

A Reputation Strategic Model of Monetary Policy in Continuous Time *

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Abstract

This paper develops a reputation strategic model of monetary policy with a continuous finite or infinite time horizon. By using the optimal stopping theory and introducing the notions of sequentially weak and strong rational expectation equilibria, we show that the time inconsistency problem may be solved with trigger reputation strategies not only for stochastic but also for non-stochastic settings even with a finite horizon. We show the existence of stationary sequentially strong rational expectation equilibrium under some condition, and there always exists a stationary sequentially weak rational expectation equilibrium. Moreover, we investigate the robustness of the sequentially strong rational expectation equilibrium behavior solution by showing that the imposed assumption is reasonable.

Keywords and Phrases: Monetary policy, time-consistency problem, reputation, continuous model.

Journal of Economic Literature Classification Number: E52, E58.

*We wish to thank two anonymous referees for helpful comments and suggestions that improved the exposition of the paper. The second author thanks the National Natural Science Foundation of China (NSFC-70773073) and the Program to Enhance Scholarly and Creative Activities at Texas A&M University as well as from Cheung Kong Scholars Program at the Ministry of Education of China for financial support.

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1 Introduction

Time inconsistency is an interesting problem in macroeconomics in general, and monetary policy in particular. Although technologies, preferences, and information are the same at different time, the policymaker's optimal policy chosen at time t_1 differs from the optimal policy for t_1 chosen at t_0 . The study of time inconsistency is important. It provides positive theories that help us to understand the incentives faced by policymakers and provide the natural starting point for attempts to explain the actual behavior of policymakers and actual policy outcomes.

This problem was first noted by Kydland and Prescott (1977). Several approaches have been proposed to deal with this problem since then. Barro and Gordon (1983) were the first to build a game model to analyze "reputation" of monetary policy. Backus and Driffill (1983) extended the work of Barro and Gordon to a situation in which the public is uncertain about the preferences of the government. Persson and Tabellini (1990) gave an excellent summarization of these models. Al-Nowaihi and Levine (1994) discussed reputation equilibrium in the Barro-Gordon monetary policy game. Haubrich and Ritter (2000) considered the decision problem of a policymaker or government who chooses repeatedly between rule and discretion in the Barro-Gordon model. The second approach is based on the incentive contracting design to monetary policy. Persson and Tabellini (1993), Walsh (1995), Svensson (1997), and Tian (2005) developed models using this approach. The third approach is built on the legislative governance. The major academic contribution in this area was by Rogoff (1985). Among these approaches, the "reputation" problem is key. If reputation consideration discourages the monetary authorities from attempting surprise inflation, then legal or contracting constraints on monetary authorities are unnecessary and may be harmful.

The main questions on reputation are when and how the government chooses inflation optimally to minimize welfare loss, and, whether the punishment on reputation loss can induce the government to keep zero inflation. The conclusions of Barro-Gordon discrete-time models are: First, there exists a zero-inflation Nash equilibrium if the punishment for the government deviating from zero-inflation is large enough. However, this equilibrium is not sequentially rational over a finite time horizon. The only sequentially rational expectation equilibrium is achieved if the government chooses discretionary inflation and the public expects it. Only over an infinite time horizon can one get a low-inflation equilibrium. Otherwise, the government would be sure in the last period to produce the discretionary outcome whatever the public's expectation were and, working backward, would be expected to do the same in the first period. Secondly, there are multiple Nash equilibria and there is no mechanism to choose among them.

Finite time horizon games are more reasonable than infinite ones in the real world. At least we see a government's terms are limited. Many experimental studies of games suggest that there are cooperation equilibrium when the players are told that the game will end. Consequently, how to induce cooperative behavior in a finitely repeated game is an interesting problem even for game theorists. Also, discrete-time methods have some limitations for reputation games. Fudenberg and Levine (1992) showed that, in every equilibrium, the government's payoff when he uses a discretionary rule becomes at least as high as the payoff when he keeps the inflation rate at zero. Cripps, Mailath, and Samuelson (2004) showed that reputation effect is temporary. In addition, in the certainty setting with discrete time, a reputation equilibrium is possible only if the horizon is infinite.

This paper extends the basic Barro-Gordon model of dynamic inconsistent monetary policy with reputation to continuous time. Continuous-time models permit a sharp characterization of the thresholds that trigger an adjustment and the level to which adjustment is made. One particularly nice result in the paper is to get reputation to work in finite time. In contrast to Barro-Gordon model, we assume that output shocks follow a Brownian motion. As Stokey (2005) pointed out, the economic effects of an aggregate productivity shock depend on the investment and hiring/firing response of firms. In situations where fixed costs are important, continuous-time models in which the stochastic shocks follow a Brownian motion or some other diffusion have strong theoretical appeal. Also, the fact that many price changes are large in magnitude suggests that the shock should be a diffusion process.

We study the time inconsistency problem in monetary policy with the continuous finite or infinite time horizon model by using the optimal stopping theory in the stochastic differential equations literature.¹ The optimal stopping theory can cover many dynamic economic applications under uncertainty. The optimal stopping theory, though relatively complete in its theoretical development, has not yet been widely applied in economics. By using the optimal stopping theory and introducing the notions of sequentially weak and strong rational expectation equilibria, we show that the time inconsistency problem may be solved with trigger reputation strategies within our setting not only for stochastic but also for non-stochastic settings even with a finite horizon. We provide the conditions for the trigger reputation strategy to be a stationary sequentially strong rational expectation equilibrium, i.e., the government will keep the inflation rate at zero when the public's behavior is characterized by strong rational expectation and uses the trigger strategy. We further show that there always exists a

¹The discussion about the optimal stopping theory can be found in Friedman (1979) and Øksendal (1998), Tsitsiklis and Roy (1999).

stationary sequentially weak rational expectation equilibrium at which the government will keep the inflation at zero.

