Reputation and Optimal Contract for Central Bankers *

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Abstract

This paper studies the time inconsistency problem on monetary policy for central banks using a unified approach that combines reputation forces and contracts. We first characterize the conditions for reputation forces to eliminate the inflation bias of discretionary policy. We then propose an optimal contract that can be used with reputation forces to implement a desired socially optimal monetary policy rule when the reputation forces alone are not large enough to discourage a central banker to use a surprise inflation policy. In contrast to most of the existing contracts that are contingent on realized inflation rates which are in turn contingent on production shocks, like the standard reputation model, a central banker in our hybrid mechanism is punished only when she uses a surprise inflation rate. Since the penalty proposed is the lowest one that discourages the central bank from attempting to cheat and the sum of the loss, reputation forces, and the penalty for the central bank to cheat is the same as the loss at the socially optimal inflation rate, our hybrid mechanism is the most efficient and robust mechanism that implement the socially optimal monetary policy rule. We also provide a upper bound of the penalty that is be lower than that of the existing contracts when realized inflation rate is greater than a certain level.

Keywords: Monetary policy, time-consistency problem, reputation, optimal contract design, a unified approach

Journal of Economic Literature Classification Number: E52, E58.

*We would like to thank Keven Huang and Tao Wu for helpful comments and suggestions. Financial support from the Private Enterprise Research Center, and the Lewis Faculty Fellowship at Texas A&M University is gratefully acknowledged.

1 Introduction

The time inconsistency problem is one of the most common problems that plague economic policy. Even though technologies, preferences, and information are the same at different times, the policymaker's optimal policy chosen at time t_1 differs from the optimal policy for t_1 chosen at t_0. One can see such a time inconsistency problem exists almost everywhere. For instance, politicians quite often announce that they will carry out a specific policy in the future, but then do something else when the time comes.[1] It is well known from Kydland and Prescott (1977) and Barro and Gordon (1983a) that the time inconsistency of optimal monetary policy may appear when a central bank faces an incentive to expand output above its equilibrium level, and the monetary policy games between the central bank and the public may result in inflation bias as a bad Nash equilibrium outcome. The society experiences a positive average inflation with no systematic improvement in output performance. Indeed, when the marginal benefit of inflation exceeds the marginal cost at a low inflation, the central bank will have an incentive to use a discretionary policy of inflationary bias, and since the public understands that it will do so, the central bank's announcement of a low inflation policy will not be credible. The public will expect a positive rate of inflation, and the central bank cannot do better than to fulfill those expectations. Thus, in order to induce the set of equilibria that lead to desired outcomes, some methods that increase the marginal cost of the central bank must be used to change the central banker's incentives.

Since the time inconsistency problem was first noted by Kydland and Prescott (1977), several solutions have been proposed to deal with this problem in monetary policy. Barro and Gordon (1983a,b) were the first to build a game theoretical model to analyze "reputation" of monetary policy. Backus and Driffill (1985) extended the work of Barro and Gordon to a situation in which the public is uncertain about the preferences of the government. Persson and Tabellini (1990) gave an excellent summarization of these models. Al-Nowaihi and Levine (1994) discussed reputation equilibrium in the Barro-Gordon monetary policy game. Li and Tian (2002) developed a reputation strategic model of monetary policy with a continuous time horizon. The second solution is built on the legislative approach. The major contribution in this area was made by Rogoff (1985). Following the legislative approach of Canzoneri (1985) and Garfinkle and Oh (1993), Lohmann (1992) showed how the welfare effects of Rogoff's conservative bankers can be improved by adding an escape clause. The third solution is based on the incentive contracting

[1] In fact, the time consistency problem can be regarded a special case of the general incentive compatibility problem in the incentive mechanism design literature (cf. Hurwicz (1972), Maskin (1999) and Tian (1989, 1990)).

approach to monetary policy. Persson and Tabellini (1993), Walsh (1995, 1998, 2003), Svensson (1997), Jansen (2000), and Huang and Padila (2002) among many others use this approach.

The basic idea of these three approaches is that if the incentives faced by a central bank in choosing how much to inflate can be affected by some means, the inflation bias may be eliminated while still leaving the central bank free to respond to aggregate output shocks. The insight is that since the inflation bias reflects the monetary authority underestimating the equilibrium cost of inflation, the bias can be eliminated if it can internalize an additional penalty to high realized inflation.

Walsh (1995) was the first to use the contracting approach to investigate the central banker's incentive problem. He showed that by tying reward of the central banker to realized inflation through a simple linear incentive contract, the inflation bias of discretionary policy is eliminated and an optimal response is achieved. Walsh's model has then been extended in several directions. Persson and Tebllinu (1993) showed how the credibility problem may be resolved by a simple performance contract that imposes a linear penalty for inflation on the central banker, and argued that this kind of contract has some resemblance to real-world institutions (also see Beestma and Jensen (1996), Herrendorf and Lockwood (1997), and Svensson(1997)).

However, a main drawback of the contracting approach in the literature is that it completely ignores the equilibrium cost of using discretion inflation rule. The existing contracting models fail to capture the fact that reputation forces can restore credibility or at least reduce the cost of contracts to some extent. Also, the existing contracting schemes are costly for use than necessary. For instance, in Walsh's setting, the government will punish the central banker by an amount that is proportional to the realized socially optimal monetary policy π_t^R. Even though a realized inflation rate does not comes from a surprise inflation, but from a production shock, the central banker will then be nevertheless penalized. Since a production shock can be arbitrary large when it is an unbounded random variable, a huge contract cost will be required for implementing the socially optimal monetary policy rule although the central bank has no incentive to use a cheating monetary policy. A central banker is normally financed by the public and making them pay a pecuniary fine would simply be a reshuffling of tax money.

In this paper, we study the time inconsistency for monetary policy by using a unified approach that combines the reputation effect and contract effect. The government designs an incentive optimal contract for the central banker, and simultaneously the public may also be able to punish the central banker by reputation forces. Each game may involve more than one period dependent on whether the central banker will cheat, and the game will be played repeatedly.

We assume that the government has complete ability to commit to the contract he proposes to the central banker. In order to focus on the nature of the incentives with which the monetary authority should be faced, like Walsh (1995), we assume that, *ex ante*, both the government and the central banker share the same preference over inflation and output fluctuations at each period. This may reflect the outcome of some appointment process that ensures a similarity of views between the government and the monetary authority. As in the standard model of the time-inconsistency monetary policy, both the government and the central bank prefer to have a low-inflation policy. When the reputation force is not big enough, the government then needs to design an additional incentive compatible contract for the central bank to ensure that, *ex post*, the central bank implements a low inflation policy.

To compare the total loss when the central banker cheats with the social loss when the central banker does not cheats, we assume that the central bank's objective is to minimize the average expected loss conditional upon the realization of information up to the present. We provide the necessary and sufficient condition for reputation forces to eliminate the discretionary policy of inflation bias. If the reputation force from the public is large enough to discourage central bankers to use a surprise inflation policy, no contract should be imposed to the central banker. Because the reputation approach does not make any transfer payment and a central banker is punished (by losing the credibility and thus increasing the social loss) only when she uses a surprise inflation rate, it is the most efficient way to implement a socially optimal monetary policy rule, and thus should be the first choice to be used. However, when reputation forces from the public alone cannot restrain the central banker from using a cheating rule, then one must impose additional cost to the central banker such as penalty determined by a contract.

