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Implementation of marginal cost pricing equilibrium allocations with transfers in economies with increasing returns to scale

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Abstract This paper considers implementation of marginal cost pricing equilibrium allocations with transfers for production economies with increasing returns to scale. We present a mechanism whose Nash equilibrium allocations coincide with the set of marginal cost pricing equilibrium allocations with transfers that characterizes Pareto efficient allocations for economies with non-convex production technologies. We allow production sets and preferences to be unknown to the planner. The mechanism has some nice properties such as feasibility, continuity, and finite-dimension of message space. Furthermore, it works not only for three or more agents, but also for two-agent economies.

Keywords Well-behaved mechanism · Implementation · Marginal cost pricing equilibrium with transfers · Increasing returns · Non-convexities

JEL Classification C72 · D61 · D71 · D82

1 Introduction

This paper considers implementation of marginal cost pricing equilibrium allocations with transfers (MCPT) for economies with increasing returns to scale, or more general types of non-convexities. We present a well-behaved mechanism that is continuous,

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feasible, and further uses a finite-dimensional message space. The reason we are interested in implementing the MCPT equilibrium principle is that Pareto efficient allocations can be characterized by this equilibrium principle for production economies. The other equilibrium principles, such as loss-free pricing equilibrium, average cost pricing equilibrium, marginal (cost) pricing equilibrium, and voluntary trading pricing equilibrium principles considered in Tian (2009a), generally do not result in Pareto efficient allocations when increasing returns to scale or other more general forms of non-convexities are present. Pareto efficiency is a highly desirable property in designing an incentive compatible mechanism. The importance of this property is attributed to what may be regarded as the minimal welfare property. Pareto optimality requires that resources be allocated efficiently. If an allocation is not efficient, the resource distribution can be improved so that at least one agent is better off without making others worse off given the resources. As such, one may desire to design a mechanism that shall implement Pareto efficient allocations in Nash equilibrium.

1.1 Related literature

Osana (1997) discussed Nash implementation of weak Pareto efficient allocations for decomposable (externality-free) economic environments. He presents a mechanism that implements a subset of weak Pareto efficient allocations. A drawback of his mechanism is that it is not individually feasible and continuous. Anderlini and Siconolfi (2004) considered implementation of Pareto-efficient and individually rational allocations for public goods economies. Again, their mechanisms are not continuous on the boundary of the feasible set, and further they considered public goods economies with only one private and one public good. Tian (2009b) gave a mechanism that fully implements the whole domain of Pareto-efficient allocations and furthermore the mechanism is well-behaved in the sense that it is continuous, feasible, market-type, uses a finite-dimensional message space, and works for any number of agents. Some of these properties have been studied systematically in Dutta et al. (1995) and Saijo et al. (1996).

1.2 Motivation

All the mechanisms mentioned above, however, are proposed only for implementing Pareto efficient allocations for convex production technologies. Non-convexities in production have practical importance and often arise from technical progress, imperfection of markets, fixed costs, increasing returns to scale, or indivisibilities. The non-convex firms can be thought of as privately owned public utilities, which are regulated (cf. Brown and Heal 1983, 1985). This type of market structure is common in the United States, and various pricing rules such as marginal cost, average cost, loss-free, voluntary trading, and quantity-taking pricing rules have been proposed in the literature. The most common justification of marginal cost pricing equilibrium with transfers is that a suitable price system and distribution of income allow the decentralization of every Pareto optimal allocation when each producer is instructed to follow the marginal cost pricing rule. Since public utilities are privately owned, any pricing rule imposed by a regulator should be incentive compatible.

Although the consideration of non-convexities in production when dealing with incentive mechanism design is of great importance, technical difficulties have limited this research. Until recently, most studies have been largely devoted to pure exchange models or economies with convex production possibility sets such as those in Hurwicz (1979), Schmeidler (1980), Hurwicz et al. (1995), Postlewaite and Wettstein (1989), Tian (1989, 1992, 1996, 1999), Hong (1995), and Peleg (1996), Suh (1995), Yoshihara (1999), Duggan (2003), among others. Recently, Tian (2009a) considered the problem of the incentive mechanism design for economies with non-convexities in production technologies. Tian (2009a) proposed specific well-behaved mechanisms that implement various pricing equilibrium allocations such as loss-free pricing equilibrium allocations, average cost pricing equilibrium allocations, marginal (cost) pricing equilibrium allocations, and voluntary trading pricing equilibrium allocations in Nash equilibrium for general non-convex production technologies. However, all of these equilibrium principles in general do not result in Pareto efficient allocations.

1.3 The results of the paper

This paper considers implementation of marginal cost pricing equilibrium allocations with transfers that contains the set of Pareto efficient allocations as a subset for production economies with increasing returns to scale or more general types of non-convexities. We present a mechanism whose Nash equilibrium allocations coincide with the set of marginal cost pricing equilibrium allocations with transfers over the class of general economies involving non-convex production technologies. We allow production sets and preferences to be unknown to the planner.

The mechanism proposed in the paper has a number of desirable properties. Firstly this is a canonical mechanism, in the sense that it is feasible, continuous and has a finite message space. Secondly, it permits the Nash implementation of the marginal cost pricing scheme without any requirement to strengthen the assumptions for the two agent economy. Hence the theorem that characterizes the Nash implementation is for economies with two or more agents. The reason for this last point is in the structure mechanism, which requires two “special” agents. The first agent is the firm’s manager, while the second agent is the principal (i.e., owner of the firm). These two agents have the ability to determine the price and output offered by the firm. They are also consumers in this market. The other agents in this economy are assumed to have the role of consumers.

The mechanism works this way. Firstly all agents, including the manager and the owner of firms, are price takers in this market: the manager sets the price for the market and the owner sets the manger’s price. Hence there is no opportunity to manipulate prices and prices are taken as given. Output is chosen by the manager to maximize the profit of the firm. This is done by benchmarking against the target set by the owner, with penalties and rewards being applied in accordance with whether or not the target is achieved. The owner is penalized if they fail to match the manager’s price vector, non-linear pricing parameter or the manager’s projected output. All consumers are penalized for failing to match the consumption bundles of their neighbor, with the penalization scheme working like a Tweed ring. The message space is finite because the manager and the owner are only required to report their production plans, but not production sets, which would result in an infinite message space. In addition, this implies that at no stage will production technology be revealed to the mechanism designer.

It may be remarked that Tian (2005) considered implementation of various pricing equilibrium allocations in production economies with increasing returns. However, it assumes that the social planner knows production sets, so it considers different situations from the present paper. In fact, the two mechanisms in these papers are very different. Because the marginal cost pricing equilibrium principle with transfers in general does not allow production optimization for non-convex production technologies, the implementation of the marginal cost pricing equilibrium principle with transfers is harder than those for the conventional Walrasian solution when production sets are unknown to the designer. The mechanism to be constructed in the paper is different from the existing ones. For instance, we need to give the managers of firms special incentives to produce in an efficient way at equilibrium, but at the same time, they do not have incentives to pursue profit maximization that results in the non-existence of pricing equilibrium in the presence of increasing returns or more general types of non-convexities in production.

The remainder of this paper is as follows. Section 2 provides a general setup of the model and gives the notation and definitions on marginal pricing equilibrium with transfers and mechanism design. Section 3 considers implementation of marginal cost pricing equilibrium allocations with transfers. The mechanism has the desired properties mentioned above. We prove that the mechanism implements marginal cost pricing equilibrium allocations with transfers. Concluding remarks are presented in Sect. 4.

2 The setup

2.1 Economic environments

We consider production economies with L commodities, $n \geqq 2$ consumers and J firms.[1] Let $N = \{1, 2, \ldots, n\}$ denote the set of consumers. Each consumer's characteristic is denoted by $e_i = (C_i, R_i)$, where $C_i = \mathbb{R}^L_+$ is the consumption set, and R_i is the preference ordering defined on $\mathbb{R}^L_+$. Let P_i denote the asymmetric part of R_i (i.e., $a \ P_i \ b$ if and only if $a \ R_i \ b$, but not $b \ R_i \ a$). We assume that R_i is continuous and convex on $\mathbb{R}^L_+$, and strictly monotonically increasing on $\mathbb{R}^L_{++}$.[2] Let $\hat{w}$ be the total initial endowment vector of commodities.

Production technologies of firms are denoted by $\mathcal{Y}_1, \ldots, \mathcal{Y}_j, \ldots, \mathcal{Y}_J$. We assume that, for $j = 1, \ldots, J$, $\mathcal{Y}_j$ is closed, contains 0 (possibility of inaction), and

[1] As usual, vector inequalities, $\geqq$, $\geq$, and $>$, are defined as follows: Let $a, b \in \mathbb{R}^m$. Then $a \geqq b$ means $a_s \geqq b_s$ for all $s = 1, \ldots, m$; $a \geq b$ means $a \geqq b$ but $a \neq b$; $a > b$ means $a_s > b_s$ for all $s = 1, \ldots, m$.

