75) - 1$, $\delta_a = 0.1$, $\delta_b = 0.5$, $x = 0.5$, and consider a uniform distribution over a 21 element grid of earnings given by $\{10^{-10}, 1.525, 2.550, 3.750, \ldots, 21.0\}$.

We will start by describing the equilibrium Ψ function. It is useful to start here because what people reveal about themselves by their actions will be key to understanding how type a individuals behave. Given our assumptions about δ_a and δ_b in A2 it follows from (8) that $\Psi^* \in [0.5, 0.9]$ so that in a stationary equilibrium a person's beginning of period score $s \in [0.5, 0.9]$.

Equilibrium Ψ Function $\Psi^*(\ell', d, \ell, s)$

- $\ell = -x$.

  1. $\Psi^*(0, 1, -x, s) < s$ for $s > 0.625$
  2. $\Psi^*(-x, 0, -x, s) = (1 - \delta_a)s + \delta_b(1 - s)$
  3. $\Psi^*(0, 0, -x, s) = (1 - \delta_a)s + \delta_b(1 - s)$
  4. $\Psi^*(x, 0, -x, s) = 1 - \delta_a$

- $\ell = 0$.

1. $\Psi^*(-x,0,0,s) < s$ for $s > 0.5125$
2. $\Psi^*(0,0,0,s) = 1-\delta_a$
3. $\Psi^*(x,0,0,s) = 1-\delta_a$

- $\ell = x$.

1. $\Psi^*(-x,0,x,s) = \delta_b$
2. $\Psi^*(0,0,x,s) = 1-\delta_a$
3. $\Psi^*(x,0,x,s) = 1-\delta_a$

Consider first the case where the individual is in debt ($\ell = -x$). While we know from Proposition 1(ii) that type b always default, in this particular equilibrium, for every s some type a default (those with low earnings) and the remaining type a choose $d = 0$ and $\ell' = x$ (those with high earnings). The fact that for every s some type a choose to pay back their loan makes equilibrium loan prices positive, i.e., $q(-x,\sigma) > 0$ for $\sigma \in [0.5, 0.9]$. If an individual defaults then $\varphi(0,1,-x,s) < s$ in accordance with the discussion following Proposition 5 since default strictly reduces the probability of an individual being of type a. Note however that because of the mean reversion in Ψ the person's score at the start of next period, $\Psi^*(0,1,-x,s)$, is lower than s only if $s > 0.625$. See Figure 2 which plots $\Psi^*(0,1,-x,s)$ against a 45° line. Next, if an individual chooses $\ell' = x$ his $\varphi(\ell',0,-x,s)$ rises to 1 because only type a people take this action. Finally, in this equilibrium a type a person with debt never chooses $\ell' \in \{-x,0\}$ so that $\mu(E_a(\ell',-x,s;q^*,\Psi^*)) = 0$ for $\ell' \in \{-x,0\}$. For these actions, off-the-equilibrium-path beliefs are assigned according to the rule specified in assumption A4.

Consider next the case where the individual does not have any assets ($\ell = 0$). In this particular equilibrium, for every s all type b borrow and for every s most type a choose $\ell' = x$, some choose $\ell' = 0$, and some with very low earnings choose $\ell' = -x$. Then, in accordance with the discussion following Proposition 5, we know that $\varphi(-x,0,-x,s) < s$. Choosing any other action (0 or x) reveals a person to be of type a and raises his φ to 1.

Finally, consider the case where the individual has assets ($\ell = x$). In this particular equilibrium, for every s all type b borrow and for every s most type a choose $\ell' = x$ and some choose $\ell' = 0$. If the individual chooses $\ell' \in \{0, x\}$, then his φ will rise to 1 since type b people never choose $\ell' \in \{0, x\}$.

We turn now to the equilibrium q function, plotted in Figure 3. The price rises linearly until $\sigma = 0.8$. Beyond this value, the price declines and then rises linearly again. Since the price on the loan is proportional to the probability of default, and since type b people always default, the decline in the price of the loan for high σ reflects a decline in the probability of repayment by type a people. Figure 4 confirms this. It shows that a type a's probability of default is constant until $\sigma = 0.8$ and then rises to a higher value. The reason for this increase is that a type a person's opportunity cost of default is *inversely* related to his score. To understand this point, it is important to remember that the *only* reason anyone cares about his score is the effect the score has on the terms of credit. We can determine part of his opportunity cost by noting what happens to his score if he defaults versus if he repays. To see this, consider Figure 5, which plots the difference in the loan price he faces tomorrow if he chooses $\ell' = x$ today relative to the price he faces if he defaults today: $q(-x, \Psi^*(x, 0, -x, s)) - q(-x, \Psi^*(0, 1, -x, s))$. As is evident, the difference is generally declining in s.