We then show how the government may present an optimal contract that can be used with reputation forces to give the central bank incentives to induce a socially optimal policy as a desired equilibrium outcome when the reputation force from the public alone cannot restrain the central bank from using a cheating rule. We present a hybrid mechanism that combines reputation forces and penalty threats. Our approach unifies the reputational approach and the contracting approach. It suggests a simple optimal incentive scheme or institution that discourages the central bank from surprise inflation and gives her enough flexibility to respond to aggregate output shocks. The length of reputation impact can reduce the penalty cost imposed to the central banker. As it will be seen, the longer the reputation effect lasts, the smaller the penalty will be required for discouraging the central bank from surprise inflation.

Our hybrid mechanism approach differs from the pure contracting approach in the following main aspects. First, the contract part in the hybrid mechanism will be used only when the reputation mechanism does not work. It is well known that the reputation mechanism is the best choice when it works since no explicit transfer payment is needed, and thus the cost of implementing a socially optimal monetary policy rule is the lowest. In this case, no contract is needed. If the reputation mechanism does not work, then one should consider other approaches.

Secondly, even if the reputation forces alone cannot eliminates the central bank's inflationary bias, it nevertheless can reduce to some extent the temptation for the central bank to cheat, and thus a contract scheme may be used together with the reputation forces to solve the central bank's incentive problem so that the contract cost will be lower. To have such a unified approach of reputation and contract, the way for the public to form their expectations in our setting is assumed to be different from the way assumed in the existing contracting approach. Like the reputation model, we assume that the public responds the central banker's cheating to the expected discretion inflation with a lag of one period while the existing contracting approach assumes that the public responds the central banker's cheating to the expected discretion inflation immediately (without any lag of time). This difference of timing in forming expectations makes our contract differs from the existing contracts.

Thirdly, unlike many existing contracts such as Walsh's contract in which the central banker will be punished as long as the realized inflation rate is not zero even though it fully results from production shocks and is out of the central banker's control, the penalty imposed to the central banker in our hybrid mechanism is independent of realized production shocks and the penalty depends only on whether or not she will use a surprise inflation rate, i.e., whether or not expected inflation rate is positive. Thus, just like the existing reputation models, the penalty determined by the both parts of reputation and contract in the hybrid mechanism depends on surprise rate, but not on the realized inflation rate that may result from some uncontrollable shocks by the central banker such as production shocks.

Fourthly, the penalty in our hybrid mechanism is bounded for preventing the central banker from cheating. While most of the existing contract mechanisms are linear in inflation rate and thus the penalty is unbounded, the penalty function in our approach is quadratic, concave, and continuous in surprise inflation rate so that the maximum penalty exists. The central banker can be punished by this upper bound of penalty if she uses a surprise inflation rate in the cheating set we will specify, and yet, this upper bound of penalty may give a lower penalty than the existing contracts. For instance, even if the reputation forces are not taken into account, the penalty is lower than the penalty determined by Walsh's contract when a realized inflation rate is greater than half of the expected optimal cheating rate $\bar{\pi}^C$ specified in (6).

Fifthly, the penalty is just the difference between temptation and enforcement for any inflation rate that makes the difference positive, and thus it the lowest penalty that just discourages the central banker from cheating, and so it reaches a lower bound of penalty payment. Thus, the penalty payment function specifies the minimum required payment that implements the socially optimal monetary policy rule, and therefore, the contract is the most efficient way to implement a socially optimal monetary policy. Hence, our hybrid mechanism provides both lower bound and upper bounded of the penalty which can discourage the central banker from using a surprise inflation monetary policy rule. Finally, since the sum of the loss, reputation forces, and the penalty for the central banker to cheat is the same as the social loss at the optimally socially optimal inflation rate even if there are production shocks, our optimal contract is a robust mechanism that implements the socially optimal monetary policy rule.

The remainder of the paper is organized as follows. Section 2 sets up the model and the time-inconstancy problem faced by the central banker. Section 3 considers the dynamic incentives and reputation forces faced by the central bank, and provides a necessary and sufficient condition for reputation forces to eliminate the discretionary policy of inflation bias. Section 4 presents an optimal contract that can be used with reputation forces to give the central banker incentives to induce a socially optimal policy as a desired equilibrium outcome when the reputation force from the public alone cannot restrain the central banker from using a cheating rule. Section 5 gives the conclusion.

2 The Setup

2.1 Economy

As a standard framework in the literature, we consider an economy characterized by the aggregate supply function:

$$y_t - \bar{y} = a(\pi_t - w_t) + x_t \quad t = 1, 2, 3, ... \tag{1}$$

where y_t is aggregate output at time t, $\bar{y}$ is the equilibrium level of output in absence of supply shocks or unanticipated inflation, π_t is the inflation rate, $w_t = \pi_t^e$ is the rate of growth of nominal wage which is equal to the public's inflationary expectations, $\{x_t\}$ are aggregate supply shocks which are assumed to be identically and independently distributed, with $E[x_t] = 0$, $var[x_t] = \sigma_x^2 < \infty$ where E is the expectations operator, and a is a positive constant that represents the effect of a money surprise on output, i.e., the rate of the output gain from the unanticipated inflation so that the larger is a, the greater is the central banker's incentives to inflate.

In order to focus on the nature of the incentives with which the monetary authority should be faced, we assume that both the government and the central bank share the same *ex ante* preference over inflation and output fluctuations at each period, which is described by a quadratic loss function of the form:

$$\pounds_t = \frac{1}{2}[\pi_t^2 + \theta(y_t - \bar{y} - k)^2], \tag{2}$$

where θ is a positive constant that represents the weight the central bank puts on output expansions relative to inflation stabilization, k is a constant that can be considered as the amounts of output that excesses the equilibrium output, and thus $\bar{y} + k$ is interpreted as the target level of output that the government and the central bank want to reach. As it will be seen below, in order to provide an incentive for the policymarker to attempt to create inflation surprises, k must be positive. Notice that the we have implicitly assumed that the central bank has a zero target inflation rate. The inflation term in (2) will be replaced by $\frac{1}{2}(\pi_t - \pi^*)^2$ if the central bank has a target inflation π^* that differs from zero inflation rate.

Substituting (1) into (2), the loss function becomes

$$\pounds(\pi_t, \pi_t^e) = \frac{1}{2}\{\pi_t^2 + \theta[a(\pi_t - \pi_t^e) + x_t - k]^2\} \quad t = 1, 2, 3, ... \tag{3}$$

We assume that the government, the central banker and the public all know the distribution of the output shocks, but only the central banker knows the current shock exactly. The government and the public only know the shocks in previous periods exactly. Thus we can think of the current output shock x_t as private information for the central banker.

