

Endogenous Information Acquisition on Opponents' Valuations in Multidimensional First Price Auctions*

Guoqiang Tian

Department of Economics

Texas A&M University

College Station, Texas, 77843

U.S.A

Mingjun Xiao

School of Economics

Shanghai University of Finance and Economics

Shanghai, 200433

China

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Abstract

This paper investigates bidder's covertly behavior of endogenous information acquisition on her opponents' valuations in first price auction model with independent private values. Such an information acquisition setting leads to a *real* multidimensionality, where the type and strategy spaces of the bidder both are bidimensional. We consider two different specifications of the environments: the *ex ante* information acquisition setting and the *interim* information acquisition setting. In equilibria the expected payoffs of the bidder under these specifications could exceed the counterpart payoffs of the corresponding standard sealed-bid auction environment without information acquisition as long as the cost is sufficiently small, but the auctioneer has lower payoffs in these models than those of the standard case. Moreover, in most cases bidder gets higher expected payoff from interim (ex ante) IA equilibrium than from exogenous IA equilibrium.

Key words: Multidimensional sealed-bid auctions, endogenous information acquisition, opponents' valuation, profitability.

JEL Classification: C70; D44; D82.

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1 Introduction

The information acquisition problem in auctions occurs when a bidder does not have a clear idea about her exact willingness to pay for the underlying auctioned object, which can happen in both common value auctions and private value auctions. Much work has been done on this problem. This paper considers another type of information acquisition problem in which the bidder knows her valuation of the auctioned object and also tries to learn the other bidders' valuations. Thus each bidder has a bidimensional type that consists of her own valuation (the valuation type) and the signal about her opponent's valuation (information type).

In realistic economic activities, like auctions or procurement, and other similar situations, bidders can have information about their own estimations of the auctioned object or project and can also have information about their opponents' evaluations of the underlying object besides their common prior. The later information may come from bidders' strategic actions or from some other parties, which helps update bidders' beliefs on opponents. In other words, this is an environment where bidders have multidimensional information, or types in our terminology. For example, in China many middle-size and small-size state-owned enterprises have been privatized by way of procurement in the past several years. There usually two or three potential buyers, the interactions between these buyers are rather involved and they may know part of their opponents' inside information. Another example is the NBA draft. The recruiting competition can be compared to an auction over the top-ranked candidates by assuming that the preferences of candidates only rely on monetary benefit. It has been observed during the period that clubs take measures to hide their own belief about candidates' value, and employ "detectives" to investigate the confidential information of other clubs' prices for recruiting their desired candidate. We can see that club could have multidimensional information, its own evaluation and possibly other clubs' evaluations of candidates in this competitive environment.

This paper tries to capture such a multidimensional information structure and to investigate the source of multidimensionality by way of information acquisition¹ on opponents. For independent private values, the bidder's own valuation does not tell her any information about other bidders' valuations. This means the bidder cannot update her belief based on her own valuation signal. A natural question, then, is whether the bidder has incentives to acquire information about her opponents' valuations. Though it is somewhat difficult to find an example in the context of independent private value such that it is fully consistent with this kind of informa-

¹For simplicity, we always use "information acquisition" to refer to the bidder acquiring information about her opponents' private valuations. Sometimes information acquisition is abbreviated as IA. This is different from the meaning elsewhere in the literature. Sometimes information acquisition, information acquiring, information collection, and collecting information are used interchangeably.

tion acquisition behavior, our paper takes the first attempt to model bidder's multidimensional information structure.

The auction format we mainly focus on is the first price sealed-bid auction (abbreviated as FPA), since this sealed-bid auctions have been widely used in practice. They are used in auctioning mineral rights in government-owned land, and the sales of artwork and real estate as well as procurement. Examples include the U.S. offshore oil and gas lease auctions, the UK Treasury securities auctions which are the multiunit equivalent of the first-price auction (every winner pays her own bid).² As Klemperer (2003, pp.2) has claimed that sealed-bid designs frequently (but not always) both attract a larger number of serious bidders and are better at deterrence of collusion than are ascending designs, see also Klemperer (1998, 2000, 2002). Maskin and Riley (1984) show that the first price sealed-bid auction is the most profitable one amongst the standard auctions when bidders are risk-averse. When considering bidders have outside options, Kirchkamp et al. (2008) show that bidders in first-price auctions have more overbidding with outside options than without, and first-price auctions yield more revenue than second-price auctions. As for second-price sealed-bid auctions (SPA) or Vickrey Auctions, it might be trivial to consider such a type of information acquisition on opponents' valuations because opponents' signals does not play any role in the formulation of strategies in SPA. These reasons lead us to consider first price auctions.