Put somewhat differently, a person's benefit from investing in reputation is high when the person's current reputation is low. As the person's reputation improves, the value of enhancing it further declines and the person repays less frequently. Returning to Figure 2, the enhancement to one's reputation from not defaulting is given by the difference between one's posterior if he doesn't default, which in this equilibrium is $\Psi^*(x, 0, -x, s) = 1 - \delta_a$, and one's posterior if he does default $\Psi^*(0, 1, -x, s)$.

It's worth emphasizing that the loan price depends on the person's updated score, and therefore depends on his ℓ and s, a point that is not evident in Figure 3. This is true because the price paid for a loan is given by $q^*(-x, \Psi^*(-x, 0, \ell, s))$. This is graphed in Figure 6 and for completeness we plot $\Psi^*(-x, 0, \ell, s)$ in Figure 7. This feature of the environment can have surprising implications. For instance, the price offered on a loan is strictly *decreasing* in the person's initial asset position. As discussed earlier, type a people with $\ell = x$ never borrow regardless of s and so the price of a loan offered to someone *with* assets is lowest because only type b borrow. But some type a people with $\ell = 0$ borrow regardless of s so the price of a loan offered to people *without* assets is higher. Finally, the price on a loan offered to someone with debt is higher than the other two because in this off-the-equilibrium-path case $\Psi^*(-x, 0, -x, s) = (1-\delta_a)s + \delta_b(1-s)$. Basically, the market views running down one's assets as a signal that a person is more likely to be of type b – hence the interest rate offered to people who take such actions is correspondingly high.

Finally, in Figure 8 we plot the invariant distribution of agents across type scores and asset holdings. As can be seen, a little under 12% of the population are borrowers and have low to medium type scores. Most people with high scores either hold positive assets (71%) or no assets (10%). Finally, there are some agents with no assets but medium type scores (a little over 7%).

At this point it is useful to summarize the extent to which a *type score* has properties similar to a *credit score* in the equilibrium studied so far. A type score is like a credit score in the following regard:

- The relationship between a type score and loan price is, for the most part, positive.
- Default reduces type-score.
- For people without assets, taking on debt reduces type-score.

In closing, one final comment is order. So far we have studied cases where one type is entirely myopic. This is, of course, a rather stringent assumption but we may view it as an approximation to the case where type b people aren't totally myopic but are greatly more impatient than type a. Specifically, we have verified that we can get an equilibrium with β^b positive but small ($\beta^b = 0.10$) in which type b people continue to behave as they did under complete myopia – they default if they have debt and borrow otherwise.

6 Conclusion

It is well known that lenders use credit scores to regulate the extension of consumer credit. People with high scores are offered credit on more favorable terms. People who default on their loans experience a decline in their scores and, therefore, lose access to credit on favorable terms. People who run up debt also experience a decline in their credit scores and have to pay higher interest rates on new loans. While credit scores play an important role in the allocation of consumer credit they have not been adequately studied in the consumption smoothing literature. This paper made an attempt to remedy this gap.

We described an economic environment in which a credit score – i.e., a person's index of creditworthiness – could be given a precise meaning. Specifically, the two types of people in our environment differed with respect to their rates of time preference. Lenders could not directly observe a person's type but made probabilistic assessments of it based on the person's financial history. We referred to the probability that a person is of a given type as a person's *type score*. We showed, via an example, that if one of the types discounted the future heavily then the probability that a person is of the *patient* type (i.e., the type that does *not* discount the future heavily) behaves like a *credit score*. That is (i) the terms of credit depend favorably (for the most part) on the probability of a person being of the patient type, (ii) this probability declines if a person defaults on a loan and (iii) this probability declines also when a person takes on new debt.

Many questions remain. First, how robust is our theory of credit scores to a richer asset space? This richness of the asset space has important implications for signalling. Second, will type scores behave like credit scores for other type differences - such as attitude to risk or occupational risk - between people? Third, do high credit scores encourage the "good risks" (the patient types in this model) to default more frequently in the data? Because maintaining a good reputation is costly, our theory implied that the "good risks" have a greater incentive to default when their scores are high. This implication seems integral to the reputation-based theory of credit scores developed in this paper. Finally, when does s fail to be a "sufficient statistic" in the updating function? This seems important since Musto documents that a person's score rises when information about a past bankruptcy leaves a person's credit history.