The results obtained in the paper are sharply contrasted to the negative results from the certainty setting with a discrete time horizon. Our results on the existence of the stationary zero inflation policy as an equilibrium solution are also true for the non-stochastic continuous finite horizon settings, which demonstrate the advantage of our continuous time model compared to the non-stochastic discrete time finite horizon model discussed in the literature. Thus, a striking advantage of using a continuous-time formulation is that it yields a solution to the time inconsistency problem whereas a discrete-time counterpart does not. Why does the much more complicated continuous-time formulation yield a positive result that the discrete-time formulation could not? Intuitively speaking, it is because, in continuous time, the government has an option to change a policy in any instant time while, in the discrete-time formulation, the government can change a policy only in each integer time. The solution to the continuous-time formulations can be viewed as the sum of the solutions to the discrete-time formulations for infinitely many small stopped subintervals. Thus, the embedded option in continuous-time formulation may appear to explain why the continuous-time formulation can yield a solution to the time inconsistency while the discrete-time versions in the existing literature fails.² Other work on time consistency in continuous-time can also found in Faingold and Sannikov (2007) and Haubrich and Ritter (2004).

We also investigate the robustness of the equilibrium behavior by showing that the imposed assumption is reasonable. We can always expect a stationary zero-inflation outcome by the sequentially strong rational expectation behavior so that the rational expectation reputation can discourage the monetary authority from attempting surprise.

The remainder of the paper is organized as follows. Section 2 will set up the model and provides a solution for the optimal stopping problem faced by the government. In Section 3, we study the equilibrium behavior. The robustness of this monetary game is discussed in Section 4. Section 5 gives the conclusion. All the proofs and how to solve the optimal stopping problem are given in Appendix.

²Such an advantage of the continuous-time formulations can be also found in other fields such as the principal-agent literature. For instance, Holmstrom and Milgrom's (1987) continuous-time Brownian model not only generate the second-best solution, but their solution is remarkably simple. Schättler and Sung (1997) provided the above explanation for the principal-agent models.

2 Model

2.1 The Setup

We consider a continuous time game theoretical model with two players: a central government and the public. Let T be the lifetime of the government that can be finite or infinite and $L[0, T]$ the class of Lebesgue integral functions defined on $[0, T]$. The government's action (strategy) space is $R_+ \times L[0, T]$, a generic element of which is denoted by $(\tau, \{\pi_t\}_{t \in T})$ where τ is the time that the government changes its monetary policy from the zero-inflation rule to a discretion rule and π_t is the inflation rate chosen by the government at time t . The public's strategy space is $L[0, T]$, a generic element of which is denoted by $(\{\pi_t^e\}_{t \in T})$ where π_t^e is the expected inflation rate formed by the public at time t .

Suppose that the government commits an inflation rate $\pi_0 = 0$ at the beginning, and the public believes it so that $\pi_0^e = \pi_0 = 0$. The government has an option to switch from the zero-inflation to a discretion rule $\pi_t \neq 0$ at any time t between 0 and T . However, once he changes his policy, he loses his reputation.

For simplicity of exposition, as standard, we assume the government's payoff function is given by a quadratic discounted expected loss function³

$$\Lambda = E \int_0^T e^{-\rho t} \left[\frac{1}{2} \theta (y_t - \bar{y}_t - k)^2 + \frac{1}{2} \pi_t^2 \right] dt \quad (1)$$

where ρ is the discount factor with $0 < \rho < 1$, y_t is aggregate output, \bar{y}_t is the economy's natural rate of output, and θ is a positive constant that represents the relative weight the government puts on output expansions relative to inflation stabilization. Without loss of generality, the target inflation rate π is assumed to be zero.⁴ (1) is a typical macro welfare function that has played an important role in the literature, and means that the government desires to stabilize both output around $\bar{y}_t + k$, which exceeds the economy's equilibrium output of \bar{y}_t by a constant k , and inflation around zero.

The government's objective is to minimize this discounted expected loss subject to the constraint imposed by a Lucas-type aggregate supply function, the so-called Phillips curve, which describes the relationship between output and inflation in each period:

$$y_t - \bar{y}_t = a (\pi_t - \pi_t^e) + X_t, \quad (2)$$

³The similar results could be obtained for a more general loss function. However, the analysis and solutions with more general loss functions are much complicated.

⁴The results obtained in the paper will continue to be true if the monetary authority has a target inflation that differs from zero.

where a is a positive constant that represents the effect of a money surprise on output, i.e., the rate of the output gain from the unanticipated inflation so that the larger is a , the greater is the central bank's incentive to inflate, and X_t is the shock at time t that follows an Ito diffusion process of the form:

$$dX_t = \sigma dB_t, \quad X_0 = x,$$

which is a special case of the general Ito diffusion:

$$dX_t = b(X_t)dt + \sigma(X_t)dB_t$$

with $b(X_t) = 0$ ⁵ and $\sigma(X_t) = \sigma$. Here, B_t is 1-dimensional Brownian Motion (or Wiener process) and σ is the diffusion coefficient with $\sigma < \infty$.⁶ A diffusion process is a continuous version of the random walk,⁷ which is a solution to a stochastic differential equation. It is a continuous-time Markov process with continuous sample paths.

The public has complete information about the policymaker's objective. It is assumed that the public forms his expectations rationally, and thus the assumption of rational expectation may implicitly define the loss function for the public as $E[\pi_t - \pi_t^e]^2$. The public's objective is to minimize this expected inflation error. Given the public's understanding of the government's decision problem, its choice of π^e is optimal. Thus, the public always accomplishes its goal by setting $\pi_t^e = E\pi_t$.

When the government controls the rate of inflation, it may deviate from the zero inflation rate. To see this, we examine the following "one-shot" game. The single-period loss function ℓ_t for the government is:

$$\begin{aligned} \ell_t(\pi_t, \pi_t^e) &= \frac{1}{2}\theta(y_t - \bar{y}_t - k)^2 + \frac{1}{2}\pi_t^2 \\ &= \frac{1}{2}\theta[a(\pi_t - \pi_t^e) + X_t - k]^2 + \frac{1}{2}\pi_t^2. \end{aligned} \quad (3)$$

⁵As for our problem, one aspect of the drift effect is the random nature of transmitting shocks from one period to the next. In aggregate shock, the drift will not have much effect in each period because the random nature of the process will tend to average out. The assumption $b(X_t) = 0$ thus may be justified.