[2] R_i is convex if, for bundles a and b, $a \ P_i \ b$ implies $\lambda a + (1 - \lambda) b \ P_i \ b$ for all $0 < \lambda \leqq 1$. Note that the term "convex" is defined as in Debreu (1959), not as in some recent textbooks.

$\{\mathcal{Y}_j - \mathbb{R}^L_+\} \subseteq \mathcal{Y}_j$ (free-disposal). It is important to note that, under these assumptions, $\partial\mathcal{Y}_j$, the boundary of the production set $\mathcal{Y}_j$, is exactly the set of (weakly) efficient production plans of the jth producer, that is,

$$\partial\mathcal{Y}_j = \{y_j \in \mathcal{Y}_j : \nexists z_j \in \mathcal{Y}_j, z_j > y_j\}.$$

We further assume that $\mathcal{Y}_j$ has a twice continuously differentiable hypersurface for each j, i.e., there exists a twice continuously differentiable function, f_j, from $\mathbb{R}^L$ into $\mathbb{R}$, such that $\mathcal{Y}_j = \{x \in \mathbb{R}^L : f_j(x) \leqq 0\}$, 0 is a regular value of f_j, and $\partial\mathcal{Y}_j = f_j^{-1}(0)$.

We also assume that there are no externalities or public goods. An economy is a profile of economic characteristics of consumers and firms. It is denoted by $e = (e_1, \ldots, e_n, \hat{w}, \mathcal{Y}_1, \ldots, \mathcal{Y}_J)$, and the set of all such economies is denoted by E which is assumed to be endowed with the product topology.

2.2 Marginal cost pricing equilibrium with transfers

An allocation of the economy e is a vector $(x_1, \ldots, x_n, y_1, \ldots, y_J) \in \mathbb{R}^{L(n+J)}$ such that: (1) $x = (x_1, \ldots, x_n) \in \mathbb{R}^{nL}_+$, and (2) $y_j \in \mathcal{Y}_j$ for $j = 1, 2, \ldots, J$. Denote by $y = (y_1, \ldots, y_J)$ the profile of production plans of firms.

An allocation (x, y) is *feasible* if

$$\sum_{i=1}^{n} x_i \leqq \hat{w} + \sum_{j=1}^{J} y_j. \tag{1}$$

Denote the aggregate endowment, consumption and production by $\hat{x} = \sum_{i=1}^{n} x_i$, $\hat{y} = \sum_{j=1}^{J} y_j$, respectively.[3] Then the feasibility condition can be written as

$$\hat{x} \leqq \hat{w} + \hat{y}.$$

An allocation (x, y) is said to be *Pareto efficient* if the following two conditions are satisfied:

(i) it is feasible;
(ii) there does not exist another feasible allocation (x', y') such that $x'_i \; R_i \; x_i$ for all $i = 1, \ldots, n$ and $x'_i \; P_i \; x_i$ for some $i = 1, \ldots, n$.

Denote by $P(e)$ the set of all such allocations.

The above two conditions for Pareto efficiency have nice geometric interpretations which are associated with the notion of supporting prices for Pareto efficient allocations and the well-known second theorem of welfare economics.

Let $\hat{\mathcal{Y}} = \sum_{j=1}^{J} \mathcal{Y}_j$ be the aggregate production set. Let

$$U_i(x_i) = \{x'_i \in X_i : x'_i \; R_i \; x_i\} \tag{2}$$

be the weak upper contour set of consumer i and let

[3] For notational convenience, "$\hat{a}$" will be used throughout the paper to denote the sum of vectors a_l, i.e., $\hat{a} = \sum a_l$.

$$U(x) = \sum_{i=1}^{n} U_i(x_i) \tag{3}$$

be the aggregate weak upper contour set. The boundary of $U(x)$ is known as a social indifference curve or a Scitovski contour through the point x.

Condition (i) for Pareto efficiency implies that $\sum_{i=1}^{n} x_i \in \{\hat{w}\} + \hat{\mathcal{Y}}$ and hence the set $U \cap [\{\hat{w}\} + \hat{\mathcal{Y}}]$ contains at least $\sum_{i=1}^{n} x_i$. Condition (ii) implies that $\{\hat{w}\} + \hat{\mathcal{Y}}$ does not intersect the interior of U. Hence, the sets U and $\{\hat{w}\} + \hat{\mathcal{Y}}$ must be "tangent" at the point $\sum_{i=1}^{n} x_i$.

When the aggregate weak upper contour set U is convex and the aggregate production set $\hat{\mathcal{Y}}$ is convex (this is true if preferences R_i, and production sets $\mathcal{Y}_j$ are convex), this tangency condition implies the existence of a hyperplane which separates the two sets U and $\{\hat{w}\} + \hat{\mathcal{Y}}$. The vector normal to this hyperplane is the vector of prices which supports the Pareto efficient allocation (x, y). However, when $\hat{\mathcal{Y}}$ is non-convex, this separation property will not in general hold but there still exists a hyperplane tangent to U and $\{\hat{w}\} + \hat{\mathcal{Y}}$ and a vector p orthogonal to the two sets if the boundaries of the two sets are smooth. Thus, in both cases, such a vector p is the supporting price vector for Pareto efficient allocations; called the efficient price vector in the literature.

It may be remarked that the smoothness of U and $\partial\hat{\mathcal{Y}}$ is not needed to obtain the existence of a price vector which supports an efficient allocation. When production sets are convex, the existence of a supporting price vector follows from the separation theorem applied to the convex sets U and $\partial\hat{\mathcal{Y}}$. The same theorem also implies the existence of a cone of normals at each point of the boundary of these sets so that one can obtain the Second Welfare Theorem that characterizes Pareto efficient allocations by decentralized competitive markets. When production sets are non-convex, it is also possible to generalize the Second Welfare Theorem using the notion of marginal cost pricing equilibrium with transfers.

To define marginal cost pricing equilibrium with transfers for non-convex economies, we first give the notion of the Clarke tangent and normal cones. The Clarke tangent normal cone to $\mathcal{Y}$ is a generalization of the notion of the marginal rate of transformation in the absence of smoothness and convexity assumptions (cf. Clarke 1975). The formal definition of the Clarke normal cone is defined as follows. For a non-empty set $Y \subseteq \mathbb{R}^L$ and $y \in Y$, the tangent cone of Y is given by $T_Y(y) = \{x \in \mathbb{R}^L$: for every sequence $y^k \in Y$ with $y^k \rightarrow y$ and every sequence $t^k \in (0, \infty)$ with $t^k \rightarrow 0$, there exists a sequence $x^k \in \mathbb{R}^L$ with $x^k \rightarrow x$ such that $y^k + t^k x^k \in Y$ for all k $\}$. The Clarke normal cone is then given by $N_Y(y) = \{x \in \mathbb{R}^L : (z, x) \leqq 0 \; \forall z \in T_Y(y)\}$.

The important properties of the Clarke normal cone are: (1) it coincides with the standard normal cone when Y is convex or when the boundary of Y is differentiable; (2) it is convex and never reduces to the null vector for any boundary point of Y, and (3) the correspondence $y \rightarrow N_Y(y)$ has a closed graph. These properties are well adapted to the economic problem of optimization and fixed points (cf. Cornet 1990; Quinzii 1992). Detailed discussions on the settings of the model in economies with increasing returns can be found in Beato (1982), Brown and Heal (1982), Cornet (1988, 1989, 1990), Bonnisseau (1988), Bonnisseau and Cornet (1988), Kamiya (1988), Vohra (1988), Brown (1990), and Brown et al. (1992).

We then have the following equilibrium notion that can be used to characterize Pareto efficient allocations.

Definition 1 An allocation $(x^*, y^*) = (x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_J^*) \in \mathbb{R}_+^{nL} \times \mathcal{Y}$ is a *marginal cost pricing equilibrium allocation* with transfers for an economy e if it is feasible and there is a price vector $p^* \in \mathbb{R}_+^L$ such that

(1) for every $i \in N$, $x_i \; P_i \; x_i^*$ implies $p^* \cdot x_i > p^* \cdot x_i^*$, i.e., x_i^* is the greatest element for R_i in the budget set $\{x_i \in \mathbb{R}_+^L : p^* \cdot x_i \leqq p^* \cdot x_i^*\}$,
(2) for $j = 1, \ldots, J$, $p^* \in N_{\mathcal{Y}_j}(y^*)$.

A marginal cost pricing equilibrium allocation with transfers for an economy e is thus a list of consumption plans (x_i^*), production plans (y_j^*), and a price vector p^* such that (a) every consumer maximizes his/her preference with respect to the budget set $\{x_i \in \mathbb{R}_+^L : p^* \cdot x_i \leqq p^* \cdot x_i^*\}$ at x_i^*, (b) firms' production plans y_j^* satisfy the first-order necessary conditions for profit maximization, i.e., at the given production plans the market prices lie in the Clarke normal cone, and (c) the excess demand over supply is zero. The main difference with the Walrasian model is Condition (b), in which firms may not maximize profits but instead behave according to the marginal cost pricing rule. Thus, the concept of a marginal cost pricing equilibrium with transfers is a natural extension of the concept of competitive equilibrium with transfers for economies in which some firms have non-convex production sets.