2.2 Three Types of Monetary Policy Rules

First note that, since the objective function is quadratic in inflation rate π_t and output y_t that in turn are linear in x_t, an optimal inflation policy rule must be a linear function in x_t.[2] That is, it belongs to the class

$$\pi_t = \bar{\pi} + a_1 x_t,$$

[2]In general, it is useful to distinguish between inflation and the central bank's policy instrument, the latter taken to be the rate of growth of a monetary aggregate directly controlled by the central bank so that inflation rate is given by $\pi_t = m_t + \nu_t + \mu_t$, where m_t is the money growth rate, ν_t is a demand (or velocity) shock, and μ_t is a "control error" in monetary policy. In this paper, we simplify the stochastic structure by setting $\nu_t = \mu_t = 0$ without affecting the results in any essential way. With these simplifications, we have $\pi_t = m_t$ and thus we have assumed that the central bank controls π_t directly.

where $\bar{\pi}$ and a_1 are constants to be determined. When $\bar{\pi} > 0$, we say the central bank has a positive inflation bias or uses a surprise inflation rate. a_1 represents the effect of production shock on the inflation rate π_t. Since the central banker knows the shock x_t exactly, to determine an optimal inflation policy rule, the central bank only needs to determine the optimal surprise inflation rate $\bar{\pi}$ and the parameter a_1 under various behavior assumptions. As in Barro and Gordon (1983a), there are three types of monetary policy rules a central banker may choose.

2.2.1 Ideal Rule: Socially Optimal Monetary Policy Rule

In this case, the central banker is assumed to be able to commit herself in advance to a linear contingent inflation rule subject to the condition that $E\pi_t = \pi_t^e$. That is, it is assumed that the public believes that the central bank follows this contingent inflation rule and the central banker does not cheat about the announcement of inflation rate. Then $\pi_t^e = E(\pi_t) = E(\bar{\pi} + a_1 x_t) = \bar{\pi}$. Substituting π_t and π_t^e into the loss function (3) and solving this unconditional expected minimization problem by choosing $\bar{\pi}$ and a_1, we get $\bar{\pi} = 0$ and $a_1 = -\frac{a\theta}{1+a^2\theta}$. Then, $\pi_t^e = 0$, and thus the optimal contingent inflation policy rule, denoted by π_t^R, which minimizes the value of social loss conditional on the realization of x_t and the constraint that $\pi_t^e = E(\pi_t) = 0$ is given by

$$\pi_t^R = -\frac{a\theta}{1 + a^2\theta} x_t, \tag{4}$$

and the corresponding expected social loss is given by

$$E[\pounds(\pi_t^R, E\pi_t^R)] = \frac{1}{2}\theta k^2 + \frac{1}{2}\frac{\theta}{1 + a^2\theta}\sigma_x^2, \tag{5}$$

which is constant for all t by the i.i.d. assumption on x_t. This is a benchmark case where the society reaches its desired socially optimal rule π_t^R. The rule, π_t^R, in this benchmark case was called an ideal rule by Barro and Gordon (1983a). (5) gives the lowest social loss for implementing this socially optimal monetary policy rule π_t^R.

2.2.2 Cheating Rule

The monetary policy rule given by (4), however, is not credible if implemented either directly by the government or by a monetary authority whose objective function is given by (3). When the public expects that the central bank will use the contingent rule π_t^R so that the expected inflation rate by the public is $\pi_t^e = E\pi_t^R = 0$, then the central bank would like to implement a positive surprise inflation rate in order to secure some benefits from a surprise inflation. Indeed, when the central bank chooses an optimal inflation policy π_t^C to minimize the value of the loss function (3) conditional on the realization of x_t and $\pi_t^e = E\pi_t^R = 0$ as given, we have

$$\pi_t^C = \frac{a\theta}{1+a^2\theta}(k - x_t), \tag{6}$$

and the corresponding expected social loss is:

$$E[\pounds(\pi_t^C, E\pi_t^R)] = \frac{1}{2}\frac{\theta}{1+a^2\theta}k^2 + \frac{1}{2}\frac{\theta}{1+a^2\theta}\sigma_x^2, \tag{7}$$

which is lower than that given by (5), and thus the central banker has an incentive to adopt a cheating monetary policy rule with a surprise inflation rate given by $\bar{\pi}^C = E(\pi_t^C) = \frac{a\theta}{1+a^2\theta}k$.

2.2.3 Discretion Rule

The discretionary rule is defined in the present context as a Nash equilibrium of a non-cooperative game between the central banker and the public. Under the assumption of rational expectations, the public will not believe that the central bank will use the contingent rule π_t^R so that the expected inflation by the public is $\pi_t^e \neq 0$.[3] Thus, the central banker will choose the optimal discretionary $\pi_t = \pi_t^D$ to minimize the value of the loss function (3) conditional on the realization of x_t and taking π_t^e as given. The equilibrium level of inflation is $\pi_t^D = \frac{a^2\theta\pi^e + a\theta(k - x_t)}{1+a^2\theta}$ and thus the expected inflation is $\pi_t^e = E(\pi_t^D) = a\theta k > 0$ which means that there exists an inflation bias on average. Thus, the discretionary inflation rule is given by

$$\pi_t^D = a\theta k - \frac{a\theta}{1+a^2\theta}x_t, \tag{8}$$

and the corresponding expected social loss is given by

$$E[\pounds(\pi_t^D, E\pi_t^D)] = \frac{1}{2}\theta(1+a^2\theta)k^2 + \frac{1}{2}\frac{\theta}{1+a^2\theta}\sigma_x^2, \tag{9}$$

which reaches a higher expected social loss than the benchmark case given by (5).

As will be seen below, since $E[\pounds(\pi_t^C, E\pi_t^R)] < E[\pounds(\pi_t^R, E\pi_t^R)] < E[\pounds(\pi_t^D, E\pi_t^D)]$, the central banker may have the incentives to cheat or deviate from the optimal policy if the time discount factor is small and as a result, she reaches a worse noncooperative Nash equilibrium outcome than the benchmark case if no additional cost is imposed to the central banker. Then, the economy suffers from a positive bias inflation with an even higher expected social loss. Thus, we have the time-inconsistency problem, which leads to an non-optimal socially optimal monetary policy.

[3]The assumption of rational expectations implicitly defines the expected loss function for the public as $\pounds_p = E[\pi_t - \pi_t^e]^2$; given the public's understanding of the central banker's choice problem, their choice of π_t^e is optimal.

The central banker then needs to be given additional incentives to implement the desired socially optimal monetary policy (4) if the reputation forces alone is not big enough to prevent the central banker from cheating. The main purpose of this paper is to solve this problem by giving an optimal hybrid mechanism that combines the reputation effect and contract enforcement. To do so, in the following, we first characterize the conditions under which the reputation mechanism alone can solve the central banker's incentive problem. We then present an optimal incentive compatible hybrid mechanism that eliminates the inflationary bias and has a lowest contract cost at the same time.

3 Reputation Mechanism and Enforcement of Ideal Rule

The idea of the reputation model is that a credible rule comes with some enforcement power that can reduce an central banker's temptation to cheat. If the central bank adopts a higher rate of inflation than people expect, then they will raise their expectations of future inflation and it results in a higher inflation in the future and a higher social loss.

The timing of the monetary policy game can be described as follows. At the beginning of each period, the central banker announces her inflation policy rule, and then the public forms their expectations (or write wage contracts) on the basis that they believe or do not believe the central banker's announcement, the output shock is then realized, and finally the central bank chooses inflation policy that obeys or does not obey her announcement on no surprise inflation. This process of the interaction between the central banker and the public will then be repeated in time, and thus it incorporates notions of reputation into a repeated-game version of the basic framework.