In our paper, the bidder can covertly incur a cost to achieve a noisy signal conveying information about her opponents' valuations. We assume neither the bidder's information acquisition choice nor the signal received by the bidder can be observed by other bidders. For simplicity, the accuracy of the signal and the cost of the signal are exogenously determined. In our private value auction model, we analyze only the two-bidder case, where each bidder's valuation of the auctioned object is independently drawn from an identical distribution that has only two possible values.

We consider two different specifications of the environments: the *ex ante* information acquisition setting where bidders invest in opponents' valuations before observing their own valuations and the *interim* information acquisition setting where bidders invest in opponents' valuations after observing their own valuations. This is because both the interim and ex-ante information acquisitions are equally interesting in daily economic activities. In some case one can obtain information about one's opponents prior to or when evaluating its own strength exactly in the competition. For example, a firm needs to know about the incumbents of the market when it considers to enter that industry. In some case one may intentionally get to know his or her com-

²See the survey paper by Porter (1995) and footnote 14 in Klemperer (2003, chapter 1). Besides, the U.S. Treasury auctions used to be run with the multiunit version of first price auctions too.

petitors, namely, an incumbent would like to expect other firms' reactions when it introduces a new round of competition since the rivals' strategies are of particular importance. In other cases, one may know partial information on opponents as a byproduct with one's own value and then decide whether to invest in more.

Most studies of information acquisition in auctions focus on how to improve the estimate of true valuation, since the bidder does not know her valuation when the auction begins, i.e., Matthews (1984) and Larson (2006) in context of common values, Persico (2000) with affiliated values, Shi and Valimaki (2007) with interdependent values, Rezende (2005), Compte and Jehiel (2007), Shi (2007) in private value settings. All these papers share the same characteristic that bidders do not know their exact valuations at the beginning of the auction. Tan (1992) and Arozamena and Cantillon (2004) who investigate the incentives for investment before auctions, and Hernando-Veciana (2006) considers the incentives of a bidder to acquire information in an auction when her information acquisition decision is observed by the other bidders before they bid.³

Our models have some similarities with that of Persico (2000), in which the bidder's strategy is bidimensional (consisting of cost to induce more accurate signal and the bidding rule). In our paper, the bidder's strategy is also two-dimensional, and the bidder has freedom to choose the probability to incur a fixed cost and receive a signal of certain accuracy. However, the basic structures of the two papers are quite different. Our paper also shares some common characteristics with Fang and Morris (2006). First, both papers study the independent private value auction model where the bidder knows exactly what her valuation is at the beginning of the auction. In addition, the costly information acquisition behavior refers to collecting information about opponents' valuations. However, these two models have a big difference. In Fang and Morris' model, the behavior of acquiring information is exogenously given, it may result in irrational behavior since the bidder could be better by taking information acquisition as a choice variable. Thus an equilibrium under their model may be only an *irrational self-restricted* equilibrium and is not an equilibrium under the rational behavior of bidders. Our paper is aimed to solve this problem under the standard rational behavior assumption by endogenizing the choice of information acquisition behavior. Though theoretically it is not guaranteed that bidder can achieve more expected payoff from endogenous information acquisition setting than from exogenous information acquisition model, this indeed turns to be true that in most cases bidder can obtain higher payoff in exogenous information acquisition models.

³There is also a strand of papers studying information acquisition problem where the auctioneer has controlled the information resources, such as Bergemann and Pesendorfer (2002) and Ganuza (2004), Eso and Szentes (2007). Milgrom and Weber (1982) also investigates the seller's information disclosure policy in affiliated setting.

The contribution of this paper is twofold. First, our model takes an initial step in endogenizing information acquisition on opponent's valuations in independent private value models. Second, in our endogenous model, both the type space and strategy space of bidders are bidimensional,⁴ which is a *rational choice* equilibrium analysis and differs from the current literature. In the model considered by Fang and Morris (2006), the bidder's type is two-dimensional, but the strategy of the bidder remains of single dimension. On the other hand, Persico (2000) constructs the auction environment in which the bidder's type is of a single dimension, but the bidder's strategy is two-dimensional, consisting of the information acquisition choice (the accuracy of the signal) and the bidding rule. Though there might not be real examples that can fully match our information acquisition model with independent private values, the multidimensionalities of type space and strategy space might capture bidders' real situations in auction activities.