Denote by $MCPT(e)$ the set of all such marginal cost pricing equilibrium allocations with transfers.

Remark 1 The term marginal cost pricing rule is, strictly speaking, inappropriate since it is not always true that the price of a good is set at its marginal cost. $p \in N_{\mathcal{Y}_j}$ implies equality between the price of a good and its marginal cost only if the input requirement sets are convex. It has been adopted because it is suggestive. With this qualification in mind we retain it.

Remark 2 Also, if production set $\mathcal{Y}_j$ is convex, then the marginal cost pricing rule given by Condition (2) implies that y_j^* maximizes firm j's profit at price p, i.e.,

$$(2') \quad \text{for } j = 1, \ldots, J, \; p^* y_j^* \geqq p^* y_j \text{ for all } y_j \in \mathcal{Y}_j.$$

Remark 3 Let (p, x, y) be a marginal cost pricing equilibrium with transfers for a production economy e. If $px_i > 0$ for consumers $i = 1, \ldots, n$, then (p, x, y) is a marginal cost pricing equilibrium for any wealth map $(r_1(w, p, y), \ldots, r_I(w, p, y))$ satisfying $r_i(w, p, y) = px_i$.

An important characterization of a Pareto optimal allocation is associated with the following version of the Second Welfare Theorem for non-convex production economies.

Lemma 1 *Suppose e is an economy such that (1) preferences R_i are continuous, convex and monotonic, and (2) production sets $\mathcal{Y}_j$ are closed and satisfy free disposal property. Let $(x^*, y^*) = \left(x_1^*, x_2^*, \ldots, x_n^*, y_1^*, y_2^*, \ldots, y_J^*\right) \in \mathbb{R}_+^{nL} \times \mathcal{Y}$ be a Pareto optimal allocation with $x_i^* > 0$ for all $i \in N$. Then (x^*, y^*) is a marginal cost pricing equilibrium allocation with transfers.*

The proof of this lemma can be found in Cornet (1990) and Quinzii (1992).

Thus, in this paper, we ignore the trivial case of zero consumption for a consumer at equilibrium and only consider implementation of marginal cost pricing equilibrium allocations with transfers whose consumption bundles for all consumers are nonzero by designing a feasible and continuous mechanism that has a finite-dimensional message space. To do so, we need to make the following indispensable assumption.

Assumption 1 (Interiority of Preferences): For all $i \in N$, $x_i \; P_i \; x_i'$ for all $x_i \in \mathbb{R}^L_{++}$ and $x_i' \in \partial\mathbb{R}^L_+$, where $\partial\mathbb{R}^L_+$ is the boundary of $\mathbb{R}^L_+$.

Remark 4 Assumption 1 cannot be dispensed. Like the Walrasian correspondence, one can show that the pricing equilibrium correspondence violates Maskin (1999) monotonicity condition on the boundary of the consumption space, and thus it cannot be Nash implemented by a feasible mechanism. However, if we modify the marginal cost pricing equilibrium with transfers to the constrained marginal cost pricing equilibrium with transfers which is obtained by changing Condition (1) in the pricing equilibrium allocation to the following condition:

Condition (1′): for every $i \in N$, $x_i \; P_i \; x_i^*$ and $x_i^* \leqq \hat{w} + \sum_{j=1}^{J} y_j^*$ imply that $p^* x_i > p^* x_i^*$,

then the constrained marginal cost pricing equilibrium allocations with transfers satisfy Maskin's monotonicity condition. In fact it can be showed that the mechanism presented below implements constrained marginal cost pricing equilibrium allocations with transfers, without Assumption 1. It is clear that every marginal cost pricing equilibrium allocation with transfers is a constrained marginal cost pricing equilibrium allocation with transfers, but the converse may not be true. However, when preferences, as summed in the paper, are convex, it can be proved that the constrained marginal cost pricing equilibrium allocations with transfers coincide with the marginal cost pricing equilibrium allocation for interior allocations.

Formally, we have the following proposition.

Proposition 1 *Suppose preferences are convex. Then the set of interior constrained marginal cost pricing equilibrium allocations with transfers coincides with the set of interior marginal cost pricing equilibrium allocations.*

Proof We only need to show that every interior constrained marginal cost pricing equilibrium allocation with transfers (CMCPT) is a marginal cost pricing equilibrium allocation with transfers (MCPT). Suppose, by way of contradiction, that there is an allocation (x^*, y^*) which is a CMCPT allocation with associated price vectors $p^* \in \mathbb{R}^L_+$, but not a MCPT allocation. Then there is x_i such that $x_i \; P_i \; x_i^*$ and $p^* x_i \leq p^* x_i^*$ for some $i \in N$. By convexity of preferences, for all $x_{i\lambda} = \lambda x_i, +(1-\lambda) x_i^*$ where $0 < \lambda \leq 1$, we have $x_{i\lambda} \; P_i \; x_i^*$ and $p^* x_{i\lambda} \leq p^* x_i^*$. Since $x* > 0$, $x_{i\lambda}^* < \sum_{i \in N} w_i$ and thus $x_{i\lambda}$ can be chosen to satisfy $x_{i\lambda}^* < \sum_{i \in N} w_i$, but this contradicts the fact that (x^*, y^*) is a CMCPT allocation, and thus (x^*, y^*) must be a MCPT allocation. □

2.3 Mechanism

In this subsection, we give some basic concepts, notations and definitions used in the mechanism design literature. Let $F : E \rightarrow\rightarrow \mathbb{R}_+^{L(n+J)}$ be a social choice correspondence to be implemented. Let M_i denote the i-th agent's message domain. Its elements are written as m_i and are called messages. Let $M = \prod_{i=1}^n M_i$ denote the message space which is assumed to be endowed with the product topology. Denote by $h : M \rightarrow \mathbb{R}_+^{L(n+J)}$ the outcome function, or more explicitly, $h(m) = (X_1(m), \ldots, X_n(m), Y_1(m), \ldots, Y_J(m))$. Then a mechanism, which is defined on E, consists of a message space M and an outcome function. It is denoted by $\langle M, h\rangle$.

A message $m^* = (m_1^*, \ldots, m_n^*) \in M$ is said to be a *Nash equilibrium* of the mechanism $\langle M, h\rangle$ for an economy e if, for all $i \in N$ and $m_i \in M_i$,

$$X_i(m^*)\ R_i\ X_i(m_i, m_{-i}^*), \tag{4}$$

where $(m_i, m_{-i}^*) = (m_1^*, \ldots, m_{i-1}^*, m_i, m_{i+1}^*, \ldots, m_n^*)$. The outcome $h(m^*)$ is then called a *Nash* (*equilibrium*) *allocation* of the mechanism for the economy e. Denote by $V_{M,h}(e)$ the set of all such Nash equilibria and by $N_{M,h}(e)$ the set of all such Nash equilibrium allocations.

A mechanism $\langle M, h\rangle$ is said to *Nash-implement* a social choice correspondence F on E, if, for all $e \in E$, $N_{M,h}(e) \subseteq F(e)$. It is said to *fully Nash-implement* a social choice correspondence F on E, if, for all $e \in E$, $N_{M,h}(e) = F(e)$.

A mechanism $\langle M, h\rangle$ is said to be *feasible*, if, for all $m \in M$, (1) $X(m) \in \mathbb{R}_+^{nL}$, (2) $Y_j(m) \in \mathcal{Y}_j$ for $j = 1, \ldots, J$, and (3) $\sum_{i=1}^n X_i(m) \leqq \hat{w} + \sum_{j=1}^J Y_j(m)$.

A mechanism $\langle M, h\rangle$ is said to be *continuous*, if the outcome function h is continuous on M.

3 Implementation of marginal cost pricing equilibrium allocations with transfers

The design of an incentive mechanism that implements marginal cost pricing equilibrium allocations with transfers is a hard job. Two difficulties are involved in implementing marginal cost pricing equilibrium allocations with transfers by a mechanism that is feasible, continuous, and uses a finite-dimensional message space. First, since production sets $\mathcal{Y}_j$ are assumed to be unknown to the designer, the designer cannot form the Clarke normal cone $N_{\mathcal{Y}_j}(y_j)$ for a reported production plan. For any subset $\tilde{\mathcal{Y}}_j$ of $\mathcal{Y}_j$, which is formed by the reported production plans, its Clarke normal cone is larger than $N_{\mathcal{Y}_j}(y_j)$ so that a price in $N_{\tilde{\mathcal{Y}}_j}(y_j)$ may not be in $N_{\mathcal{Y}_j}(y_j)$. Thus, the marginal cost pricing rule cannot be obtained by using $\tilde{\mathcal{Y}}_j$. Secondly, since production sets may be non-convex, the marginal cost pricing rule may not coincide with the profit maximization rule, and we cannot use the profit maximization approach used in Hong (1995) and Tian (1999) to consider the problem of implementation of marginal cost pricing equilibrium allocations with transfers.