The public is assumed to use the following behavior strategy in this repeated monetary policy game to form their expectations. If the central bank uses the socially optimal monetary policy rule in the previous period, the public trusts the central bank will continue to use the rule in the current period, and forms their expectations by this belief which equals $\pi_t^e = E\pi_t^R = 0$. But, if the central bank departs from the socially optimal monetary rule π_{t-1}^R last period by using a cheating rule π_{t-1} so that $E\pi_{t-1} \neq \pi_{t-1}^e = E\pi_{t-1}^R$, the public then loses their trust and does not expect the central banker to follow her rule. With a lag of one period (the contract length), the public expects the central bank to pursue the discretionary policy π_{t+i}^D, and responds to a deviation $\pi_{t+i}^e = E\pi_{t+i}^D = \theta a k$ for the next P periods where $i = 1, \ldots, P$ and P may be interpreted as the punishment length for the cheating or the negotiation power over wages of a monopoly union. The punishment length P is assumed to be exogenously given and fixed.[4] The credibility is restored as the P periods punishment. That is, it is assumed that the public has the following form of expectation mechanism:

$$\pi_t^e = E\pi_t^R \quad \text{if } E\pi_{t-1} = \pi_{t-1}^e = E\pi_{t-1}^R \tag{10}$$

$$\pi_{t+i}^e = E\pi_{t+i}^D \quad \text{if } E\pi_{t-1} \neq \pi_{t-1}^e = E\pi_{t-1}^R \tag{11}$$

for $i = 0, 1, \ldots, P-1$. Notice that the form of expectations above is different from the form of expectations used in the contracting literature in which the public is assumed to respond the central banker's cheating to the expected discretion inflation without any lag of time, i.e., $\pi_t^e = E\pi_t$ for any period of time t so that the cheating monetary policy rule π_t^Cdoes not appear in the contracting model. This difference of timing in forming expectations makes our contract differs from the existing contracts.

Accordingly, the central banker can maintains its reputation or credibility in each period if she wants. On the other hand, if the central banker cheats during period t, the expectations are the ones associated with the discretionary rule π_{t+i}^D for next P periods. Notice that the assumption of rational expectations implies that there is at least one period punishment to the central banker if she cheats.

The government wants to eliminate the inflation bias of discretionary policy while still preserving the ability of the central bank to respond to aggregate output shocks. Thus, he wants the central bank to implement the socially optimal contingent rule π_t^R for any length of periods. While the reputation game can be repeated independently as many times as desired, it is assumed that the future is discounted by $0 < \beta < 1$ so that the government's total loss for $1+P$ periods has present values $\sum_{i=1}^{P} \beta^i E[\pounds(\pi_{t+i}^R, E\pi_{t+i}^R)]$ under the socially optimal monetary policy rule π_t^R. For the convenience of discussion, we may consider the average loss, and may also want to discount the time, so the total time having this total loss has the present value $\sum_{i=0}^{P} \beta^i$. Thus the government's average expected social loss for the length of $P+1$ periods at π_t^R is given by

$$\pounds_G = \sum_{i=0}^{P} \frac{\beta^i E[\pounds(\pi_{t+i}^R, E\pi_{t+i}^R)]}{\sum_{i=0}^{P} \beta^i} = \frac{1}{2}\theta k^2 + \frac{1}{2}\frac{\theta}{1+a^2\theta}\sigma_x^2. \tag{12}$$

However, the central bank may have a different *ex-post* average objective function. Since the central bank's decision may cheat at any time t, the central banker's average expected loss under

[4] If P is an endogenous variable, there exists a multiplicity of reputational equilibria that can be supported as subgame-perfect equilibria on the part of the public sector. al-Nowaihi and Levine (1994) considered this problem with solution.

the cheating is given by

$$\lambda(\pi_t) = \frac{E_t[\pounds(\pi_t, E\pi_t^R) + \sum_{i=1}^P \beta^i \pounds(\pi_{t+i}^D, E\pi_{t+i}^D)]}{\sum_{i=0}^P \beta^i} \tag{13}$$

where E_t denotes the expectations conditional upon the realization of all information up to and including period t. This objective function shows that the central bank has an option to use a discrete monetary policy in period t when she thinks it is necessary. We now investigate under which conditions, reputation forces will prevent the central banker from using a cheating monetary policy.

Note that $E_t(\pounds(\pi_t^C, E\pi_t^R)) \leq E_t(\pounds(\pi_t, E\pi_t^R))$ for any linear contingent inflation rate π_t, and thus the central banker does not have any incentive to cheat at π_t if she does not have an incentive to cheat at π_t^C. So we only need to consider the loss at the optimal cheating policy π_t^C.

Note that

$$\begin{aligned} \lambda(\pi_t^C) - \lambda(\pi_t^R) &= \frac{E_t\{[\pounds(\pi_t, E\pi_t^R) - \pounds(\pi_t^R, E\pi_t^R)] + \sum_{i=1}^P \beta^i [\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R)]\}}{\sum_{i=0}^P \beta^i} \\ &= \frac{a^2\theta^2 k^2}{2\sum_{i=0}^P \beta^i}(\sum_{i=1}^P \beta^i - \frac{1}{1+a^2\theta}). \end{aligned} \tag{14}$$

Thus, if $\sum_{i=1}^P \beta^i \geq \frac{1}{1+a^2\theta}$, then the central banker does not have the incentives to cheat or deviate from the socially optimal monetary policy. This result can be regarded as a version of the folk theorem for infinite-horizon repeated games studied in Fudenberg and Maskin (1986), which suggests that the central bank has an incentive to cheat when the discount factor is small. The smaller is β, the larger is the reputation force P needed. However, when β is too small so that $\sum_{i=1}^P \beta^i < \frac{1}{1+a^2\theta}$, the reputation force alone cannot solve the central bank's cheating problem. Indeed, when $\beta < \frac{1}{2+a^2\theta}$, $\sum_{i=1}^P \beta^i = \beta\frac{1-\beta^P}{1-\beta} < \frac{1}{1+a^2\theta}$ for any P, and thus the minimum discount factor for the central bank to keep the socially optimal monetary policy rule is $\beta = \frac{1}{2+a^2\theta}$.

The intuition behind this is that: if the central banker uses a cheating policy, she will receive a benefit or gain of $E_t[\pounds(\pi_t^R, E\pi_t^R)) - \pounds(\pi_t, E\pi_t^R))]$ from reneging arising from the one period before the public can retaliate, but the public will then retaliate the central bank for P periods and the penalty will be $\sum_{i=1}^P \beta^i E_t[\pounds(\pi_{t+1}^D, E\pi_{t+1}^D) - \pounds(\pi_{t+1}^R, E\pi_{t+i}^R))]$ which arises from the P periods of punishment. As in Barro and Gordon (1983a), we may call the benefit of cheating as temptation and the cost of cheating as enforcement to renege on the socially optimal monetary policy rule π_t^R. When the enforcement is greater than the temptation, i.e., $\sum_{i=1}^P \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))] > E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))]$, the reputation mechanism implements the socially optimal contingent monetary policy π_t^R.

Thus, we have the following proposition.

Proposition 1 *For the reputation repeated game of monetary policy between the central bank and the public, there is no inflation bias for any period of time if and only if* $\sum_{i=1}^{P} \beta^i \geq \frac{1}{1+a^2\theta}$, *and further the socially optimal monetary policy rule* $\{\pi_t\}$ *cannot be supportable for any length of reputation forces if* $\beta < \frac{1}{2+a^2\theta}$.