The remainder of this paper is organized as follows. Section 2 describes the basic setting of the model. Section 3 discusses bidders' behavior when bidders can make ex ante information acquisition choices. Section 4 considers a more relaxed environment in which bidders make their information acquisition choices at an interim stage. Section 5 discusses the implications: efficiency, surplus, and revenue across FPA and SPA formats; and the notions of endogenous information acquisition versus exogenous information acquisition. Section 6 discusses some variants of the model. Section 7 concludes. All the proofs are relegated to the appendix.

2 Basic Setting of the Model

There are two risk neutral bidders indexed by $i \in \mathcal{N} = \{1, 2\}$ who compete for an underlying object. The object is worth zero to the seller. Every bidder $i \in \mathcal{N}$ has a valuation $v_i \in \mathcal{V} = \{V_l, V_h\}$ for the object, where $0 < V_l < V_h$. The valuations v_i are private and independently drawn from identical distribution. The prior distribution of bidder i 's valuation v_i is given by $Pr\{v_i = V_l\} = p_l$ ($0 < p_l < 1$), and $Pr\{v_i = V_h\} = p_h = 1 - p_l$ for $i \in \mathcal{N}$. Then $v = (v_1, v_2) \in \mathcal{V} \times \mathcal{V}$ is a profile of possible valuations.

The auction format is the first price sealed-bid auction. The reservation price of the seller is normalized to zero. Each bidder i can incur an exogenous cost c to get a noisy signal s_i which induces information about her opponent's private valuation. If the bidder does not want

⁴In our independent private value auction models, the type of the bidder is determined by the bidder's valuation type and the signal received, and the strategy of the bidder includes the information acquisition choice and bidding rule. As we will show in the appendix A, the metric dimension of strategy space is at least as big as the metric dimension of type space for each bidder in interim IA equilibrium. Therefore, our model relaxes the restrictive approach of encoding multidimensional types into a single-dimensional variable and is a real multidimensional model. From the point of view in general mechanism design (cf. Hurwicz (1979), Mount and Reiter (1974), and Tian (1994, 2004, 2006)), the information smuggling of changing multi-dimensional types (say, by the inverse Peano curve) into a single-dimensional variable is essentially equivalent to one-dimension type problem.

to collect information about her opponent's valuation, she cannot receive any signal and does not incur any cost. To clearly describe the information structure, we assume neither the bidder's information acquisition choice nor the signal received by the bidder can be observed by her opponent.

The signal s_i received by bidder i is drawn from the support $\mathcal{S} = \{L, H\}$. For convenience, we denote s_0 as receiving no signal, and let $\bar{\mathcal{S}} = \{L, H, s_0\}$. Given bidder j 's valuation, the probability of the transmitted signal s_i taking on each possible value is characterized by

$$\Pr\{s_i = L|v_j = V_l\} = \Pr\{s_i = H|v_j = V_h\} = q, \quad (1)$$

for $i \neq j$, $i, j \in \mathcal{N}$; note that $\Pr\{s_i = L|v_j = V_l\} + \Pr\{s_i = H|v_j = V_l\} = 1$ and $\Pr\{s_i = H|v_j = V_h\} + \Pr\{s_i = L|v_j = V_h\} = 1$. Since all the signals are characterized by one parameter q , we call q the accuracy of the signal. Without loss of generality, we assume $0.5 < q \leq 1$. Bidder j updates her belief about i 's information type s_i according to the signal distribution specified by equation (1).

The auction game we will analyze is a Bayesian game of incomplete information. The type space of each bidder is $\mathcal{T} \equiv \{V_l, V_h\} \times \{L, H, s_0\}$. One representative type of bidder i is $t_i = (v_i, s_i) \in \mathcal{T}$. We call v_i the *valuation type*, and s_i the *information type*. Given bidder i 's signal s_i , bidder i updates her belief on her opponent's valuation type according to the Bayes' rule as follows. For $s_i \in \{L, H\}$, $i \neq j$, $i, j \in \mathcal{N}$,

$$\begin{aligned} \Pr\{v_j = V_l|s_i = L\} &= \frac{p_l q}{p_l q + p_h(1-q)}, \\ \Pr\{v_j = V_l|s_i = H\} &= \frac{p_l(1-q)}{p_l(1-q) + p_h q}; \end{aligned} \quad (2)$$

and the conditional probability is also a probability distribution, i.e. $\Pr\{v_j = V_l|s_i = L\} + \Pr\{v_j = V_h|s_i = L\} = 1$ and $\Pr\{v_j = V_l|s_i = H\} + \Pr\{v_j = V_h|s_i = H\} = 1$.