⁶A standard Brownian motion (or a standard Wiener process) is a stochastic process $\{B_t\}_{t \geq 0+}$ if (i) $B_0 = 0$, (ii) it is almost surely continuous, (iii) it has stationary independent increments, and (iv) $B_{t+s} - B_s$ has the normal distribution with expected value 0 and variance t . The term *independent increments* means that for any choice of nonnegative real numbers, $0 \leq s_1 \leq t_1 \leq s_2 \leq t_2$, $B_{t_1} - B_{s_1}$ and $B_{t_2} - B_{s_2}$ are independent random variables. The term *stationary increments* for any $0 < s, t < \infty$ the distribution of the increment $B_{t+s} - B_s$ has the same distribution as $B_t - B_0 = B_t$. Thus, X_t is a continuous-time Markov process with continuous sample paths.

⁷A Brownian motion can be viewed as the limit of discrete-time random walks as the time interval and the step size shrink together in a certain way.

The discretionary solution is given by Nash equilibrium. Then the government minimizes ℓ_t by taking π_t^e as given, and thus we have the best response function for the policymaker:

$$\pi_t^D = \frac{a\theta}{1 + a^2\theta} (a\pi_t^e - X_t + k). \quad (4)$$

The public is assumed to understand the incentive facing the government so they use (4) in forming their expectations about inflation so that

$$\pi_t^e = E\pi_t^D = \frac{a\theta}{1 + a^2\theta} (a\pi_t^e - EX_t + k). \quad (5)$$

Solving for π_t^e and π_t^D , we get the unique Nash equilibrium:

$$\pi_t^D = a\theta k - \frac{a\theta}{1 + a^2\theta} (X_t + a^2\theta EX_t) \quad (6)$$

$$\pi_t^{e*} = E\pi_t^{D*} = a\theta(k - EX_t) \quad (7)$$

Thus, as long as $EX_t \neq k$, the policymaker has incentives to use the discretion rule although the loss at $\pi_t^e = \pi_t = 0$ is lower than at $\pi_t^{e*} = E\pi_t^{D*}$.

Note that, if $X_t = k$ a.s. for $t \geq 0$, the unique Nash equilibrium of the “one-shot” game for the public and the government, is $\pi_t^{e*} = \pi_t^* = 0$ a.s., and thus, the time-inconsistency problem will not appear. To make the problem non-trivial, without loss of generality, we assume that $X_t \neq k$ a.s. for $t \geq 0$ in the rest of the paper.

A potential solution to the above time inconsistency problem is to force the government to bear some consequence penalties if it deviates from its announced policy of low inflation. One of such penalties that may take is a loss of reputation, and so, in this paper, we will adopt the reputation approach that incorporates notions of reputation into a repeated-game framework to avoid this time consistency problem. If the government deviates from the low-inflation solution, credibility is lost and the public expects high inflation in the future. That is, the public expects zero-inflation as long as government has fulfilled the inflation expectation in the past. However, if actual inflation exceeds what was expected, the public anticipates that the policymaker will apply discretion in the future. So the public forms their expectation according to the trigger strategy: Observing “good” behavior induces the expectation of continued good behavior and a single observation of “bad” behavior triggers a revision of expectations.

2.2 The Optimal Stopping Problem for Government

In order to solve the time inconsistency problem by using the reputation approach, we first incorporate the government’s loss minimization problem into a general optimal stopping time problem. During any time in $[0, T]$, the policymaker has the right to reveal his type (discretion

or zero-inflation). Since he has the right but not the obligation to reveal his type, we can think it is an option for the policymaker. So the policymaker's decision problem is to choose a best time $\tau \in [0, T]$ to exercise this option.

The policymaker considers the following time-inhomogeneous optimal stopping problem: Find τ^* such that

$$L^*(x) = \inf_{\tau} E^x \left[\int_0^{\tau} f(t, X_t) dt + g(\tau, X_{\tau}) \right] = E^x \left[\int_0^{\tau^*} f(t, X_t) dt + g(\tau^*, X_{\tau^*}) \right], \quad (8)$$

where E^x denotes the expectations conditional upon the realization of all information at time 0,

$$f(s, X_t) = \frac{1}{2} \theta e^{-\rho s} (X_t - k)^2 \quad (9)$$

is the policymaker's instantaneous loss function that is clearly Lipschitz continuous when he uses the zero-inflation rate and

$$g(s, X_{\tau}) = e^{-\rho s} E^{X_{\tau}} \left[\int_s^T e^{-\rho(t-s)} \left[\frac{\theta}{2} [a(\pi_t^D - \pi_t^e) + X_t - k]^2 + \frac{\pi_t^{D^2}}{2} \right] dt \right] \quad (10)$$

is the policymaker's expected loss function in which he begins to use the discretion rule at time s . Here $E^{X_{\tau}}$ denotes the expectations conditional upon the realization of all information up to time τ . Note that $g(\cdot) \geq 0$ since the loss function $\ell_t \geq 0$.

We assume that the public uses stationary strategy: $\pi_t^e = \pi^e$ for $t \geq \tau$. To compute $g(\tau, X_{\tau})$, putting (4) into (10), we have

$$\begin{aligned} g(\tau, X_{\tau}) &= e^{-\rho\tau} E^{X_{\tau}} \left[\int_{\tau}^T e^{-\rho(t-\tau)} \left[\frac{\theta}{2} [a(\pi_t^D - \pi^e) + X_t - k]^2 + \frac{1}{2} \pi_t^{D^2} \right] dt \right] \\ &= \frac{1}{2} \frac{\theta}{1 + a^2\theta} e^{-\rho\tau} E^{X_{\tau}} \left[\int_{\tau}^T e^{-\rho(t-\tau)} (-X_t + k + a\pi^e)^2 dt \right] \\ &= \frac{\theta}{2(1 + a^2\theta)} e^{-\rho\tau} \cdot \left\{ E^{X_{\tau}} \left[\int_{\tau}^T e^{-\rho(t-\tau)} X_t^2 dt \right] - 2(k + a\pi^e) E^{X_{\tau}} \left[\int_{\tau}^T e^{-\rho(t-\tau)} X_t dt \right] \right. \\ &\quad \left. + (k + a\pi^e)^2 E^{X_{\tau}} \left[\int_{\tau}^T e^{-\rho(t-\tau)} dt \right] \right\} \quad (11) \end{aligned}$$

In Appendix, we will solve the optimal stopping problem (8) and give the condition for $\tau^* = T$ to be the optimal stopping time.

3 The Equilibrium Behavior of the Monetary Policy Game

In order to study the equilibrium behavior of the monetary policy game, we first give the following lemma that shows that the government will keep the zero-inflation policy when the public uses trigger strategies and reputation penalties imposed by the public are large enough.