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Fig. 2: Posterior for Defaulters

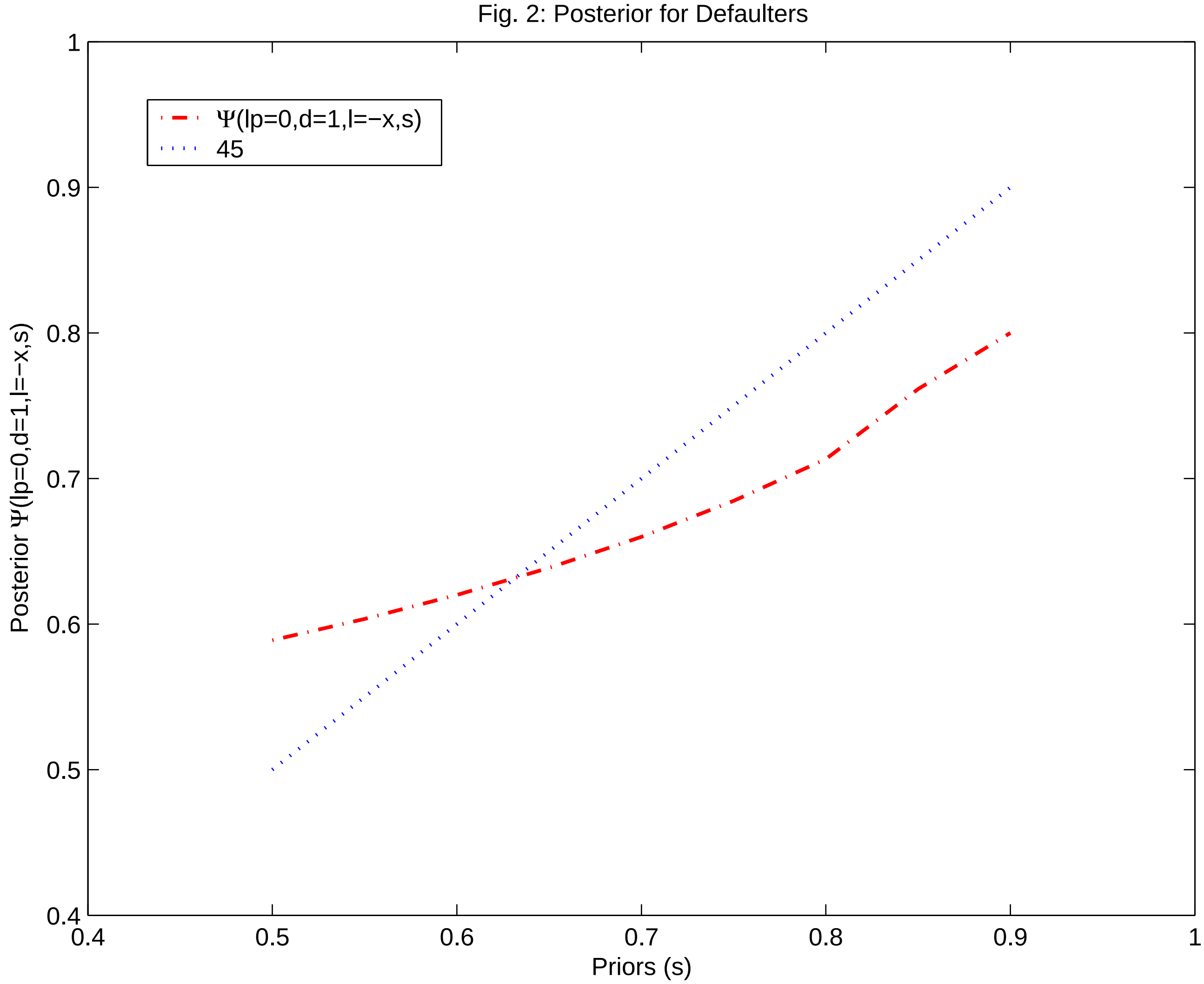
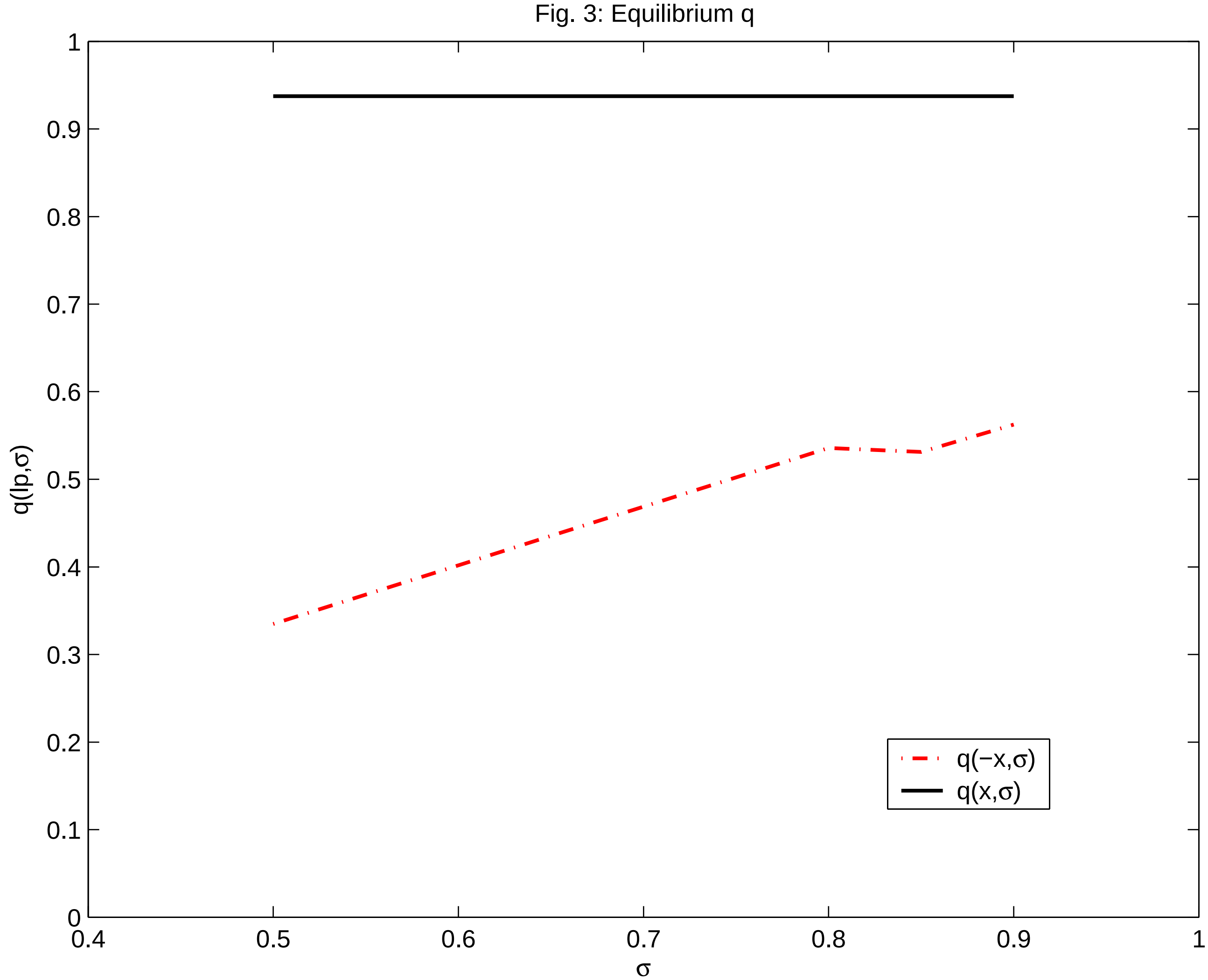