Note that, when β is large, we only need a small P to keep this inequality held. In particular, when β is close to one, we have $\sum_{i=1}^{P} \beta^i > \frac{1}{1+a^2\theta}$ and thus the central bank does not have an incentive to cheat or deviate from the optimal policy in any period t for any $P \geq 1$. Then, we have the following corollary.

Corollary 1 *For the reputation repeated game of monetary policy between the central bank and the public, when* β *is close to one, there is no inflation bias for any positive length of reputation.*

Hence, reputation forces discourage the central banker from attempting to cheat and the legal, institution, or contracting constraint on the central banker is unnecessary and will impose unnecessary contract cost. In particular, all contracts such those in Walsh (1995) are unnecessary for use.

Remark 1 The conclusion of Corollary 1 is based on the quadratic specification of objective function in (2). However, as shown in Barro and Gordon (1983a), when the objective function is replaced by the quadratic-linear objective function

$$\pounds_t = \frac{1}{2}[\pi_t^2 - \theta(y_t - \bar{y} - k)], \tag{15}$$

the zero average inflation policy rule is no longer supportable by the reputation mechanism for any discount factor $0 \leq \beta \leq 1$ although a non-zero but less than the discretionary inflation rate may be sustainable. In this case, we need to adopt other means such as the contracting approach we will discuss below to solve the central bank's incentive problem.

4 A Hybrid Mechanism of Optimal Contracts and Reputation

The above proposition shows that, when $\sum_{i=1}^{P} \beta^i < \frac{1}{1+a^2\theta}$, the reputation force alone is not large enough to discourage the central bank from surprise inflation. The repeated monetary policy game between the central bank and the public yields inflation bias as a bad noncooperative Nash equilibrium outcome so that the society experiences a positive average inflation without systematic improvement in output performance and suffering a higher social loss. The government then needs to step in and may play an important role of providing an incentive compatible mechanism that induces a desired socially optimal monetary policy rule. In this section, we use the principal-agent framework (a simple case of general mechanism design) to determine how the optimal contract is designed and combined with the reputation punishment together to solve the central bank's incentive compatibility problem. In this framework, the principal is the government whose goal is to implement the socially optimal monetary policy rule π_t^R, and the agent is the central bank, to which the government delegates the task of implementing the goal. In this section, we show that by combining the reputation pressure with an additional incentive contract, one can induce the central bank to eliminate the inflation bias of discretionary policy, and give the central bank the right incentives to induce the socially optimal monetary policy π_t^R as a desired equilibrium outcome. In addition, the hybrid mechanism has the lowest contract cost in the sense that the transfer payment is the lowest to implement the socially optimal monetary policy for any rate of surprise inflation.

Our hybrid mechanism approach differs from the pure contracting approach in the following main aspects. First, a contract part in the hybrid mechanism will be used only when the reputation mechanism does not work, i.e., only when $\sum_{i=1}^{P} \beta^i < \frac{1}{1+a^2\theta}$. It is well known that the reputation mechanism is the best choice when it works since no explicit transfer payment is needed, and thus the cost of implementing a socially optimal monetary policy rule is the lowest. In this case, no contract is needed. If the reputation mechanism does not work, then one should consider other approaches. Secondly, even if the reputation forces alone cannot eliminates the central bank's inflationary bias, it nevertheless can reduce in some extent the temptation for the central banker to cheating, and thus a contract scheme may be used together with the reputation forces to solve the central bank's incentive problem so that the contract cost will be lower. Thirdly, unlike many existing contracts such as Walsh's contract in which the central banker will be punished as long as the realized inflation rate is not zero even though it fully results from production shocks, the penalty function for the central banker in our hybrid mechanism is independent of realized production shocks x_t and the penalty depends only on whether or not she will use a surprise inflation rate, i.e., whether or not expected inflation rate $\bar{\pi} > 0$. Fourthly, while most existing contract mechanisms are linear in inflation rate and thus the penalty is unbounded, the penalty function in our approach is quadratic, concave, and continuous in surprise inflation rate so that the maximum penalty exists. Finally, the penalty is just the difference between temptation and enforcement: $E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))] -$

$\sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))]$, for any π_t that makes the difference positive. Thus, the penalty payment function specifies the minimum required payment that implements the socially optimal monetary policy rule $\{\pi_t^R\}$.

Now, we formally present the optimal hybrid mechanism below. In the hybrid mechanism, the central banker receives a penalty payment (which may be zero) from the government when the central banker uses a surprise inflation rate $\bar{\pi}$ at time t. The payment could be considered as a direct cost of the central banker or more broadly as legal constraints for the central bank, denoted by $W(\bar{\pi})$. Notice that, the penalty function is the function of surprise inflation, but not a function of the actual inflation rate π_t that is given by $\pi_t = \bar{\pi} + a_1 x_x$ which depends on both surprise inflation rate $\bar{\pi}$ and production shock x_t. This specification makes our contract be significantly different from one in Walsh (1985) that is linear and depends on the inflation rate π_t which in turn depends on production shock x_t.

The problem faced by the government (principal) is to design a penalty function $W(\bar{\pi})$ that makes the central banker have no incentives to cheat and thus induces the central bank to choose the socially optimal monetary policy $\{\pi_t^R\}$, and further minimizes the expected value of the loss

$$\lambda(\pi_t, W) = \frac{\pounds(\pi_t, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \pounds(\pi_{t+i}^D, E\pi_{t+i}^D) + W(\bar{\pi})}{\sum_{i=0}^{P} \beta^i} \tag{16}$$

conditional on the realization of x_t.

Like the first term in (13), the loss function $\pounds(\cdot)$ is valued at $(\pi_t, E\pi_t^R) = (\pi_t, 0)$, but not at $(\pi_t, E\pi_t)$ when the central banker cheats at the current period t. Notice that this is the main difference between our unified approach and the existing contracting approach. In the existing contract models, the public is assumed to respond the central bank's cheating to the expected discretion inflation $E\pi_{t+i}^D = a\theta k$ without any lag of time so that the cheating monetary policy rule π_t^C does not appear in the central bank's objective function.

When $\pounds(\pi_t, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \pounds(\pi_{t+i}^D, E\pi_{t+i}^D) > \pounds(\pi_t^R, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \pounds(\pi_{t+i}^R, E\pi_{t+i}^R)$ or equivalently, when enforcement is greater than temptation:

$$\sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R)] > E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R)],$$

it is more costly for the central banker to use the discretionary policy π_t, and thus she does not have an incentive to cheat. Thus, the reputation mechanism alone cannot solve the central bank's incentive problem. So we only need to consider the case where $\sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))] < E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))]$. Define the set of inflation rates in which the central banker has an incentive to deviate from the socially optimal monetary policy rule π_t^R by

$$\Pi_t = \{\pi_t : \sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))] - E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))] < 0\},$$

which may be called the cheating set at time t. Note that $E_t[\lambda(\pi_t^R, 0)] = E_t[\pounds(\pi_t^R, E\pi_t^R)]$. The contract $W(\bar{\pi})$ implements the optimal policy $\{\pi_t^R\}$ if $E_t[\lambda(\pi_t, W)] \geq E_t[\pounds(\pi_t^R, E\pi_t^R)]$ for all $\pi_t \in \Pi_t$ and for all t. It is clear that there are many such contracts which implement the optimal policy $\{\pi_t^R\}$. For instance, any $W(\bar{\pi})$ that makes $E_t[\lambda(\pi_t, W)] \geq E_t[\pounds(\pi_t^R)]$ for all t can be used as such a contract. Hence, our interest here is to find an optimal one which has the lowest penalty cost.