To overcome these difficulties, note that, as shown by Cornet (1990), although the marginal cost pricing rule does not coincide with the profit maximizing rule for the linear profit function $\pi_j = py_j$, it is equivalent to a profit maximizing rule under which every marginal cost pricing production plan maximizes a quadratic profit function and the first order conditions for maximizing the linear profit function and the quadratic profit function coincide at the marginal cost pricing production plan. Thus, to show that a price vector satisfies the marginal cost pricing rule under a proposed mechanism, it is sufficient to show that the proposed production plan maximizes some quadratic profit function for every firm under Nash equilibrium of the mechanism.

To do this, define the perpendicular set

$$\perp_{\mathcal{Y}_j}\left(y_j^*\right) = \left\{p \in \mathbb{R}_+^L : \exists \rho \geqq 0, \forall y_j \in \mathcal{Y}_j, py_j^* \geqq py_j - \rho \|y_j - y_j^*\|^2\right\}. \quad (5)$$

When $p \in \perp_{\mathcal{Y}_j}(y_j^*)$, p is said to be a perpendicular vector (or proximal normal vector) to $\mathcal{Y}_j$ at y_j^*. It can be verified that $\perp_{\mathcal{Y}_j}(y_j^*) \subseteq N_{\mathcal{Y}_j}(y_j^*)$. The condition that p is a perpendicular vector is a necessary condition for profit maximization for py_j over $\mathcal{Y}_j$. Indeed, if y_j^* maximizes the profit py_j over $\mathcal{Y}_j$, it also maximizes the quadratic function $py_j - \rho\|y_j - y_j^*\|^2$ over the production set $\mathcal{Y}_j$ for every $\rho \geqq 0$ so that $py_j^* \geqq py_j - \rho\|y_j - y_j^*\|^2$ for all $y_j \in \mathcal{Y}_j$, and thus $p \in \perp_{\mathcal{Y}_j}(y_j^*)$. One can also see that if $\mathcal{Y}_j$ is convex, this necessary condition for profit maximization is also sufficient by taking $\rho = 0$.

The perpendicular set $\perp_{\mathcal{Y}_j}(y_j^*)$ has a clear economic and geometric interpretation. $p \in \perp_{\mathcal{Y}_j}(y_j^*)$ means that, given the price system p, the production plan y_j^* maximizes the quadratic function $py_j - \rho\|y_j - y_j^*\|^2$ over the production set $\mathcal{Y}_j$, i.e., the profit py_j^* up to the perturbation "$-\rho\|y_j - y_j^*\|^2$." This condition can also be interpreted in terms of "nonlinear prices" by noting that it is equivalent to saying that y_j^* maximizes the quadratic function

$$\pi_j(y_j) = \left[p - \rho\left(y_j - y_j^*\right)\right]\left(y_j - y_j^*\right)$$

over the production set $\mathcal{Y}_j$. As such, ρ can be interpreted as the nonlinear price parameter, which is used to determine the marginal pricing rule for non-convex production technologies. Like the linear price vector p, each firm takes it as given and it will be determined by the mechanism.

In general, we do not have $\perp_{\mathcal{Y}_j}(y_j^*) = N_{\mathcal{Y}_j}(y_j^*)$. However, under the assumption that $\mathcal{Y}_j$ has a twice continuously differentiable hypersurface, the gradient of f_j at y_j, denoted by $\nabla f_j(y_j)$, defines the marginal cost pricing rule for a firm with a smooth technology. Then, if $f_j(y_j) = 0$ and $\nabla f_j(y_j) \neq 0$, by Proposition 2 in Cornet (1990), we have the following set equivalences.

$$\perp_{\mathcal{Y}_j}\left(y_j^*\right) = N_{\mathcal{Y}_j}\left(y_j^*\right) = \left\{\lambda \nabla f_j\left(y_j^*\right) : \lambda \geqq 0\right\}$$

for all $y \in \partial\mathcal{Y}$. Thus, the marginal cost pricing rule, profit maximizing prices for the quadratic profit function (or for the profit function with nonlinear prices), and the first order condition for the linear profit function are all equivalent.[4] We will use this fact to show our implementation results on marginal cost pricing equilibrium allocations with transfers.

3.1 The description of the mechanism

In the mechanism proposed below, the designer does not need to know firms' true production sets. However, it is well known by now that, to guarantee the feasibility even at disequilibrium points, while the designer does not necessarily need to know firms' true production sets,[5] the designer must have some information about production sets. The existing mechanisms require all individual agents report price vectors and production plans of firms. Note that, when goods are physical, the requirement of reporting a production plan can be guaranteed by asking the manager to reveal the production plan to the designer. This requirement results in not only a high dimension of message space, but it is also highly unrealistic since not everyone in the real world knows prices of all commodities and production technologies of all firms. To the contrary, the mechanisms constructed below only require two agents to announce price vectors and production plans of firms. One agent can be regarded as the manager or CEO of firms who is asked to announce production plans from the firm's production set, and the other may be regarded as the owner or a group of investors who may not be involved directly in production, but may oversee or govern the manager in her production activities.

Thus, it is assumed that the manager of a firm knows the firm's production possibility set while others may or may not have this information. Although we can assume that there are a pair of consumers who serve as the manager and the owner for each firm, which may be a more realistic assumption,[6] to simplify the exposition, without loss of generality, it is assumed that consumer 1 is the manager of all firms and consumer 2 is the owner of all firms. Thus, all the mechanisms proposed in this paper rely on two special kinds of agents: "the manager", consumer 1, who is assumed to know production possibilities of firms, and "the owner", consumer 2, who has some information about that, but perhaps not as complete as the managers have.

The message space of the mechanism is defined as follows. For each $i \in N$, let the message domain of agent i be of the form

[4] Under the convexity of production sets which we do not assume in this paper, all the three conditions are also equivalent.

[5] If this assumption is relaxed and one is willing to give up the requirement of feasibility of a mechanism like most mechanisms that implement Walrasian or Lindahl allocations in the economics literature, it is much easier to construct a mechanism that implements marginal pricing equilibrium allocations with transfers.

[6] For instance, for an economy with three consumers and two firms, if consumer 1 and consumer 2 are the manager and owner of firm 1, and consumer 2 and consumer 3 are the manager and owner of firm 2, then consumer 2's preliminary consumption bundle would be a consumption in his feasible consumption set that is closest to his announced consumption bundle multiplied by the sum of penalties/rewards of serving as the manager of firm 1 and owner of firm 2, which are specified in next section, for each mechanism that implements corresponding pricing equilibrium allocations.

$$M_i = \begin{cases} \Delta_{++}^{L-1} \times \tilde{\mathcal{Y}} \times \mathcal{Y} \times \mathcal{Y} \times \mathbb{R}_+ \times Z_i \times \mathbb{R}_{++} \times (0, 1] & \text{if } i = 1 \\ \Delta_{++}^{L-1} \times \mathbb{R}^{JL} \times \mathbb{R}_+ \times Z_i \times \mathbb{R}_{++} \times (0, 1] & \text{if } i = 2 \\ Z_i \times \mathbb{R}_{++} \times (0, 1] & \text{if } i \geqq 3 \end{cases}$$

where

$$\tilde{\mathcal{Y}} = \left\{ y \in \mathcal{Y} : \hat{w} + \sum_{j=1}^{J} y_j > 0 \right\}, \tag{6}$$

and

$$Z_i = \left\{ (z_{i1}, \ldots, z_{in}) \in \mathbb{R}_+^{nL} : z_{i,i+1} \neq 0 \right\} \tag{7}$$

for $i = 1, \ldots, n$, where $n + 1$ is read to be 1.

Remark 5 Note that, unlike the existing mechanisms, we do not require all consumers, except the first two consumers, to announce prices and production profiles. Thus, the dimension of the message space can be lower than that of the existing mechanisms that implement Walrasian allocations for convex production economies with more than two agents if this technique is used.