Fig. 3: Equilibrium q



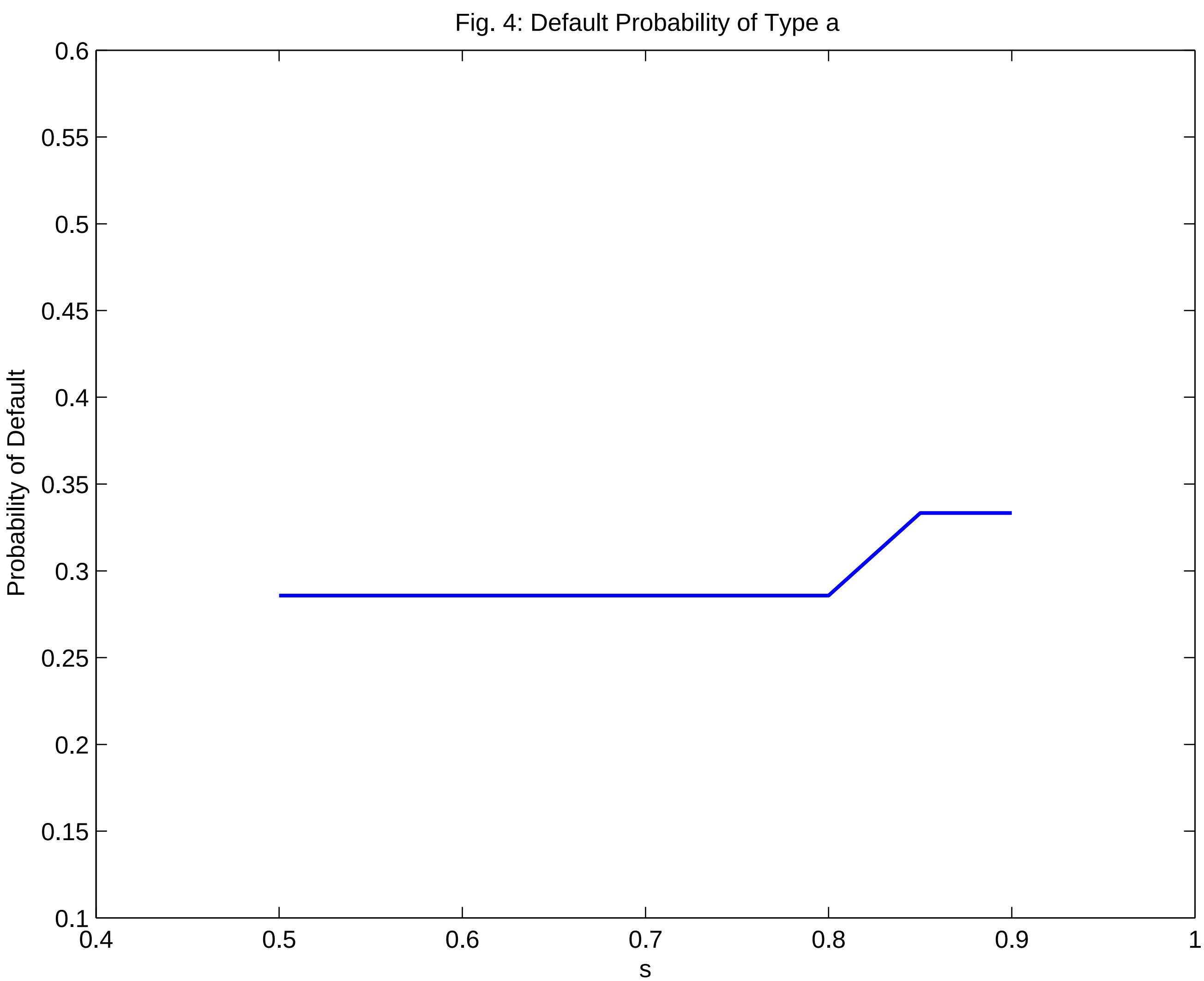


Fig. 4: Default Probability of Type a
Probability of Default
s
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig. 5: (Partial) Value of a Reputation