Then, we have the following definition about the optimal contract.

Definition 1 A contract $W(\bar{\pi})$ is said to be an optimal contract which implements the optimal policy $\{\pi_t^R\}$ if it satisfies the following three conditions.

(1) (Incentive Compatibility): $E_t[\lambda(\pi_t, W)] \geq E_t[\pounds(\pi_t^R, E\pi_t^R)]$ for any $\pi_t \in \Pi_t$.

(2) (Optimal Choice): π_t minimizes $E_t[\lambda(\pi_t, W)]$ for all $\pi_t \in \Pi_t$.

(3) (Efficient Contract): $E[\lambda(\pi_t, W)] = E[\pounds(\pi_t^R, E\pi_t^R)]$ for all $\pi_t \in \Pi_t$.

In the above definition, Condition 1 is known as the incentive compatibility requirement that discourages the central bank deviating from the socially optimal monetary policy $\{\pi_t^R\}$ so that the central bank's interest is compatible with the government's interest. Condition 2 is known as the central banker's rational (optimal) choice condition on monetary policy rule. Condition 3 is known as the efficient contract condition under which the penalty determined by the contract is the lowest penalty that just discourages the central banker from cheating, i.e., the sum of the loss at any discretionary monetary policy rule, reputation forces, and the penalty is exactly equal to the loss at the socially optimal monetary policy π_t^R. Notice that Condition 2 is not the same as Condition 3. For instance, when a big constant term is added into the penalty function W, the central banker's original optimal choice on π_i remains optimal, but the $\lambda(\pi_t, W)$ becomes larger. Also notice that expectations in the first two conditions are taken conditional on the realization of x_t since, by assumption, it is known by the central bank.

As we mentioned earlier, any optimal monetary policy rule of the central bank belongs to the class of linear contingent function in x_t: $\pi_t = \bar{\pi} + a_1 x_t$, the central banker wants to choose the optimal $\bar{\pi}$ and a_1 so that π_t minimizes $E_t[\lambda(\pi_t, W)]$ for all $\pi_t \in \Pi_t$. Then the first order conditions for the central bank's problem are obtained by differentiating (16) with respect to a_0

and $\bar{\pi}$, respectively

$$(1+\theta a^2)\pi_t x_t + \theta a(x_t - k)x_t = 0 \tag{17}$$

and

$$(1+\theta a^2)\pi_t + \theta a(x_t - k) + \frac{\partial W(\bar{\pi})}{\partial \bar{\pi}} = 0. \tag{18}$$

Taking the unconditional expectation for equation (17) and solving for a_1, the optimal a_1 is given by $a_1 = -\frac{a\theta}{1+a^2\theta}$. Thus, in the hybrid mechanism, the optimal monetary policy rule of the central bank has the form of $\pi_t = \bar{\pi} - \frac{a\theta}{1+a^2\theta}x_t = \bar{\pi} + \pi_t^R$, where $\bar{\pi}$ is a surprise inflation bias deviating from the socially optimal inflation policy rule π_t^R by the central banker.

To find out the penalty function $W(\pi_t)$, substituting $\pi_t = \bar{\pi} - \frac{a\theta}{1+a^2\theta}x_t = \bar{\pi} + \pi_t^R$ into (18) and solving for $\frac{\partial W(\bar{\pi})}{\partial \bar{\pi}}$, we have

$$\frac{\partial W(\bar{\pi})}{\partial \bar{\pi}} = \theta a k - (1+\theta a^2)\bar{\pi}. \tag{19}$$

Thus, we have

$$W(\bar{\pi}) = W^0 + \theta a k \bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2. \tag{20}$$

To make $W(\bar{\pi})$ be the optimal incentive compatible contract, we need to determine the constant term W^0 so that $E_t[\lambda(\pi_t, W) - \pounds(\pi_t^R, E\pi_t^R)] \geq 0$ and $E[\lambda(\pi_t, W) - \pounds(\pi_t^R, E\pi_t^R)] = 0$.

Note that, by substituting $W(\bar{\pi})$ into (16), we have

$$\begin{aligned}\lambda(\pi_t, W) - \lambda(\pi_t^R) &= \frac{[\frac{1}{2}(1+\theta a^2)\bar{\pi} - \theta a k)]\bar{\pi} + \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^P \beta^i + W(\bar{\pi})}{\sum_{i=0}^P \beta^i} \\ &= \frac{\frac{1}{2}a^2\theta^2k^2\sum_{i=1}^P \beta^i + W^0}{\sum_{i=0}^P \beta^i}.\end{aligned} \tag{21}$$

Then, when $W^0 = -\frac{1}{2}a^2\theta^2k^2\sum_{i=1}^P \beta^i$, we have $\lambda(\pi_t, W)) = \pounds(\pi_t^R, E\pi_t^R)$. Therefore, we have $E_t[\lambda(\pi_t, W) - \pounds(\pi_t^R, E\pi_t^R)] \geq 0$ and $E[\lambda(\pi_t, W) - \pounds(\pi_t^R, E\pi_t^R)] = 0$, which means that $W(\bar{\pi})$ is the optimal contract under which the cost of using a cheating monetary policy rule π_t is the same as the cost for the central banker to use the socially optimal monetary policy rule. Thus, the central banker cannot benefit from the cheating, although it is not worse off either.

Thus, the optimal incentive compatible contract that is contingent on the surprise inflation rate $\bar{\pi}$ and discourages the central banker from using the discretionary monetary policy rule π_t is given by

$$W(\bar{\pi}) = \theta a k \bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 - \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^P \beta^i \quad \forall\, \pi_t \in \Pi_t. \tag{22}$$

The central banker will be penalized by the amount of $W(\bar{\pi})$ if she uses a surprise inflation rate $\pi_t \in \Pi_t$ at time t, and will not be penalized if she uses a socially optimal monetary policy rule

π_t^R or uses a surprise inflation rate π_t which is not in the cheating set Π_t.[5] Thus, the optimal contract penalty payment $W(\bar{\pi})$ is solely based on whether the central banker uses the surprise inflation rate $\bar{\pi}$, but not based on the inflation rate which in turn depends on the magnitude of production shocks x_t.

To determine the cheating set Π_t, note that $W(\bar{\pi}) \geq 0$ if and only if

$$g_1 \equiv \frac{a\theta k\left[1-\sqrt{1-(1+\theta a^2)\sum_{i=1}^{P}\beta^i}\right]}{1+\theta a^2} < \bar{\pi} < \frac{a\theta k\left[1+\sqrt{1-(1+\theta a^2)\sum_{i=1}^{P}\beta^i}\right]}{1+\theta a^2} \equiv g_2.$$

Thus the central banker does not have an incentive to cheat at the cheating set $\Pi_t = [g_1, g_2]$ and the optimal contract that implements the socially optimal monetary policy rule $\{\pi_t^R\}$ can be written as

$$W(\bar{\pi}) = \begin{cases} \theta a k\bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 - \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i & \text{if } g_1 < \bar{\pi} < g_2 \\ 0 & \text{otherwise} \end{cases}. \tag{23}$$

Remark 2 The term $\sum_{i=1}^{P}\beta^i[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R)] = \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i$ in the penalty function $W(\bar{\pi})$ in (23) is the reputation effect. Thus, this hybrid mechanism is the combination of the reputation mechanism and the contract $W(\bar{\pi})$. When $1 < (1+\theta a^2)\sum_{i=1}^{P}\beta^i$, the reputation force is enough to solve the central bank's incentive problem by Proposition 1, and thus no penalty is needed so that the contract cost is zero.