Lemma 1 Let $\tau = \inf\{s > 0 : \pi_s \neq 0\}$. Then, for any trigger strategy of the public, $\{\pi_t^e(x)\}$, which has the form of

$$\pi_t^e = \begin{cases} 0 & \text{if } 0 \leq t < \tau \\ \pi^e(x) \in \{h : (x - k - ah)^2 > (1 + a^2\theta)(x - k)^2\} & \text{if } t \geq \tau \end{cases},$$

discourages the policymaker from attempting surprise inflation.

There is an intuition for the condition in Lemma 1. The RHS is the loss when the government undertakes discretion. The LHS is the loss when the public thinks the government will inflate, but instead the government continues with zero inflation. In effect, this condition rules out the perverse case that, when contemplating a switch, the bigger temptation is to stick with zero inflation.

Although there are (infinitely) many trigger strategies given in Lemma 1 that can discourage the policymaker from attempting surprise inflation, most of them are not optimal for the public in terms of minimizing the public's expected inflation error: $E(\pi_t - \pi_t^e)^2$. To rule out non-optimal strategies, we have to impose some assumptions how the public form an expectation and what an equilibrium solution should be used to describe the public's self-interested behavior. Different assumptions on the public's behavior may result in different optimal solutions. In the following, we introduce two types of sequentially rational expectation equilibrium solution concepts.

Let $\{\mathcal{F}_t\}$ be a filtration, i.e., a nondecreasing family $\{\mathcal{F}_t : t \geq 0\}$ of sub- σ -fields of \mathcal{F} : $\mathcal{F}_s \subset \mathcal{F}_t \subset \mathcal{F}$ for $0 \leq s < t < \infty$, which is assumed to be generated by the process itself, i.e., $\mathcal{F}_t := \sigma(X_s : 0 \leq s \leq t)$. Then, \mathcal{F}_t can be regarded the set of accumulated information up to time t .

Suppose the government knows the distribution of the shock, X_t , exactly, that is,

$$d\tilde{P}^G = dP,$$

where \tilde{P}^G is the belief of the government for the movement of the shock, P is the measure of the shock.

We suppose that the public does not know the distribution of the shock, but its belief \tilde{P}^P is absolutely continuous with respect to P , which means that if an event does not occur in probability, then the public will believe that this event will not happen.⁸

⁸Formally, $\tilde{P}^P(A) = 0$ for each $A \in \mathcal{F}_t$ such that $P(A) = 0$. This condition is needed to guarantee the existence of Radon-Nikodym derivative and apply Radon-Nikodym Theorem below.

Then, by Radon-Nikodym Theorem (Lipster & Ahiryaev 2001, p. 13), there exist Radon-Nikodym derivatives, $M(t)$, such that

$$d\tilde{P}^P = M(t)dP, (a.s.),$$

and $M(t)$ is a martingale. This means that, whenever new information becomes available, the belief of the public is adjusted. We can interpret $M(t)$ as the information structure of the society, it is a measurement of how the public knows the real shock.

We suppose that $M(t)$ is P -square-integrable and X_t is \tilde{P}^P -integrable. We also suppose that $\langle X_t, M(t) \rangle = 0^9$. Heuristically, this assumption can be interpreted as: the history of the shock can't help the public to predict the movement of the future shock.¹⁰

We denote by \tilde{E} the expectation operator with respect to \tilde{P}^P .

A strategy $(\tau, \{\pi_t, \pi_t^e\})$ is said to be a *sequentially strong rational expectation equilibrium strategy* for the dynamic model defined above if

(1) the belief of the public for the movement of the natural rate X_t , \tilde{P}^P , satisfies

Bayes' rule:

$$\tilde{E}[X_t | \mathcal{F}_s] = \frac{1}{M(s)} E[X_t M(t) | \mathcal{F}_s] \quad (12)$$

for all $s < t$;

(2) The expectation of the public is strong rational: $\pi_t^e = E^{X_s} \pi_t^D := \tilde{E}[\pi_t^D | \mathcal{F}_s]$ for all $s < t$;

(3) it optimizes the objectives of the public and the government.

A strategy $(\tau, \{\pi_t, \pi_t^e\})$ is said to be a *sequentially weak rational expectation equilibrium strategy* for the dynamic model defined above if

(1) the belief of the public for the movement of the natural rate X_t , \tilde{P}^P , satisfies

Bayes' rule:

$$\tilde{E}[X_t | \mathcal{F}_0] = \frac{1}{M(0)} E[X_t M(t) | \mathcal{F}_0]; \quad (13)$$

(2) The expectation of the public is weak rational: $\pi_t^e = E^x \pi_t^D := \tilde{E}[\pi_t^D | \mathcal{F}_0]$ for all t ;

(3) it optimizes the objectives of the public and the government.

⁹ $\langle X, Y \rangle$ is cross-variation, which is defined by $\langle X_t, Y_t \rangle = \lim_{|\Pi| \rightarrow 0} \sum_{1 \leq k \leq m} (X_{t(k)} - X_{t(k-1)})(Y_{t(k)} - Y_{t(k-1)})$, where X_t and Y_t are square-integrable, and $\Pi = [t_0, t_1, \dots, t_m]$ is a partition of $[0, t]$.

¹⁰Note that, if one assumes that the public knows the distributions of the shocks, X_t , exactly, then $M(t) = 1$. This is a usual assumption made in the literature.

The difference between sequentially strong rational expectation equilibrium and sequentially weak rational expectation equilibrium is that the sequentially weak rational expectation equilibrium uses the information only at time 0 to form the public's belief and expectation on the government's policy while the sequentially strong rational expectation equilibrium uses accumulated information up to the present to form the public's belief and expectation on the government's policy. Thus, the sequentially weak rational expectation equilibrium, in general, is a weaker equilibrium solution concept to describe the public's behavior. This implies that every sequentially strong rational expectation equilibrium is clearly a sequentially weak rational expectation equilibrium, but the reverse may not be true. However, when the natural rates, $\{X_t\}$, are non-stochastic, these two equilibrium solutions are equivalent.