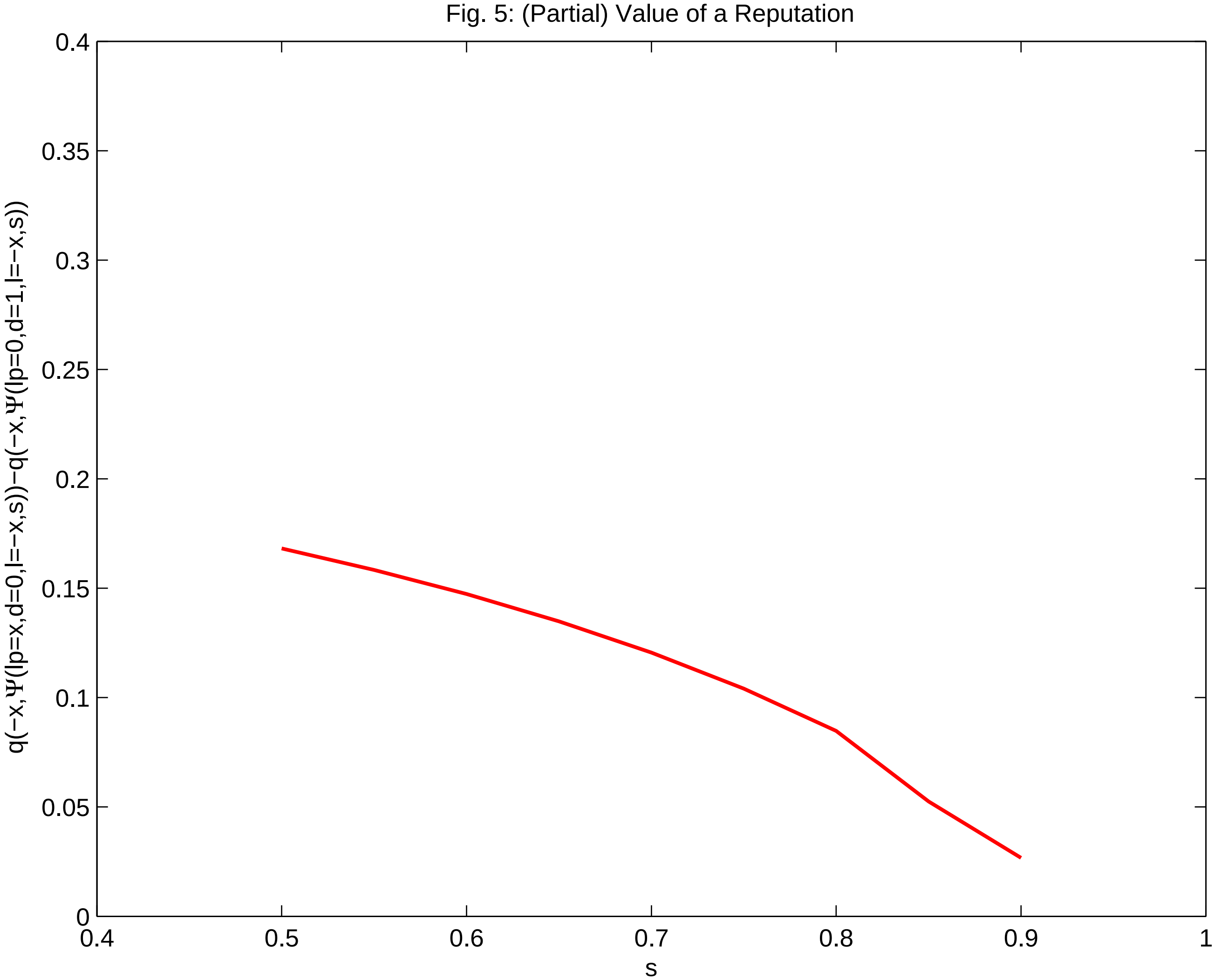


Fig. 6: Equilibrium q for Different Initial Assets