Remark 3 The penalty function $W(\bar{\pi})$ depends not only on the discount factor but also the length of reputation P. Furthermore, $W(\bar{\pi})$ is a decreasing function in P, which means the length of the reputation impact from the public can reduce the contract cost.

Remark 4 When future is perfectly discounted, i.e., β approaches to zero, there is no reputation forces imposed on the central bank. In this case, the cheating set has a simple form which is given by $\Pi_t = (0, \frac{2\theta a k}{1+\theta a^2})$, and the reputation term disappears in the above penalty function. Thus, our hybrid mechanism becomes a pure contract mechanism that is given by

$$W(\bar{\pi}) = \begin{cases} \theta a k\bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 & \text{if } \bar{\pi} \in (0, \frac{2\theta a k}{1+\theta a^2}) \\ 0 & \text{otherwise} \end{cases}, \tag{24}$$

which implements the socially optimal monetary policy rule $\{\pi_t^R\}$. This is a new contract scheme and is different from the existing contract schemes. The hybrid mechanism defined in (23) is the sum of the reputation forces given by $\sum_{i=1}^{P}\beta^i[\pounds(\pi_{t+i}^D) - \pounds(\pi_{t+i}^R)] = \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i$ and the pure contract mechanism defined by (24).

[5] Since the central banker is assumed to be rational, she will never have an incentive to choose an inflation rate not in Π_t, otherwise she will be worse off.

Remark 5 Notice that, if the central banker chooses a cheating inflation rule according to the rule specified by (6): $\pi_t = \pi_t^C = \frac{1}{1+\theta a^2}\theta a(k - x_t)$, then $\bar{\pi}^C = \frac{1}{1+\theta a^2}\theta ak$ and thus $W(\bar{\pi}^C)$ is given by

$$W(\bar{\pi}^C) = \frac{1}{2}\frac{\theta^2 a^2 k^2}{1 + \theta a^2} - \frac{1}{2}a^2\theta^2 k^2 \sum_{i=1}^{P} \beta^i. \tag{25}$$

In fact, since the penalty function is linear-quadratic (i.e., the first term in the penalty function is linear and the second term is quadratic), continuous, and concave in $\bar{\pi}$, the maximum penalty will be reached at $\bar{\pi}^C = E\pi_t^C = \frac{1}{1+\theta a^2}\theta ak$. The intuition behind this is that, since we are looking for the minimal penalty for the central banker to keep the socially optimal monetary policy π_t^R for any level of surprise inflation and the bank's loss will be minimized at π_t^C, the maximum penalty needs to be imposed to make the sum of the loss, the reputation forces, and the penalty as big as the loss at the socially optimal policy π_t^R in order for the central banker to have no incentives to cheat. Thus, the penalty function is an increasing function when $\bar{\pi} < \frac{1}{1+\theta a^2}\theta ak$ and a decreasing function when $\bar{\pi} > \frac{1}{1+\theta a^2}\theta ak$. Note that the penalty will be zero when $\bar{\pi} \le g_1$ or $\bar{\pi} \ge g_2$. However, since the sum of the loss and the penalty equals the loss at the socially optimal monetary policy π_t^R over the interval (g_1, g_2), the central banker does not have the incentives to use a discretionary monetary policy and thus the optimal contract is robust so that we can allow the price fluctuations due to measurement error or uncontrollable production shocks.

Remark 6 The maximum penalty specified in (25) gives a upper bound of penalty when the central banker adopts a surprise inflation rate in the cheating set. One may simply use this upper bound of penalty to punish the central banker when she cheats. That is, the penalty will be equal to this upper bound and so it is constant for any $\bar{\pi} = (g_1, g_2)$ and zero otherwise. Yet, this penalty may be lower than the one determined by Walsh's penalty function since his payment is continent on observed inflation rate which in turn is implicitly dependent on production x_t that can be arbitrarily larger. Indeed, since Walsh's penalty is linear which is given by $\theta ak\pi_t$ and our penalty $W(\bar{\pi}) < \frac{\theta^2 a^2 k^2}{2(1+\theta a^2)}$. Then, when $\pi_t > \bar{\pi}^C/2$, $\frac{\theta^2 a^2 k^2}{2(1+\theta a^2)} = \frac{1}{2}\theta ak\bar{\pi}^C < \theta ak\pi_t$ by noting that $\bar{\pi}^C = \frac{\theta ak}{1+\theta a^2}$. Thus, even if we do not consider the reputation effect, the penalty from the pure mechanism specified in (24) is lower the penalty in Walsh's contract when $\pi_t > \bar{\pi}^C/2$.

Thus, the optimal contract $W(\bar{\pi})$ combined with the reputation enforcement implements the socially optimal monetary policy rule $\{\pi_t^R\}$ given by (4) and has the lowest cost for preventing the central banker from using an inflation bias of discretionary policy. Also, the total cost for using a cheating monetary policy under this hybrid mechanism is the same as the social cost at π_t^R, and thus the hybrid mechanism is the most efficient way to implement the socially optimal monetary policy rule π_t^R. Hence, when the reputation enforcement alone cannot restrain the central banker from using a cheating rule, this simple hybrid contingent mechanism may be used.

Summarizing the above discussion we have the following proposition.

Proposition 2 *In the monetary policy repeated game among the public, the central banker and the government, suppose the reputation enforcement lasts for P periods so that* $\sum_{i=1}^{P}\beta^i < \frac{1}{1+a^2\theta}$*, and suppose the penalty function* $W(\bar{\pi})$ *is given by (23). Then, the hybrid mechanism is an optimal mechanism that implements the socially optimal monetary policy rule* π_t^R.

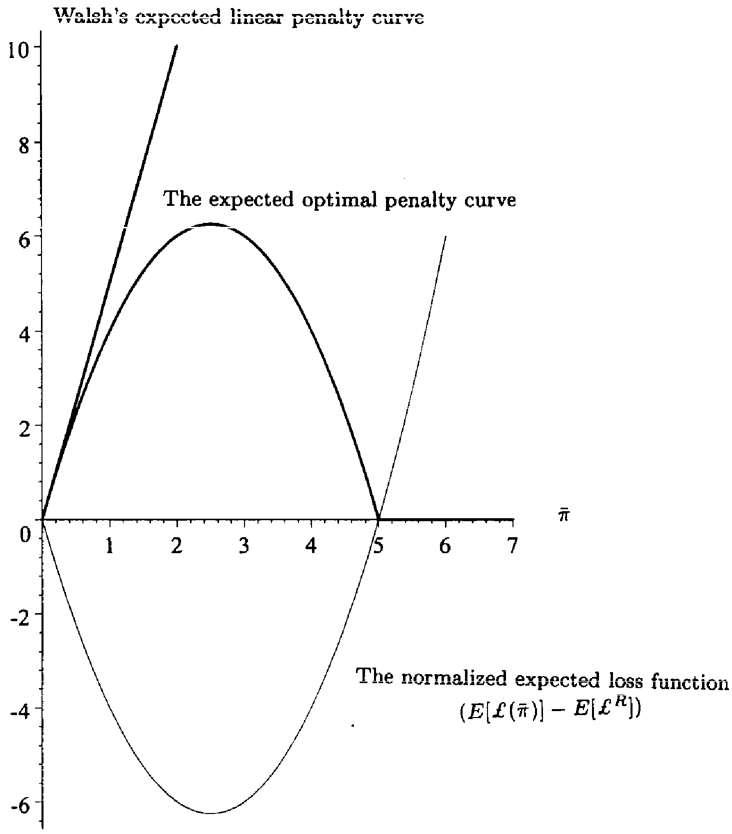

Figure 1: Parameters: $\theta = 1, a = 1, k = 5, P = 1, \beta = 0.2$

The above results on the optimal contract with the quadratic loss function can be illustrated using Figure 1. In Figure 1, the quadratic and convex curve is determined by the equation