A generic element of M_i is $m_i = (p_i, y_i, t_i, s_i, \rho_i, z_i, \gamma_i, \eta_i)$ for $i = 1$, $m_i = (p_i, y_i, \rho_i, z_i, \gamma_i, \eta_i)$ for $i = 2$, and $m_i = (z_i, \gamma_i, \eta_i)$ for $i = 3, \ldots, n$. The component p_i is the price vector proposed by agent i and is used as a price vector by the other agents $k \neq i$. The components $y_1 = (y_{11}, \ldots, y_{1J})$, $t_1 = (t_{11}, \ldots, t_{1J})$, and $s_1 = (s_{11}, \ldots, s_{1J})$ are production profiles announced by agent 1 who works as the manager (CEO) of firms, and will be used to determine feasible allocations, efficient production profiles, and marginal cost pricing rule, respectively. The component $y_2 = (y_{21}, \ldots, y_{2J})$ is a production profile announced by agent 2 who may be interpreted as the owner or a group of investors of firms, and will be used to induce the efficient production profile and marginal pricing rule. While y_1, t_1, and s_1 are required to be production profiles in $\mathcal{Y}$,[7] the production profile y_2 is not necessarily in $\mathcal{Y}$ since agent 2 is not assumed to know production sets. Also, it is worthwhile to point out that the production plans, y_1, t_1, and s_1, selected by the manager need not be equal and that equality will happen at equilibrium, as shown in Lemma 4 below. The reason will be discussed in Remark 6 below.

Note that in order for a mechanism to have a finite-dimensional message space, unlike the mechanism constructed in Hurwicz et al. (1995), that requires agents to report production sets that result in an infinite-dimensional message space, we only require agents to report production plans for firms, but not the production sets. The component ρ_i is a non-linear pricing parameter in the quadratic profit function, that is proposed by agent i ($i = 1, 2$) and is used to determine the marginal cost pricing

[7] Actually, we require y_1 be a feasible production profile so that aggregate consumptions for consumers are positive.

rule. The component $z_i = (z_{i1}, \ldots, z_{in})$ is an allocation proposed by agent i, where z_{ik} is the consumption bundle of agent k proposed by agent i. The component γ_i is a shrinking index of agent i used to shrink the consumption of other agents in order to have a feasible outcome and be better off. When an outcome is not a Nash equilibrium of the mechanism, some agent can declare a larger enough number so that the resulting outcome improves his/her utility. The component η_i is the penalty index of agent i for announcing different consumption or production profiles. If she announces different production profiles or she proposes consumption bundles for the others that are different from those announced by agent $i+1$, agent i can always choose a smaller $\eta_i < \eta_i^*$ so that her consumption becomes larger and she would be better off. Hence, no choice of η_i could constitute part of the Nash equilibrium strategy when they are different.

Before we formally define the outcome function of the mechanism, we give a brief description and explain why the mechanism works. For each announced message $m \in M$, the price vector facing consumer 1 is determined by p_2 announced by agent 2, and the price vector of the others is determined by p_1 announced by agent 1. Thus, each individual takes prices as given and cannot change them by changing his own message. Also, since the non-linear price parameter used to construct the marginal pricing is determined by ρ_2 announced by agent 2, the manager of firms cannot change marginal pricing by changing her own message. The feasible production outcome $Y(m) \in \tilde{\mathcal{Y}}$ is then determined by the feasible production profile y_1 proposed by agent 1, who is the manger of the firms. Also to give the manager of firms an incentive to produce at a profit maximizing level for the quadratic profit function at equilibrium, a special compensation τ_1 is provided to the manager of firm j; she will receive a positive amount of compensation if she proposes a more profitable production plan y_1 than one that is announced by agent 2. Her preliminary consumption outcome is determined by her consumption in $B_1(m)$ that is closest to the consumption bundle z_1 announced by the consumer 1 with a reward/penalty scale given by $[v_1(m) + \tau_1(m) + \frac{1}{1+\eta_1(\|t_1-y_2\|+\|s_1-y_2\|+\|z_{12}-x_2'(m)\|+\|z_{1,-1}-z_{2,-1}\|)}]$ when she announces an inefficient production profile, or a production profile that does not follow marginal pricing rule, or a consumption profile $z_{1,-1}$ different from $z_{2,-1}$, where $z_{j,-i} = (z_{j,1}, \ldots, z_{j,i-1}, z_{j,i+1}, \ldots, z_{j,n})$ which denotes consumption bundles proposed by agent j for all agents except for agent i.

To give incentives to agent 2, the owner of firms, to match the price vector, non-linear pricing parameter, and production profile announced by the manager of firms and match the consumption bundles announced by agent 3, agent 2's preliminary consumption outcome is determined by a consumption in $B_2(m)$ that is closest to the consumption bundle z_2 announced by the consumer 2 with a penalty scale given by $\frac{1}{1+\|p_1-p_2\|+\|\rho_1-\rho_2\|+\|y_1-y_2\|+\eta_2(\|z_{23}-x_3'(m)\|+\|z_{2,-2}-z_{3,-2}\|)}$. For agent $i \geqq 3$, the preliminary consumption outcome $x_i(m)$ is determined by her consumption in $B_i(m)$ that is closest to the consumption bundle z_i announced by the consumer i with a penalty scale given by $\frac{1}{1+\eta_i(\|z_{i,i+1}-x_{i+1}'(m)\|+\|z_{i,-i}-z_{i+1,-i}\|)}$ if she announces a consumption profile $z_{i,-i}$ different from $z_{i+1,-i}$. To obtain the feasible outcome consumption $X(m)$, we need to shrink the preliminary consumption $x_i(m)$ in the way specified below. We will show that the mechanism constructed in such a way has the properties we desire, and fully implements marginal cost pricing equilibrium allocations with transfers over the class of production economies under consideration.

Now we formally present the outcome function of the mechanism.

In order for each agent to take prices as given, define agent i's proposed price vector $p_i : M \to \Delta_{++}^{L-1}$ by

$$p_i(m) = \begin{cases} p_2 & \text{if } i = 1 \\ p_1 & \text{otherwise} \end{cases}.$$

Define the feasible production outcome function $Y : M \to \tilde{\mathcal{Y}}$ by

$$Y(m) = y_1. \tag{8}$$

To give the manager incentives to produce efficiently and follow the marginal cost pricing rule, some special compensation schemes ν_1 and τ_1 will be provided to her according to the following formulas.

Define $\nu_1 : M \to \mathbb{R}_+$ by

$$\nu_1(m) = \sum_{j=1}^{J} \nu_{1j}(m) \tag{9}$$

where

$$\nu_{1j}(m) = \prod_{l=1}^{L} \max\left\{0, t_{1j}^l - y_{2j}^l\right\}. \tag{10}$$

The compensation formula ν_1 means that agent 1, as the manager of firm j, will receive a positive amount of compensation $\nu_{1j}(m)$, if she can propose a more efficient production plan t_{1j} than y_{2j} proposed by agent 2 for firm j. In other words, $\nu_{1j}(m) > 0$ if and only if $t_{1j} > y_{2j}$.

Define $\tau_1 : M \to \mathbb{R}_+$ by

$$\tau_1(m) = \sum_{j=1}^{J} \tau_j(m) \tag{11}$$

where

$$\tau_{1j}(m) = \max\{0, p_1(m)(s_{1j} - y_{2j}) - \rho_2\|s_{1j} - y_{2j}\|^2\}. \tag{12}$$

The compensation formula τ_1 means that agent 1 will receive a positive amount of compensation $\tau_{1j}(m)$ if the manager can announce a more profitable production plan s_{1j} than a production plan y_{2j} proposed by agent 2, i.e., $p_1(m)s_{1j} - \rho_2\|s_{1j} - y_{2j}\|^2 > p_1(m)y_{2j}$, or he will receive a zero amount of compensation. In other words, $\tau_{1j}(m) > 0$ if and only if $p_1(m)s_{1j} - \rho_2\|s_{1j} - y_{2j}\|^2 > p_1(m)y_{2j}$.

Remark 6 The reason we want agent 1 to report a production profile s_1 that may be different from y_1 is the following. In determining the marginal cost pricing rule, if the production plan y_{2j}, announced by agent 2, does not maximize $p_1(m)y_j - \rho_2\|y_j - y_{2j}\|^2$ over production set $\mathcal{Y}_j$, then there is a 'test' production plan $y_j \in \mathcal{Y}_j$ such that $p_1(m)y_j - \rho_2\|y_j - y_{2j}\|^2 > p_1(m)y_{2j}$. Such a test production plan y_j may not be in the feasible set $\mathcal{Y}_1(m)$, and if so, we cannot use such a production plan in $\mathcal{Y}_1(m)$ to ensure that the proposed price vector at Nash equilibrium satisfies the marginal cost pricing rule. For detail, see the proof of Lemma 8.

Agent i's feasible consumption correspondence $B_i : M \rightarrow\rightarrow \mathbb{R}^L_+$ is then defined by

$$B_i(m) = \left\{x_i \in \mathbb{R}^L_+ : p_i(m) \cdot x_i = p_i(m) \cdot z_{i-1,i} \ \& \ x_i \leq \hat{w} + \hat{Y}(m)\right\}, \tag{13}$$

which is a continuous correspondence with non-empty, compact, and convex values.

Define x_i': $M \rightarrow B_i$ by

$$x_i'(m) = \left\{x_i : \min_{x_i \in B_i(m)} \| x_i - z_{ii} \|\right\}, \tag{14}$$

which is closest to z_{ii}.