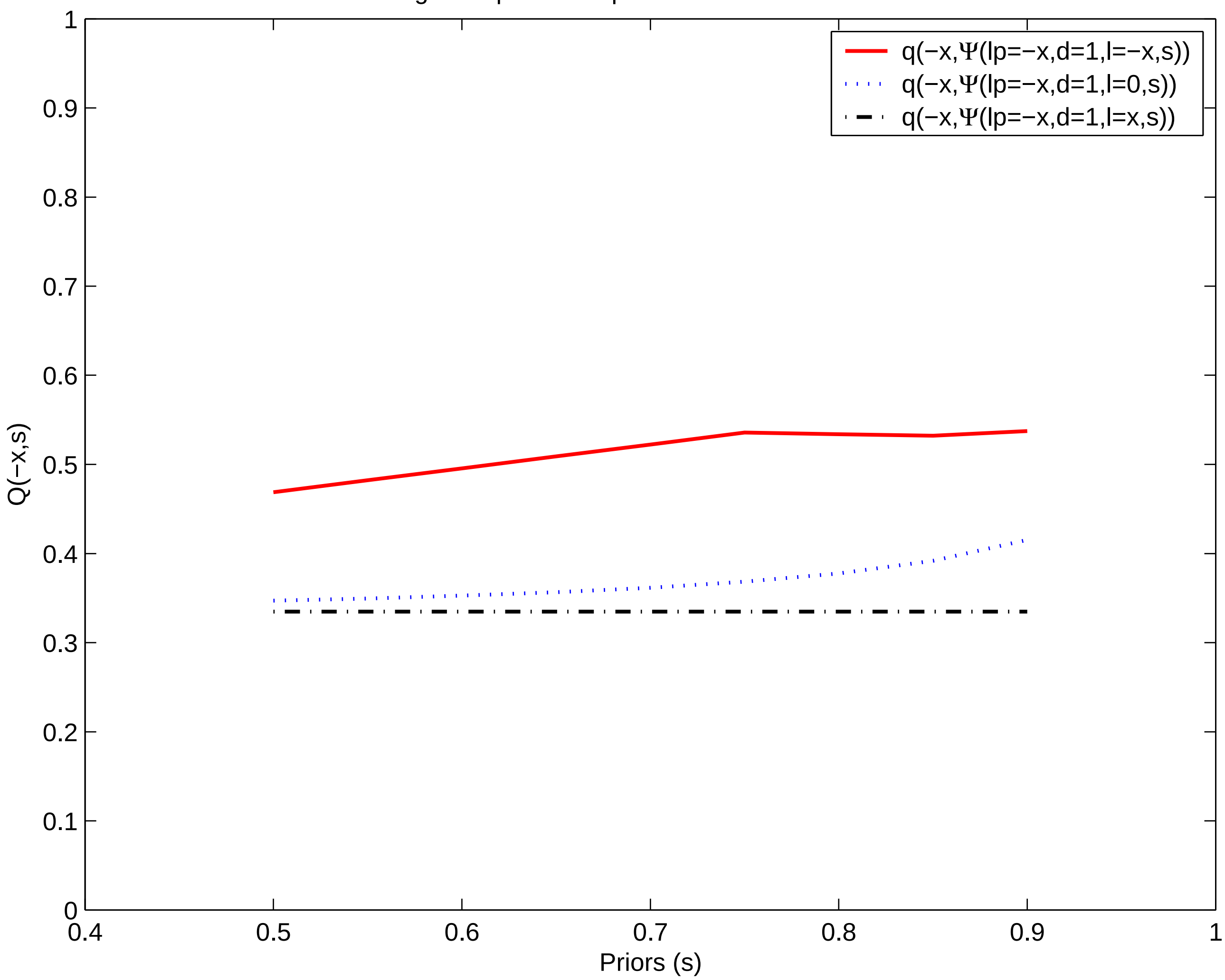
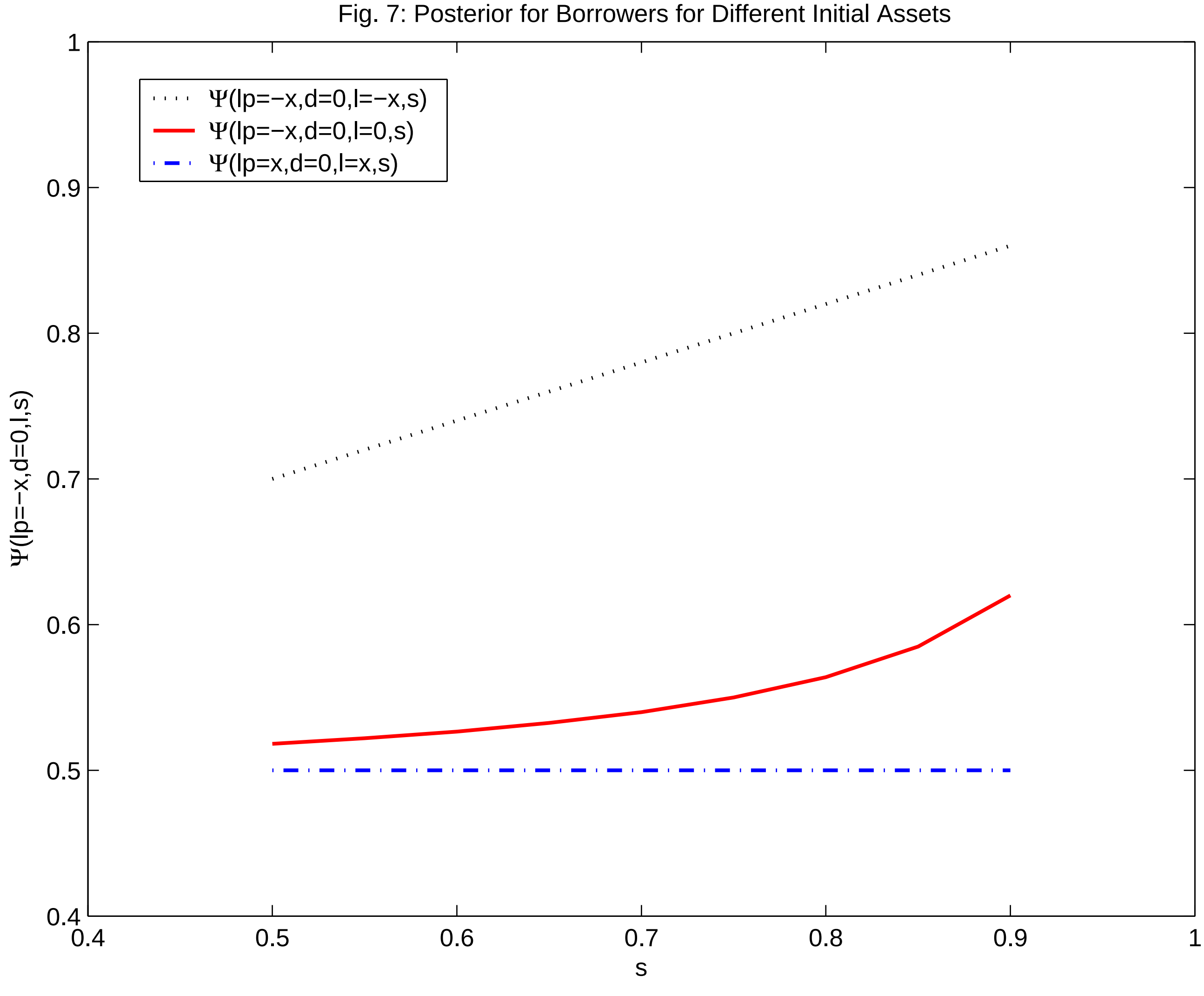