$$\begin{aligned} L_\beta(\bar{\pi}) &= \sum_{i=1}^{P} E_t\beta^i[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R)] - E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R)] \\ &= \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 - \theta ak\bar{\pi} + \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i < 0, \qquad (26) \end{aligned}$$

which is the difference between the reputation enforcement and the temptation to cheat for $\pi_t \in \Pi_t$. The penalty function is given by the quadratic and concave curve which is the mirror image (reflection mapping) of $L_\beta(\bar{\pi})$ for $g_1 < \bar{\pi} < g_2$ and equals zero otherwise. The penalty reaches the maximum while $L_\beta(\bar{\pi})$ reaches its minimum at $\bar{\pi} = \frac{\theta ak}{1+a^2\theta}$. Since $W(\bar{\pi}) + L_\beta(\bar{\pi}) = 0$ for all $g_1 < \bar{\pi} < g_2$ due to their mirror image each other, the penalty for the central banker to cheat just equals the difference between the temptation to cheat and the reputation enforcement. When $\bar{\pi} \geq g_2$ or $\bar{\pi} \leq g_1$, the central banker does not have an incentive to cheat since $\lambda(\bar{\pi}, 0)$ is greater than $\pounds(\pi_t^R)$. Thus, no penalty is necessarily imposed to the central bank.

Thus, our optimal contract has some advantages that the existing contracts do not share. Our approach answers some criticisms for the contracting approach. One criticism is that the incentive contract is costly to use. In Walsh's model, since only the socially optimal monetary policy rule π_t^R satisfies the first order condition and the linear contract is contingent on π_t^R which in turn is contingent on production shocks, the central banker will be nevertheless penalized even though the central banker has no incentive to use a cheating monetary policy. Since the production shock x_t can be arbitrary large when x_t is an unbounded random variable, a huge contract cost may be required for implementing a socially optimal monetary policy rule for a very large production shock. A central banker is normally financed by the public and making them pay a pecuniary fine would simply be a reshuffling of tax money. In our approach, however, a contract penalty will be imposed only when the reputation forces alone are not big enough to prevent the central banker from cheating and further she uses an inflation bias of monetary policy $\pi_t \in \Pi_t$. The magnitude of the penalty is in fact the difference between the temptation to cheat and the reputation enforcement, and thus it is the lowest penalty that just discourages the central banker from cheating. Since the central banker has no incentive to cheat, the penalty is actually zero when $\pi_t = \pi_t^R$. In addition, since the sum of the social loss, reputation cost, and contract penalty for the central banker to cheat is the same for any rate of surprise inflation and just equal the social cost at the socially optimal monetary policy rule π_t^R, our contract is robust. Thus, our optimal contract is the most efficient and stable mechanism that implements the socially optimal monetary policy rule, and at the same time it freely responds to production shocks.

It may be remarked that, just like an agent in the usual principal-agent model that satisfies the binding participation constraint, the central banker, as an agent, is indifferent for both the socially optimal monetary policy rule π_t^R and a cheating monetary policy. This, however, may not a problem. Since the central banker in any case cannot benefit from using a surprise inflation monetary policy, but more likely she will be hurt by cheating due to the loss of credibility or pressure of reputation, influence on future promotion, the central banker will choose the socially optimal monetary policy. Even if the credibility, reputation pressure, or negative influence on future promotion does not work, one may give the central banker additional (any positive) amounts of penalty when she cheats, and then the socially optimal monetary policy π_t^R becomes the unique optimal choice to the central banker.

Thus our optimal hybrid mechanism approach suggests that there may be a simple optimal incentive scheme that can solve the time inconsistency problem in monetary policy. When the reputation forces alone do not work, our hybrid mechanism approach presents an optimal contract that has some nice properties that an existing pure contract mechanism may not share. In any case, we provide a lower bound of the penalty for the central banker to implement a socially optimal monetary policy rule. The contract is an optimal contract that has the lowest cost of implementing the socially optimal monetary policy rule. The penalty function W depends solely on surprise inflation rate.

We know that among the approaches that solve the time inconsistency problem, the "reputation" problem is key. If reputation consideration discourages the central banker from attempting surprise inflation, then legal or contracting constraints on central bankers are unnecessary and may be harmful. The result in this section, however, suggests that one can reduce the contract cost of maintaining the stationary inflation policy by combining the reputation impact with the contracting penalty when the reputation enforcement alone cannot solve the time inconstancy problem.

5 Conclusion

In this paper, we have studied the time-inconsistency problem for central bankers. When the reputation enforcement from the public is not large enough to discourage the central bank to use a surprise inflation policy, a contract can be used together with reputation forces to implement the socially optimal monetary policy rule. We presented an optimal hybrid mechanism that combine the reputation approach and contracting approach. This hybrid mechanism discourages the central bank from surprise inflation and gives her full flexibility to respond to output shocks.

Our unified approach of combing reputation and contract has some nice properties that the existing mechanisms may not share. Our results answer the concern that using the incentive contract is very costly. The results obtained in the paper suggest that one can reduce the contract cost of implementing a socially optimal inflation policy by combining the reputational approach with the contracting approach if the reputation enforcement alone cannot solve the time inconstancy problem. Also, unlike the existing optimal contracts, our contract is only contingent on surprise inflation rate, but not on production shocks. The central banker will be punished only when she has an incentive to use an inflation bias of monetary policy. The magnitude of the penalty is in fact the difference between the temptation to cheat and the lowest penalty which just discourages the central banker from cheating, and so it reaches a lower bound of penalty payment. Thus, our hybrid mechanism is an optimal mechanism that has the lowest cost of implementing the socially optimal monetary policy rule, and it therefore is the most efficient way to implement a socially optimal monetary policy. In addition, since the sum of the loss, reputation forces, and the penalty for the central banker to cheat is the same as the social loss at the optimally socially optimal inflation rate even if there are production shocks, our optimal contract is a robust mechanism that implements the socially optimal monetary policy rule. We have also provided a upper bound of penalty that may be lower than the linear contract when inflation rate is greater than $\bar{\pi}^C/2$.

Of course, like the reputation approach, a weakness of our hybrid mechanism is that the penalty is the function of surprise inflation rate, but not contingent on realized inflation rates. Thus, it imposes a stronger information requirement to the government than the existing contracts in the literature since the surprise inflation rate may be hard to be verifiable exactly although one can estimate easily by various existing econometric methods in the literature. In any case, we can similarly give a hybrid mechanism that combines reputation forces and a contract that is contingent on realized inflation rates, but not on surprise rates if one is willing to increase the contract cost.

6 References

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