Define agent i's preliminary consumption outcome function x_i: $M \rightarrow \mathbb{R}^L_+$ by

$$x_i(m) = \begin{cases} \left[\nu_1(m) + \tau_1(m) + \frac{1}{1+\eta_1(\|t_1-y_2\|+\|s_1-y_2\|+\|z_{12}-x_2'(m)\|+\|z_{1,-1}-z_{2,-1}\|)}\right] x_1'(m) & \text{if } i = 1 \\ \frac{1}{1+\|p_1-p_2\|+\|\rho_1-\rho_2\|+\|y_1-y_2\|+\eta_i(\|z_{23}-x_3'(m)\|+\|z_{2,-2}-z_{3,-2}\|)} x_2'(m) & \text{if } i = 2 \\ \frac{1}{1+\eta_i(\|z_{i,i+1}-x_{i+1}'(m)\|+\|z_{i,-i}-z_{i+1,-i}\|)} x_i'(m) & \text{if } i \geqq 3 \end{cases},$$

where $z_{i,-i} = (z_{i,1}, \ldots, z_{i,i-1}, z_{i,i+1}, z_{i,n})$.

Define the γ-correspondence A: $M \rightarrow 2^{\mathbb{R}_+}$ by

$$A(m) = \left\{\gamma \in \mathbb{R}_+ : \gamma\gamma_i \leq 1 \ \forall i \in N \ \& \ \gamma \sum_{i=1}^{n} \gamma_i x_i(m) \leqq \hat{w} + \sum_{j=1}^{J} Y_j(m)\right\}. \tag{15}$$

Let $\bar{\gamma}(m)$ be the largest element of $A(m)$, i.e., $\bar{\gamma}(m) \in A(m)$ and $\bar{\gamma}(m) \geq \gamma$ for all $\gamma \in A(m)$.

Finally, define agent i's outcome function for consumption goods X_i: $M \rightarrow \mathbb{R}^L_+$ by

$$X_i(m) = \bar{\gamma}(m)\gamma_i x_i(m), \tag{16}$$

which is agent i's consumption resulting from the strategic configuration m.

Thus the outcome function $(X(m), Y(m))$ is continuous and feasible on M since, by the construction of the mechanism, $(X(m), Y(m)) \in \mathbb{R}^{nL}_{+} \times \mathcal{Y}$, and

$$\hat{X}(m) \leqq \hat{w} + \hat{Y}(m) \tag{17}$$

for all $m \in M$.

Remark 7 The above mechanism works not only for three or more agents, but also for a two-agent world.

3.2 The implementation result

The remainder of this section is devoted to proving the following theorem.

Theorem 1 *For the class of non-convex production economic environments E specified in Sect. 2, if the following assumptions are satisfied*:

(1) $\hat{w}$;
(2) *For each $i \in N$, preference orderings, R_i, are continuous on $\mathbb{R}^L_+$, strictly increasing on $\mathbb{R}^L_{++}$, and satisfies the interiority assumption*;
(3) *Production sets $\mathcal{Y}_j$ are nonempty, closed, $0 \in \mathcal{Y}_j$, $\{\mathcal{Y}_j - \mathbb{R}^L_+\} \subseteqq \mathcal{Y}_j$, and have twice continuously differentiable hypersurfaces for all j.*

then the mechanism defined in the above subsection, which is continuous, feasible, and uses a finite-dimensional message space, fully implements marginal cost pricing equilibrium allocations with transfers in Nash equilibrium on E.

The proof of Theorem 1 consists of the following two propositions which show the equivalence between Nash equilibrium allocations and marginal cost pricing equilibrium allocations with transfers. Lemmas 2–8 provide the groundwork for Propositions 2 and 3. Proposition 2 below proves that every Nash equilibrium allocation is a marginal cost pricing equilibrium allocation with transfers, and thus the mechanism constructed in the previous section implements marginal cost pricing equilibrium allocations with transfers. If $\mathcal{Y}_j$ has a twice continuously differentiable hypersurface for all j, Proposition 3 below proves that every marginal cost pricing equilibrium allocation with transfers is a Nash equilibrium allocation. Therefore, combining these two propositions, we know that the mechanism fully implements marginal cost pricing equilibrium allocations with transfers.

Lemma 2 *Suppose $x_i(m)\ P_i\ x_i$. Then agent i can choose a very large γ_i such that $X_i(m)\ P_i\ x_i$.*

Proof If agent i declares a large enough γ_i, then $\bar{\gamma}(m)$ becomes very small (since $\bar{\gamma}(m)\gamma_i \leqq 1$) and thus almost nullifies the effect of other agents in $\bar{\gamma}(m)\sum_{i=1}^{n}\gamma_i x_i$

$(m) \leqq \hat{w} + \sum_{j=1}^{J} Y_j(m)$. Thus, $X_i(m) = \bar{\gamma}(m)\gamma_i x_i(m)$ can arbitrarily approach $x_i(m)$ as agent i wishes. From $x_i(m)\ P_i\ x_i$, and continuity of preferences, we have $X_i(m)\ P_i\ x_i$ if agent i chooses a very large γ_i. □

Lemma 3 *If $m^* \in V_{M,h}(e)$, then $X(m^*) \in \mathbb{R}^{nL}_{++}$, $x_i(m^*) \in \mathbb{R}^{nL}_{++}$ and $x'_{ii}(m^*) \in \mathbb{R}^{nL}_{++}$ for all $i \in N$.*

Proof We argue by contradiction. Suppose $X(m^*) \in \partial\mathbb{R}^{nL}_{+}$. Then there is some $i \in N$ such that $X_i(m^*) \in \partial\mathbb{R}^{L}_{+}$. Since $\hat{w} + \sum_{j=1}^{J} Y_j(m^*) > 0$, $p_i(m^*) > 0$, and $z^*_{i-1,i} \neq 0$ by construction, we have $p_i(m^*)z^*_{i-1,i} > 0$. Thus there is some $x_i \in \mathbb{R}^{L}_{++}$ such that $p_i(m^*)x_i \leqq p_i(m^*)z^*_{i-1,i}$, $x_i \leqq \hat{w} + \sum_{j=1}^{J} Y_j(m^*)$, and $x_i\ P_i\ X_i(m^*)$ by interiority of preferences. Now suppose that agent i chooses $z_{ii} = x_i$, $\gamma_i > \gamma_i^*$, and keeps the other components of the message unchanged. Then, $z_{ii} \in B_i(m_i, m^*_{-i})$, and thus $x'_i(m_i, m^*_{-i}) = z_{ii}$. Since $x_i(m_i, m^*_{-i})$ is proportional to $x'_i(m_i, m^*_{-i})$, $x_i(m_i, m^*_{-i}) > 0$, we have $x_i(m_i, m^*_{-i})\ P_i\ X_i(m^*)$ by interiority of preferences. Therefore, by Lemma 2, agent i can choose a very large γ_i such that $X_i(m_i, m^*_{-i})\ P_i\ X_i(m^*)$. This contradicts $m^* \in V_{M,h}(e)$ and thus we must have $X_i(m^*) \in \mathbb{R}^{L}_{++}$ for all $i \in N$. Since $X_i(m^*)$ is proportional to $x_i(m^*)$ and $x'_{ii}(m^*)$, $x_i(m^*) \in \mathbb{R}^{nL}_{++}$ and $x'_{ii}(m^*) \in \mathbb{R}^{nL}_{++}$ for all $i \in N$. □

Lemma 4 *If m^* is a Nash equilibrium, then $p_1^* = p_2^*$, $\rho_1^* = \rho_2^*$, $y_1^* = t_1^* = s_1^* = y_2^*$, $\nu_1(m^*) = 0$, $\tau_1(m^*) = 0$, $z^*_{i,i+1} = x_{i+1}(m^*)$, and $z^*_{i,-i} = z^*_{i+1,-i}$ for all $i \in N$. Consequently, $p(m^*) \equiv p_i(m^*) = p_1^* = p_2^*$ for $i \in N$, $Y(m^*) = y_1^* = t_1^* = s_1^* = y_2^*$, $x_i(m^*) = x'_i(m^*)$ for all $i \in N$, and $z_1^* = z_2^* = \cdots = z_n^* = x'(m^*) = x(m^*)$.*

Proof We first show that $p_1^* = p_2^*$, $\rho_1^* = \rho_2^*$, and $y_1^* = y_2^*$. Suppose, by way of contradiction, that $p_1^* \neq p_2^*$, $\rho_1^* \neq \rho_2^*$, or $y_1^* \neq y_2^*$. Since $x'_i(m^*) > 0$ for all agent i by Lemma 3, agent 2 can choose $p_2 = p_1^*$, $\rho_1^* = \rho_2^*$, or $y_2 = y_1^*$ so that her consumption becomes larger and she would be better off by monotonicity of preferences. Hence, m^* is not a Nash equilibrium strategy if $p_1^* \neq p_2^*$, $\rho_1^* \neq \rho_2^*$, or $y_1^* \neq y_2^*$. Thus, we must have $p_1^* = p_2^*$, $\rho_1^* = \rho_2^*$, and $y_1^* = y_2^*$ at Nash equilibrium.