Remark 1 While the sequentially rational expectation equilibrium concepts share the two common basic properties of discrete time equilibrium concepts such as Sequential or Perfect Bayes Equilibrium or other recent continuous time equilibrium concepts such as the public sequential equilibria introduced in Faingold and Sannikov (2007), which require at equilibrium that (1) the system of beliefs be derived through Bayes's rule whenever possible and (2) the strategy of each player maximizes his expected payoffs, our equilibrium concepts also require that rational expectation should be satisfied.

Now we use these two types of sequentially rational expectation equilibria to study the time consistency problem in monetary policy. Propositions 1 and 2 below show the existence of such equilibria in which the government will keep the inflation rate at zero.

$$A = \{z : [(x - k) + a^2\theta(z - k)]^2 > (1 + a^2\theta)(x - k)^2\}. \quad (14)$$

As we see below, an element z in set A is used to determine the public's expected inflation rate that is given by $\pi_t^e = a\theta(k - z)$ for $t \geq \tau$ where τ is the first time that the government changes its policy from zero inflation to a discretion rule. When $X_\tau \in A$, it means that public's expected inflation rate $\pi_t^e = a\theta(k - z)$ is big enough in absolute value so that the government has no incentive to make a surprise inflation.

Proposition 1 *Let $(\tau, \{\pi_s\})$ be the strategy of the government, where τ is the first time that the government changes its policy from zero-inflation to discretion rule, i.e., $\tau = \inf\{s > 0 : \pi_s \neq 0\}$. Let the strategy of the public $\{\pi_t^e\}$ be given by*

$$\pi_t^e = \begin{cases} 0 & \text{if } 0 \leq t < \tau \\ a\theta(k - X_\tau) & \text{if } t \geq \tau \end{cases}. \quad (15)$$

If $X_\tau \in A$, then $(\tau^*, \{\pi_t^*, \pi_t^{e*}\})$ with $\tau^* = T$, $\pi_t^* = 0$ and $\pi_t^{e*} = 0$ for all $t \geq 0$ is a sequentially strong rational expectation equilibrium strategy for the policymaker and the public.

Thus, Proposition 1 implies that, as long as $|X_\tau|$ is big enough, the public can use the trigger strategy given by (15) to induce a stationary zero-inflation sequentially strong rational expectation equilibrium. Note that the conclusion is based on the assumption that $X_\tau \in A$. If this assumption is not satisfied, we may not have the stationary zero-inflation result. Nevertheless, Proposition 3 in the next section will show that the stationary zero-inflation sequentially strong rational expectation equilibrium is stochastically stable in the sense that the expected exiting time from A is infinity, i.e., as $x \in A$, the public and the government will have a strong belief $X_i \in A$ for all $t \in (0, T]$.

The sequentially strong rational expectation equilibrium has imposed a restrictive assumption on the public's self-interested behavior. If the public's self-interested behavior is described by the sequentially weak rational expectation equilibrium, the stationary zero inflation strategy is always an equilibrium that prevents the government deviating from the zero-inflation rate.

Proposition 2 *Let $(\tau, \{\pi_s\})$ be the strategy of the government, where $\tau = \inf\{s > 0 : \pi_s \neq 0\}$. Let the strategy of the public $\{(\pi_t^e)\}$ be given by*

$$\pi_t^e = \begin{cases} 0 & \text{if } 0 \leq t < \tau \\ a\theta(k - x) & \text{if } t \geq \tau \end{cases}.$$

Then, the stationary zero-inflation policy, i.e., τ^ , $\pi_t^* = 0$ and $\pi_t^{e*} = 0$ for all $t \geq 0$, is a sequentially weak rational expectation equilibrium strategy for the policymaker and the public.*

Thus, the public can use a trigger strategy to induce a stationary zero-inflation equilibrium outcome that is sequentially weak rational.

When shocks X_t becomes nonstochastic, i.e., $X_t = X_0 = x$, the sequentially strong rational expectation equilibrium and sequentially weak rational expectation equilibrium are the same. Thus, from Propositions 1 and 2, we have the following corollary that shows the existence of a stationary zero inflation equilibrium.

Corollary 1 *Let $(\tau, \{\pi_s\})$ be the strategy of the government, where $\tau = \inf\{s > 0 : \pi_s \neq 0\}$. Let the strategy of the public $\{(\pi_t^e)\}$ be given by*

$$\pi_t^e = \begin{cases} 0 & \text{if } 0 \leq t < \tau \\ a\theta(k - x) & \text{if } t \geq \tau \end{cases}.$$

Then, the stationary zero-inflation policy, i.e., τ^ , $\pi_t^* = 0$ and $\pi_t^{e*} = 0$ for all $t \geq 0$, is a sequentially (strong) rational expectation equilibrium strategy for the policymaker and the public.*

This possibility result on the existence of the stationary zero inflation policy as an equilibrium shows the advantage of our nonstochastic continuous finite horizon setting compared to the nonstochastic discrete time finite horizon settings discussed in the literature. It is well known that, in the certainty setting with discrete time, a reputation equilibrium is possible only if the horizon is infinite. Otherwise, the government would be sure in the last period to produce the discretionary outcome whatever the public's expectation were and, working backward, would be expected to do the same in the first period. Thus, our results are sharply contrasted to the negative results from the certainty setting with discrete time horizon.

4 Robustness of Equilibrium Solutions

In this section we study the robustness of sequentially strong rational expectation equilibrium. In order to get the stationary zero-inflation sequentially strong rational expectation equilibrium, we imposed the assumption that $X_\tau \in A$ where τ is the first time that the government changes its policy from zero-inflation to discretion rule. It might appear that the result in Proposition 1 is sensitive to this assumption. Is this assumption reasonable? The following proposition shows that the result is quite robust in the sense that the expected first exiting time from A will be infinite.

Proposition 3 *Let $A = \{z : [(x - k) + a^2\theta(z - k)]^2 > (1 + a^2\theta)(x - k)^2\}$, and let $\eta = \inf\{t > 0 : X_t \notin A\}$ be the first time X_t exits from A . Then, we have*

$$E^x[\eta] = \infty$$

for all $x \in R$.

Proposition 3 implies that, because the expected exiting time from A is infinite, i.e., $E^x[\eta] = \infty$ for all $x \in R$, the policymaker will have the belief that the future shocks will stay in A forever, and consequently they will likely make decisions and behave according to this belief. As a result, the stationary zero-inflation sequentially strong rational expectation equilibrium will likely appear in the game when the public has the same belief as the government.