Fig. 7: Posterior for Borrowers for Different Initial Assets



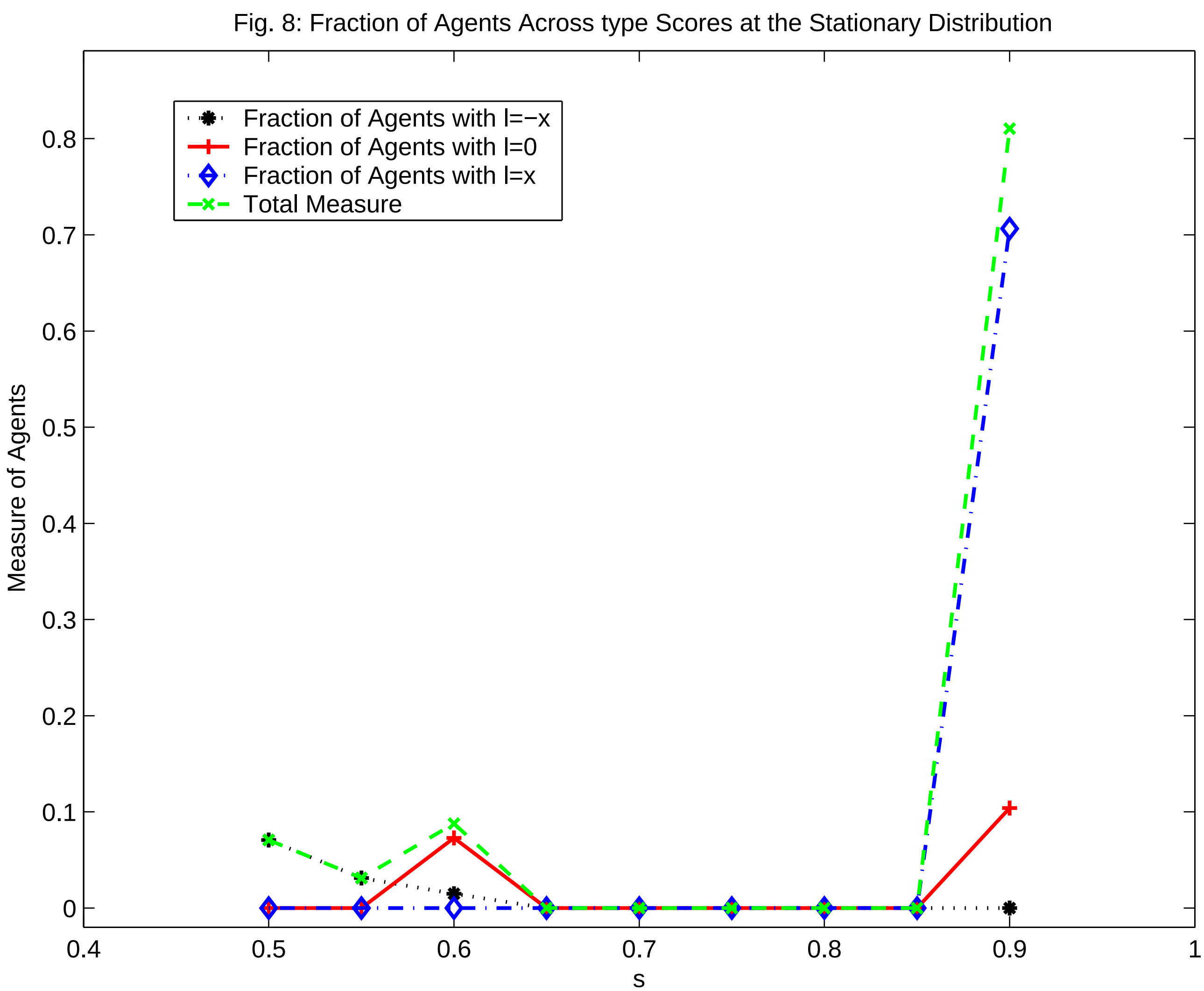


Fig. 8: Fraction of Agents Across type Scores at the Stationary Distribution
Fraction of Agents with l=−x
Fraction of Agents with l=0
Fraction of Agents with l=x
Total Measure
Measure of Agents
s
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.4
0.5
0.6
0.7
0.8
0.9
1