We now show that $t_1^* = s_1^* = y_2^*$, $z^*_{i,i+1} = x_{i+1}(m^*)$, and $z^*_{i,-i} = z^*_{i+1,-i}$ for all $i \in N$. Suppose not. Then agent i can choose a smaller $\eta_i < \eta_i^*$ in $(0, 1]$ so that her consumption becomes larger and she would be better off by monotonicity of preferences. Hence, no choice of η_i could constitute part of the Nash equilibrium strategy when $t_1^* \neq y_2^*$, $s_1^* \neq y_2^*$, $z^*_{i,i+1} = x_{i+1}(m^*)$, or $z^*_{i,-i} \neq z^*_{i+1,-i}$. Thus, we must have $t_1^* = s_1^* = y_2^*$, $z^*_{i,i+1} = x_{i+1}(m^*)$, and $z^*_{i,-i} = z^*_{i+1,-i}$ at the Nash equilibrium.

Consequently, by the construction of the mechanism, $p(m^*) \equiv p_i(m^*) = p_1^* = p_2^*$ for all $i \in N$, $Y(m^*) = y_1^* = y_2^*$, $\nu_1(m^*) = 0$, $\tau_1(m^*) = 0$, $x_i(m^*) = x'_{ii}(m^*)$ for all $i \in N$, $z_1^* = z_2^* = \cdots = z_n^* = x(m^*)$. □

Lemma 5 *Suppose there is $x_i \in \mathbb{R}^{L}_{+}$ for some $i \in N$ such that $p(m^*)x_i \leqq p(m^*)$ $X_i(m^*)$ and $x_i\ P_i\ X_i(m^*)$. Then there is some $m_i \in M_i$ such that $X_i(m_i, m^*_{-i})\ P_i$ $X_i(m^*)$.*

Proof Since $X(m^*) > 0$ by Lemma 3, $X_i(m^*) < \hat{w} + \sum_{j=1}^{J} Y_j(m^*)$. Let $x_{\lambda i} = \lambda x_i + (1-\lambda)X_i(m^*)$. Then, by convexity of preferences, we have $x_{\lambda i}\ P_i\ X_i(m^*)$ for any

$0 < \lambda < 1$. Also $x_{\lambda i} \in \mathbb{R}^L_{++}$, $p(m^*)x_{\lambda i} \leqq p(m^*)x_i(m^*)$, and $x_{\lambda i} < \hat{w} + \sum_{j=1}^J Y_j(m^*)$ when λ is sufficiently close to 0. Then, if agent i chooses $z_{ii} = x_{\lambda i}$, $\gamma_i > \gamma_i^*$, and keeps the other components of the message unchanged. Then, $z_{ii} \in B_i(m_i, m^*_{-i})$, and thus $x_i(m_i, m^*_{-i}) = x'_{ii}(m_i, m^*_{-i}) = z_{ii} = x_{\lambda i}\ P_i\ X_i(m^*)$. Therefore, by Lemma 2, agent i can choose a very large γ_i such that $X_i(m_i, m^*_{-i})\ P_i\ X_i(m^*)$. This contradicts $(X(m^*), Y(m^*)) \in N_{M,h}(e)$. □

Lemma 6 *If $(X(m^*), Y(m^*)) \in N_{M,h}(e)$, then $\bar{\gamma}(m^*)\gamma_i^* = 1$ for all $i \in N$ and thus $X(m^*) = x(m^*) = x'(m^*)$.*

Proof Suppose, by way of contradiction, that $\bar{\gamma}(m^*)\gamma_i^* < 1$ for some $i \in N$. Then $X_i(m^*) = \bar{\gamma}(m^*)\gamma_i^* x_i(m^*) < x_i(m^*)$, and thus $x_i(m^*)\ P_i\ X_i(m^*)$ by monotonicity of preferences. Therefore, by Lemma 2, agent i can choose a very large γ_i such that $X_i(m_i, m^*_{-i})\ P_i\ X_i(m^*)$. This contradicts $m^* \in V_{M,h}(e)$. Thus we must have $\bar{\gamma}(m^*)\gamma_i^* = 1$, and therefore $X(m^*) = x(m^*) = x'(m^*)$. □

Lemma 7 *If m^* is a Nash equilibrium, then $Y(m^*) \in \partial\mathcal{Y}$ and thus $\nu_{1j}(m_1, m^*_{-1}) = \nu_{1j}(m^*) = 0$ for all $m_1 \in M_1$.*

Proof First note that, under the assumptions that $\mathcal{Y}_j$ is closed, contains 0, and $\{\mathcal{Y}_j - \mathbb{R}^L_+\} \subseteq \mathcal{Y}_j$ for $j = 1, \ldots, J$, $\partial\mathcal{Y}_j$, the boundary of the production set $\mathcal{Y}_j$, is exactly the set of weakly efficient production plans. Suppose, by way of contradiction, that $Y_j(m^*) \notin \partial\mathcal{Y}_j$ for some firm j. Then, there is a production plan $y_j \in \mathcal{Y}_j$ such that $y_j > Y_j(m^*)$. Now suppose agent 1 chooses $t_{1j} = y_j$, $\gamma_1 > \gamma_1^*$, $\eta_1 < \eta_1^*$, and keeps the other components of the message unchanged. We then have $\nu_1(m_1, m^*_{-1}) = \prod_{l=1}^L (y_j^l - Y_j^l(m^*)) > 0$, and thus $x_1(m_1, m^*_{-1}) = [\nu_1(m_1, m^*_{-1}) + \frac{1}{1+\eta_1\|t_{1j} - Y(m^*)\|}]x'(m^*)$. Note that, when $\eta_1 \to 0$, $[\nu_1(m_1, m^*_{-1}) + \frac{1}{1+\eta_1\|t_{1j} - Y(m^*)\|}]$ $x'(m^*) \to [\nu_1(m_1, m^*_{-1}) + 1]x'(m^*) > x(m^*) = X(m^*)$. Thus, when η_1 is sufficiently close to zero, from $[\nu_1(m_1, m^*_{-1}) + 1]x'(m^*) > x(m^*) = X(m^*)$, we have $x_1(m_i, m^*_{-1})\ P_1\ X_1(m^*)$ by continuity of preferences. Then, by Lemma 2, agent 1 can choose a very large γ_1 such that $X_1(m_1, m^*_{-1})\ P_1\ X_1(m^*)$, which contradicts the fact m^* is a Nash equilibrium. Hence $Y_j(m^*)$ must be a weakly efficient production plan, and thus $Y(m^*) \in \partial\mathcal{Y}$. Finally, since there is no production plan $y_j \in \mathcal{Y}_j$ such that $y_j > Y_j(m^*)$ for all j, we must have $\nu_1(m_1, m^*_{-1}) = \nu_1(m^*) = 0$ for all $m_1 \in M_1$. □

Lemma 8 *If m^* is a Nash equilibrium, then $p(m^*)Y_j(m^*) \geqq p(m^*)y_j - \rho_2^*\|y_j - Y_j(m^*)\|^2$ for all $y_j \in \mathcal{Y}_j$ and thus $p(m^*) \in \perp_{\mathcal{Y}_j} (Y_j(m^*)) \subseteq N_{\mathcal{Y}_j}(Y_j(m^*))$ for $j = 1, \ldots, J$. Consequently, $p(m^*)$ satisfies the marginal cost pricing rule and $\tau_{1j}(m_1, m^*_{-1}) = \tau_{1j}(m^*) = 0$ for all $m_1 \in M_1$.*

Proof Suppose, by way of contradiction, $Y_j(m^*)$ does not maximize the quadratic profit function $p(m^*)y_j - \rho_2^*\|y_j - Y_j(m^*)\|^2$ in $\mathcal{Y}_j$ for some firm j. Then, there exists a production plan $y_j \in \mathcal{Y}_j$ such that $p(m^*)Y_j(m^*) < p(m^*)y_j - \rho\|y_j - Y_j(m^*)\|^2$.