Summarizing the above discussion, we conclude that for any initial shock x , one can always expect all future shocks X_t are in B and thus can expect a stationary zero-inflation outcome by the sequentially strong rational expectation behavior. Thus, for this continuous time dynamic stochastic game, the sequentially strong rational expectation equilibrium behavior may be well justified.

5 Conclusion

In this paper, we examine the equilibrium behavior of the time inconsistency problem in a continuous time stochastic world. We introduce the notions of sequentially weak and strong rational expectation equilibria, and show that the time inconsistency problem may be solved with trigger reputation strategies not only for stochastic but also for non-stochastic settings even with finite horizon. We provide the condition for the existence of stationary sequentially strong rational expectation equilibrium, and showed that there always exists a stationary sequentially weak rational expectation equilibrium, at which the government will always keep the inflation at zero. Thus, the reputation can discourage the monetary authority from attempting surprise inflation. Furthermore, we investigate the robustness of the sequentially strong rational expectation equilibrium behavior solution by showing that the imposed assumption is reasonable and the sequentially rational expectation equilibrium is stochastically stable.

We end the paper by mentioning some possible extension. Our model is mainly in the framework of the Barro-Gordon monetary policy game. However, much of monetary policy is discussed today around one version or another of the New Keynesian model. By carrying the same discussion in the framework of the New Keynesian model, we may get some interesting results. For instance, the paper rests on the specific assumption by which the public has the rational expectation. In the New Keynesian model, it is the forward looking nature of the price mechanism and the fact that private agents are bound to form expectations about monetary policy that opens the door to surprises from the central bank and reintroduces that way the issue of time inconsistency.

Appendix

Solving the Optimal Stopping Problem

We first calculate the conditional expectation for X_t^2 and X_t . Let A be the characteristic operator of Ito diffusion $dX_t = b(X_t)dt + \sigma(X_t)dB$ (with $b = 0$). Then

$$\begin{aligned} Af &= \sum_i b_i \frac{\partial f}{\partial x_i} + \frac{1}{2} \sum_{i,j} (\sigma\sigma^T)_{i,j} \frac{\partial^2 f}{\partial x_i \partial x_j} \\ &= \frac{1}{2} \sum_{i,j} (\sigma\sigma^T)_{i,j} \frac{\partial^2 f}{\partial x_i \partial x_j} \end{aligned}$$

Then, by Dynkin formula (Øksendal 1998, p. 118), we have

$$E^{X_\tau} [X_t] = X_\tau + E^{X_\tau} \left[\int_\tau^t AX_s ds \right] = X_\tau \quad (16)$$

$$E^{X_\tau} [X_t^2] = X_\tau^2 + E^{X_\tau} \left[\int_\tau^t AX_s^2 ds \right] = X_\tau^2 + \sigma^2(t - \tau). \quad (17)$$

Substituting (16) and (17) into (11), we have

$$\begin{aligned} g(\tau, X_\tau) &= \frac{1}{2} \frac{\theta}{1 + a^2\theta} \left\{ \sigma^2 \left[\frac{1}{\rho^2} (e^{-\rho\tau} - e^{-\rho T}) - \frac{1}{\rho} (T - \tau) e^{-\rho T} \right] \right. \\ &\quad \left. + (X_\tau - k - a\pi^e)^2 \frac{1}{\rho} (e^{-\rho\tau} - e^{-\rho T}) \right\}. \end{aligned} \quad (18)$$

Note that, if we define

$$\begin{aligned} f_1(s, X_t) &= -f(s, X_t), \\ g_1(s, X_\tau) &= -g(s, X_\tau), \end{aligned}$$

then the loss minimization problem in (8) can be reduced to the following maximization problem:

Find τ^* such that

$$\begin{aligned} G_0^*(x) &= \sup_{\tau \in [0, T]} E^x \left[\int_0^\tau [-f(t, X_t)] dt - g(\tau, X_\tau) \right] \\ &= \sup_{\tau \in [0, T]} E^x \left[\int_0^\tau f_1(t, X_t) dt + g_1(\tau, X_\tau) \right]. \end{aligned} \quad (19)$$

Since $E^x \left[\int_0^\tau f(t, X_t) dt + g(\tau, X_\tau) \right] < \infty$ for all $\tau \in [0, T]$ and

$$\left\{ \int_0^\tau f(t, X_t) dt + g(\tau, X_\tau); \tau \text{ stopping time} \right\}$$

is uniformly integrable, the problem (19) is well-defined and can be solved by the standard optimal stopping theory (cf. Øksendal, 1998, p. 207).

We now use the optimal stopping approach to solve the optimization problem (19).

In order to solve the government's optimization problem (19) by using a standard framework of the optimal stopping problem involving an integral (cf. Øksendal (1998, p. 213)), we make the following transformations: Let

$$W_\tau = \int_0^\tau f_1(t, X_t) dt + w, \quad w \in R$$

and define the Ito diffusion $Z_t = Z_t^{(s,x,w)}$ in R^3 by

$$Z_t = \begin{bmatrix} s + t \\ X_t \\ W_t \end{bmatrix}$$

for $t \geq 0$. Then

$$dZ_t = \begin{bmatrix} dt \\ dX_t \\ dW_t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -\frac{1}{2}\theta e^{-\rho t}(X_t - k)^2 \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sigma \\ 0 \end{bmatrix} dB_t, \quad Z_0 = (s, x, w).$$

So Z_t is an Ito diffusion starting at $z := Z_0 = (s, x, w)$. Let $R^z = R^{(s,x,w)}$ denote the probability distribution of $\{Z_t\}$ and let $E^z = E^{(s,x,w)}$ denote the expectation with respect to R^z . In terms of Z_t the problem (19) can be written

$$G_0^*(x) = G^*(0, x, 0) = \sup_\tau E^{(0,x,0)}[W_\tau + g_1(\tau, X_\tau)] = \sup E^{(0,x,0)}[G(Z_\tau)]$$

which is a special case of the problem

$$G^*(s, x, w) = \sup_\tau E^{(s,x,w)}[W_\tau + g_1(\tau, X_\tau)] = \sup E^{(s,x,w)}[G(Z_\tau)]$$

with

$$G(z) = G(s, x, w) := w + g_1(s, x).$$

Then, for

$$\begin{aligned} f_1(s, x) &= -\frac{1}{2}\theta e^{-\rho \cdot s}(x - k)^2 \\ g_1(s, x) &= -\frac{1}{2} \frac{\theta}{1 + a^2\theta} \left\{ \sigma^2 \left[\frac{1}{\rho^2} (e^{-\rho s} - e^{-\rho T}) - \frac{1}{\rho} (T - s)e^{-\rho T} \right] \right. \\ &\quad \left. + (x - k - a\pi e)^2 \frac{1}{\rho} (e^{-\rho s} - e^{-\rho T}) \right\} \end{aligned}$$

and

$$G(s, x, w) = w + g_1(s, x),$$

the characteristic operator A_Z of Z_t is given by

$$\begin{aligned}
A_Z G &= \frac{\partial G}{\partial s} + \frac{1}{2} \sigma^2 \frac{\partial^2 G}{\partial x^2} - \frac{1}{2} \theta e^{-\rho s} (x-k)^2 \frac{\partial G}{\partial w} \\
&= \frac{1}{2} \frac{\theta}{1+a^2\theta} (x-k-a\pi^e)^2 e^{-\rho s} - \frac{1}{2} \theta (x-k)^2 e^{-\rho s} \\
&= \frac{1}{2} \frac{\theta}{1+a^2\theta} [(x-k-a\pi^e)^2 - (1+a^2\theta)(x-k)^2] e^{-\rho s}.
\end{aligned} \tag{20}$$

Let

$$U = \{(s, x, w) : G(s, x, w) < G^*(s, x, w)\}$$

and

$$V = \{(s, x, w) : AG(x) > 0\}.$$

Then, by (20) we have

$$\begin{aligned}
V &= \{(s, x, w) : A_Z G(s, x, w) > 0\} \\
&= R \times \{x : (x-k-a\pi^e)^2 > (1+a^2\theta)(x-k)^2\} \times R.
\end{aligned} \tag{21}$$

Remark 2 Øksendal (1998, p. 205) shows that: $V \subset U$, which means that it is never optimal to stop the process before it exits from V . If we choose a suitable $\pi^e(x)$ such that $(x-k-a\pi^e(x))^2 > (1+a^2\theta)(x-k)^2$, then we have $U = V = R \times \{(-\infty, k) \cup (k, \infty)\} \times R$. Therefore, any stopping time less T will not be optimal for all $(s, x, w) \in V$, and thus $\tau^* = T$ is the optimal stopping time. We will use this fact to study the time inconsistency problem of the monetary policy game in the following sections.

Proof of Lemma 1

For each $x \in R$, if we choose any $\pi^e \in \{h : (x-k-ah)^2 > (1+a^2\theta)(x-k)^2\}$, we have

$$(x-k-a\pi^e(x))^2 > (1+a^2\theta)(x-k)^2 \quad \text{for all } x \in R.$$

Then, V in (21) becomes $V = R \times \{(-\infty, k) \cup (k, \infty)\} \times R$, and thus any stopping time less T is not optimal for the government. Hence, $\tau^* = T$. Thus, when the public applies this trigger strategy, it is never optimal for government to stop the zero-inflation policy. Q.E.D

Proof of Proposition 1

To prove $(\tau, \{\pi_t, \pi_t^e\})$ results in a sequentially strong rational expectation equilibrium, $\tau^* = T$, $\pi_t^* = 0$ and $\pi_t^{e*} = 0$ for all $t \geq 0$, we need to show that (1) it satisfies Bayes' rule, (2) the strong rational expectation condition holds: $\pi_t^e = E^{X_\tau} \pi_t^D := \tilde{E}[\pi_t^D | \mathcal{F}_\tau]$, (3) $\pi_t^e \in \{h : (x-k-$

$ah)^2 > (1 + a^2\theta)(x - k)^2\}$, and (4) $(\tau^*, \{\pi_t^*, \pi_t^{e*}\})$ optimizes the objectives of the public and the government.

We first claim that the public updates its belief by Bayes' Rule. Indeed, since $M(t)$ is a martingale and, for $s < t$, X_t is a \tilde{P}^P -integrable random variable, then, by Lemma of Shreve & Kruzhilin (1999, p. 438), the Bayes' Rule holds:

$$\tilde{E}[X_t|\mathcal{F}_s] = \frac{1}{M(s)}E[X_tM(t)|\mathcal{F}_s].$$

To show $\pi_t^e = E^{X_\tau}\pi_t^D$, first note that X_t and $M(t)$ are square-integrable martingale, using the fact that $X_tM(t) - \langle X_t, M(t) \rangle$ is a martingale (Karatzas & Shreve(1991, p. 31)) and the assumption $\langle X_t, M(t) \rangle = 0$. We can get that $X_tM(t)$ is a martingale, by Bayes' Rule:

$$\tilde{E}[X_t|\mathcal{F}_\tau] = \frac{1}{M(\tau)}E[X_tM(t)|\mathcal{F}_\tau] = \frac{1}{M(\tau)}X_\tau M(\tau) = X_\tau.$$

which means $\{X_t\}$ is also a martingale under \tilde{P}^P . Since the policymaker's best response function is given by

$$\pi_t^D = \frac{a\theta}{1 + a^2\theta}(a\pi_t^e - X_t + k),$$

$\{X_t\}$ is a martingale under \tilde{P}^P , and $\pi_t^e = a\theta(k - X_\tau)$ is complete information at time t , we have

$$\begin{aligned} E^{X_\tau}\pi_t^D &= E^{X_\tau}\frac{a\theta}{1 + a^2\theta}(a\pi_t^e - X_t + k) \\ &= \frac{a\theta}{1 + a^2\theta}(a\pi_t^e - E^{X_\tau}X_t + k) \\ &= \frac{a\theta}{1 + a^2\theta}(a\pi_t^e - X_\tau + k). \end{aligned} \tag{22}$$

Substituting $\pi_t^e = a\theta(k - X_\tau)$ into (22), we have $E^{X_\tau}\pi_t^D = \frac{a\theta}{1 + a^2\theta}[a^2\theta(k - X_\tau) - X_\tau + k] = a\theta(k - X_\tau) = \pi_t^e$.