Now suppose agent 1 chooses $s_{1j} = y_j$, $\gamma_1 > \gamma_1^*$, $\eta_1 < \eta_1^*$, and keeps the other components of the message unchanged. We have $\tau_1(m_1, m^*_{-1}) = p(m^*)y_j - \rho_2^*\|y_j - Y_j(m^*)\|^2 - p(m^*)Y_j(m^*) > 0$, and thus $x_1(m_1, m^*_{-1}) = [\tau_1(m_1, m^*_{-1}) +$

$\frac{1}{1+\eta_1\|s_{1j}-Y(m^*)\|}]x'(m^*)$. Note that, when $\eta_1 \to 0$, $[\tau_1(m_1, m^*_{-1}) + \frac{1}{1+\eta_1\|s_{1j}-Y(m^*)\|}]$ $x'(m^*) \to [\tau_1(m_1, m^*_{-1}) + 1]x'(m^*) > x(m^*) = X(m^*)$. Thus, when η_1 is sufficiently close to zero, from $[\tau_1(m_1, m^*_{-1}) + 1]x'(m^*) > x(m^*) = X(m^*)$, we have $x_1(m_i, m^*_{-1})\ P_1\ X_1(m^*)$ by continuity of preferences. Then, by Lemma 2, agent 1 can choose a very large γ_1 such that $X_1(m_1, m^*_{-1})\ P_1\ X_1(m^*)$, which contradicts the fact m^* is a Nash equilibrium. Hence, we must have $p(m^*)Y_j(m^*) \geqq p(m^*)y_j - \rho^*_2\|y_j - Y_j(m^*)\|^2$ for all $y_j \in \mathcal{Y}_j$, and thus $p(m^*) \in \perp_{\mathcal{Y}_j} (Y_j(m^*)) \subseteq N_{\mathcal{Y}_j}(Y_j(m^*))$ for $j = 1, \ldots, J$, which means $p(m^*)$ satisfies the marginal cost pricing rule. Finally, since $Y_j(m^*)$ maximizes the quadratic profit function $p(m^*)y_j - \rho^*_2\|y_j - Y_j(m^*)\|^2$ over the production set $\mathcal{Y}_j$ for all j, we must have $\tau_1(m_1, m^*_{-1}) = \tau_1(m^*) = 0$ for all $m_1 \in M_1$, by the definition of $\tau_1(\cdot)$. □

Proposition 2 *If the mechanism defined above has a Nash equilibrium m^*, then the Nash equilibrium allocation $(X(m^*), Y(m^*), p(m^*))$ is a marginal pricing equilibrium allocation with transfers, i.e., $N_{M,h}(e) \subseteq MPCT(e)$.*

Proof Let m^* be a Nash equilibrium. We need to prove that $(X(m^*), Y(m^*), p(m^*))$ is a marginal cost pricing equilibrium allocation with transfers. Note that the feasibility condition is satisfied, all individuals take $p(m^*)$ as given by the construction of the mechanism, and $p(m^*)$ satisfies the marginal cost pricing rule by Lemma 8. So we only need to show that $X_i(m^*)$ is the greatest element for R_i in the budget set $\{x_i \in \mathbb{R}^L_+ : p(m^*)x_i \leqq p(m^*)X_i(m^*)\}$ for all $i \in N$.

Suppose, by way of contradiction, that there is some $x_i \in \mathbb{R}^L_+$ such that $x_i\ P_i\ X_i(m^*)$, and $p(m^*)x_i \leqq p(m^*)X_i(m^*)$. Since $X(m^*) \in \mathbb{R}^{nL}_{++}$ by Lemma 3, there is some $m_i \in M$ such that $X_i(m_i, m^*_{-i})\ P_i\ X_i(m^*)$ by Lemma 5. This contradicts $(X(m^*), Y(m^*)) \in N_{M,h}(e)$. Thus, $(X(m^*), Y(m^*))$ is a marginal cost pricing equilibrium allocation with transfers. □

Proposition 3 *If $\mathcal{Y}_j$ has a twice continuously differentiable hypersurface and (x^*, y^*) is a marginal cost pricing equilibrium allocation with transfers with $p^* \in \Delta^{L-1}_+$ as the equilibrium price system such that $x^*_i > 0$ for all $i \in N$, then there is a Nash equilibrium m^* of the above mechanism such that $Y(m^*) = y^*$, $p(m^*) = p^*$, and $X_i(m^*) = x^*_i$ for $i \in N$, i.e., $MCPT(e) \subseteq N_{M,h}(e)$.*

Proof We first note that by the strict monotonicity of preference orderings, the normalized price vector p^* must be in Δ^{L-1}_{++}. Since $\mathcal{Y}_j$ has a twice continuously differentiable hypersurface, $p^* = \nabla f_j(y^*_j)/\|\nabla f_j(y^*_j)\|_1 \in \perp_{\mathcal{Y}_j} (y^*_j) = N_{\mathcal{Y}_j}(y^*_j)$, for $j = 1, 2, \ldots, J$, where $\|\cdot\|_1$ is the l_1-norm. Consequently, there exists a $\rho^* \geqq 0$ such that $p^*y^*_j \geqq p^*y_j - \rho^*\|y_j - y^*_j\|\}$ for all $y_j \in \mathcal{Y}_j$. We need to show that there is a message m^* such that (x^*, y^*) is a Nash equilibrium allocation. For each $i \in N$, define m^*_i by $p^*_i = p^*$, $\rho^*_1 = \rho^*_2 = \rho^*$, $y^*_1 = t^*_1 = s^*_1 = y^*_2 = y^*$, $z^*_i = x^*$, $\gamma^*_i = 1$, and $\eta^*_1 = 1$. Then, it can be easily verified that $Y(m^*) = y^*_1 = y^*_2 = t^*_1 = s^*_1 = y^*$, $p_i(m^*) = p^*$, and $X_i(m^*) = x^*_i$ for all $i \in N$. Then, $p_i(m_i, m^*_{-i}) = p_i(m^*)$, $p(m^*)Y_j(m^*) \geqq p(m^*)y_j - \rho^*_2\|y_j - Y_j(m^*)\|^2$ for all $y_j \in \mathcal{Y}_j$. Thus $\nu_1(m_1, m^*_{-1}) = \nu_1(m^*) = 0$, $\tau_1(m_1, m^*_{-1}) = \tau_1(m^*) = 0$ for all $m_1 \in M_1$, and

$p(m^*) = \nabla f_j(Y_j(m^*))/\|\nabla f_j(Y_j(m^*))\|_1 \in \perp_{\mathcal{Y}_j}(Y_j(m^*)) = N_{\mathcal{Y}_j}(Y_j(m^*))$ for $j = 1, 2, \ldots, J$. Hence

$$p(m^*) \cdot X_i(m_i, m^*_{-i}) \leqq p(m^*) \cdot X_i(m^*), \tag{18}$$

for all $m_i \in M_i$. Thus, $X_i(m_i, m^*_{-i})$ satisfies the budget constraint for all $m_i \in M_i$. Thus, we must have $X_i(m^*) \ R_i \ X_i(m_i, m^*_{-i})$, or it will contradict the fact that $(X(m^*), Y(m^*))$ is a marginal cost pricing equilibrium allocation with transfers. Therefore, $(X(m^*), Y(m^*))$ is a Nash equilibrium allocation. □

Proof of Theorem 1 By Proposition 2, $N_{M,h}(e) \subseteq MCPT(e)$, which means the mechanism implements $MCPT_c$ equilibrium allocations with transfers in Nash equilibrium. When $\mathcal{Y}_j$ has a twice continuously differentiable hypersurface, by Proposition 3, we know that $MCPT(e) \subseteq N_{M,h}(e)$. Thus, by combining Proposition 2 and Proposition 3, the mechanism fully implements $MCPT$ equilibrium allocations with transfers in Nash equilibrium. The proof of Theorem 1 is completed. □

Remark 8 When a production set does not have twice continuously differentiable hypersurfaces, our mechanism may not have a Nash equilibrium. Indeed, Cornet (1990) gave an example for which $\perp_{\mathcal{Y}_j}(y^*_j) = \{0\}$. Cornet (1990) provided the conditions under which a production set may not have twice continuously differentiable hypersurfaces, but that guarantee

$$\perp_{\mathcal{Y}_j}(y^*_j) = N_{\mathcal{Y}_j}(y^*_j) = \{\lambda \nabla f_j(y^*_j) : \lambda \geqq 0\}$$

for all $y \in \partial\mathcal{Y}$. If so, the existence of Nash equilibria of the mechanism is guaranteed. Also, by Cornet (1990), any Pareto efficient allocation can be supported by a $MCPT$ equilibrium with $p^* \in \perp_{\mathcal{Y}_j}(y^*_j) \subseteq N_{\mathcal{Y}_j}(y^*_j)$. Thus, our mechanism achieves Pareto efficient allocations when production sets have twice continuously differentiable hypersurfaces or when the conditions specified in Lemma 1 in Cornet (1990) are satisfied.

4 Concluding remarks

In this paper we consider implementation of marginal cost pricing equilibrium allocations with transfers that contains the set of Pareto optimal allocations as a subset, for general non-convex production economies. We present a specific mechanism that uses a finite-dimensional message space and fully implements marginal cost pricing equilibrium allocations with transfers when preferences and productions sets are unknown to the designer.

The mechanism constructed in the paper is well-behaved and has several desired properties. It is continuous, feasible, and has a finite-dimensional message space. It is also a market type mechanism. The price and quantity are components of the message spaces. In addition, it works not only for three or more agents, but also for two-agent economies and thus it is a unified mechanism irrespective of the number of agents.

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