Now, since $X_\tau \in A$, we have $(x - k - a\pi_t^e)^2 > (1 + a^2\theta)(x - k)^2$ and thus $\pi_t^e \in \{h : (x - k - ah)^2 > (1 + a^2\theta)(x - k)^2\}$ for all $x \in R$. Then, by Lemma 1, and the optimal stopping time is $\tau^* = T$. Therefore, we must have $\pi_t^* = 0$ for all $t \in [0, T]$.

Since the public only cares about his inflation prediction errors, so $\pi_t^e = \theta(k - X_\tau)$ minimizes the public's expected loss when the policy change occurs at time t in this game. Hence, we must have $\pi_t^{e*} = 0$ for all $t \in [0, T]$ since $\tau^* = T$. Thus, we have shown that the trigger strategies $(\tau, \{\pi_t, \pi_t^e\})$ result in a sequentially strong rational expectation equilibrium, i.e., $\tau^* = T$, $\pi_t^* = 0$, and $\pi_t^{e*} = 0$ for all $t \geq 0$. Q.E.D.

Proof of Proposition 2

Substituting $\pi_t^e = a\theta(k - x)$ for $t \geq \tau$ into $(x - k - a\pi_t^e)^2$, we have

$$\begin{aligned} (x - k - a\pi_t^e)^2 &= (1 + a^2\theta)^2(x - k)^2 \\ &> (1 + a^2\theta)(x - k)^2 \end{aligned}$$

for all $x \in R$. Then, we have

$$U = V = R \times \{(-\infty, k) \cup (k, \infty)\} \times R \quad (23)$$

and thus the optimal stopping time is given by $\tau^* = T$. Hence, $\tau^* = T$, $\pi_t^* = 0$ and $\pi_t^{T*} = 0$ for all $t \in [0, T]$. The proofs of the other parts are the same as those in Proposition 1. Therefore, the trigger strategies $(\tau, \{\pi_t, \pi_t^e\})$ result in a stationary zero-inflation sequentially weak rational expectation equilibrium, which is given by τ^* , $\pi_t^* = 0$ and $\pi_t^{e*} = 0$ for all $t \geq 0$. Q.E.D.

Proof of Proposition 3

First note that $x \in A$. Solving $[(x - k) + a^2\theta(z - k)]^2 > (1 + a^2\theta)(x - k)^2$ for z , we have

$$z > \frac{1}{a^2\theta} \left[(1 + a^2\theta)k - x + \sqrt{1 + a^2\theta}(x - k) \right]$$

when $x \geq k$, and

$$z < \frac{1}{a^2\theta} \left[(1 + a^2\theta)k - x - \sqrt{1 + a^2\theta}(k - x) \right]$$

when $x < k$.

Let $C = \frac{1}{a^2\theta} \left[(1 + a^2\theta)k - x + \sqrt{1 + a^2\theta}|x - k| \right]$ and $D = \frac{1}{a^2\theta} \left[(1 + a^2\theta)k - x - \sqrt{1 + a^2\theta}|x - k| \right]$.

Since $X_0 = x \in A$ for all $x \in R$, there are two cases to be considered: (1) $x \geq k$ so that $x > C$ and (2) $x < k$ so that $x < D$.

Case 1. $x > C$. Let $\eta_c = \inf\{t > 0: X_t \leq C\}$, and let η_n be the first exit time from the interval

$$\{X_t : C \leq X_t \leq n\}$$

for all integers n with $n > C$. We first show that $P^x(X_{\eta_n} = C) = \frac{n-x}{n-C}$ and $P^x(X_{\eta_n} = n) = \frac{x-C}{n-C}$.

Consider function $h \in C_0^2(R)$ defined by $h(x) = x$ for $C \leq x \leq n$ ($C_0^2(R)$ means the functions in $C^2(R)$ with compact support in R). By Dynkin's formula,

$$E^x [h(X_{\eta_n})] = h(x) + E^x \left[\int_0^{\eta_n} Ah(X_s) ds \right] = h(x) = x, \quad (24)$$

we have

$$CP^x(X_{\eta_n} = C) + nP^x(X_{\eta_n} = n) = x.$$

Thus,

$$P^x(X_{\eta_n} = C) = \frac{n-x}{n-C}$$

and

$$P^x(X_{\eta_n} = n) = 1 - P^x(X_{\eta_n} = C) = \frac{x-C}{n-C}.$$

Now Consider $h \in C_0^2(R)$ such that $h(x) = x^2$ for $C \leq x \leq n$. Applying Dynkin's formula again, we have

$$E^x [h(X_{\eta_n})] = h(x) + E^x \left[\int_0^{\eta_n} Ah(X_s) ds \right] = x^2 + \sigma^2 E^x [\eta_n], \quad (25)$$

and thus

$$\sigma^2 E^x [\eta_n] = C^2 P^x(X_{\eta_n} = C) + n^2 P^x(X_{\eta_n} = n) - x^2.$$

Hence, we have

$$E^x [\eta_n] = \frac{1}{\sigma^2} \left[C^2 \frac{n-x}{n-C} + n^2 \frac{x-C}{n-C} - x^2 \right].$$

Letting $n \rightarrow \infty$, we conclude that $P^x(X_{\eta_n} = n) = \frac{x-C}{n-C} \rightarrow 0$ and $\eta_c = \lim \eta_n < \infty$ a.s.

Therefore, we have

$$E^x[\eta_c] = \lim_{n \rightarrow \infty} E^x [\eta_n] = \infty.$$

Case 2. $X_0 = x < D$. Define $\eta_D = \inf\{t > 0; X_t \geq D\}$. Let η_n be the first exit time from the interval

$$\{X_t : -n \leq X_t \leq D\}$$

for all integers n with $-n < D$. By the same method, we can prove that

$$E^x [\eta_n] = \frac{1}{\sigma^2} \left[D^2 \frac{n+x}{n+D} + n^2 \frac{D-x}{n+D} - x^2 \right].$$

Letting $n \rightarrow \infty$, we conclude that $P^x(X_{\eta_n} = n) = \frac{D-x}{n+D} \rightarrow 0$ and $\eta_D = \lim \eta_n < \infty$ a.s., and thus

$$E^x[\eta_D] = \lim_{n \rightarrow \infty} E^x [\eta_n] = \infty.$$

Thus, in either case, we have $E^x[\eta] = \infty$. Q.E.D.

6 References

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