3 Ex Ante Endogenous Information Acquisition

In this section we consider an auction economy in which bidders can acquire information on their opponents' valuations *ex ante*. That is, the bidder can decide whether to collect information before knowing her own valuation.

The procedure of this auction game is as follows:

- Stage 1. Bidders simultaneously decide whether to incur a cost c to collect information on her opponent's valuation, and this decision is not observed by her opponent.

- Stage 2. Nature draws a valuation for each bidder and tells the bidder only what her own valuation is. Every bidder receives a signal s_i revealing her opponent's valuation if she incurs a cost c in Stage 1. Otherwise, she receives no signal. Bidders' signal is unobservable for her opponent.
- Stage 3. Bidders submit their bids simultaneously based on their own valuations, and possibly their updated belief from the signal according to (2).
- Stage 4. The bidder whose bid is the highest receives the object and pays what she bids.⁵

The unique equilibrium for this FPA is summarized in the following proposition.

Proposition 1 *When the informative cost is sufficiently small, i.e. $c \leq p_l p_h^2 (V_h - V_l)(2q - 1)$, if the bidder can acquire information ex ante, then the unique equilibrium of the FPA is symmetric and is described as follows. For $i = 1, 2$*

1. *With probability π_e , the bidder collects information about her opponent's private valuation; with probability $(1 - \pi_e)$ the bidder does not collect information.*
2. $b_i(V_l, s) = V_l$ for $s \in \{s_0, L, H\}$.
3. *Type- (V_h, s_0) bidder mixes over $[\delta_1, \delta_2]$ according to the cumulative distribution function(c.d.f.) $J^{s_0}(\cdot)$ specified by*

$$J^{s_0}(b) = \frac{p_l + \pi_e(1 - q)p_h}{p_h(1 - \pi_e)} \cdot \frac{b - \delta_1}{V_h - b}, \quad b \in [\delta_1, \delta_2]. \quad (3)$$

4. *Type- (V_h, L) bidder mixes over $[V_l, \delta_1]$ according to the c.d.f. $J^l(\cdot)$ specified by*

$$J^l(b) = \frac{p_l q}{\pi_e p_h (1 - q)^2} \cdot \frac{b - V_l}{V_h - b}, \quad b \in [V_l, \delta_1]. \quad (4)$$

5. *Type- (V_h, H) bidder mixes over $[\delta_2, \delta_3]$ according to the c.d.f. $K^h(\cdot)$ specified by*

$$J^h(b) = \frac{p_l(1 - q) + p_h q(1 - \pi_e q)}{\pi_e p_h q^2} \cdot \frac{b - \delta_2}{V_h - b}, \quad b \in [\delta_2, \delta_3]. \quad (5)$$

⁵When there are ties, both bidder are asked to report a number, either 0 or ϵ^* where ϵ^* is a sufficiently small positive number. If only one bidder reports ϵ^* , then this bidder gets the object and pays his bid; else the bidders get the object with equal probability and the bidder having the object pays her bid plus her reported number. This tie-breaking rule guarantees whenever a bidder with higher valuation submits the same bid as the bidder with lower valuation, the former wins. This tie-breaking rule is called Vickrey rule. Actually it does not matter which tie-breaking rule is used here, we just choose the Vickrey one for convenience.

Here

$$\delta_1 = \frac{p_l q V_l + \pi_e p_h (1 - q)^2 V_h}{p_l q + \pi_e p_h (1 - q)^2}, \quad (6)$$

$$\delta_2 = \frac{p_h (1 - \pi_e) V_h + [p_l + \pi_e (1 - q) p_h] \delta_1}{p_l + p_h (1 - \pi_e q)}, \quad (7)$$

$$\delta_3 = \frac{\pi_e p_h q^2 V_h + [p_l (1 - q) + p_h q (1 - \pi_e q)] \delta_2}{p_l (1 - q) + p_h q}, \quad (8)$$

and π_e is determined by the following condition:

$$\begin{aligned} & p_h \{ p_l q (V_h - V_l) + [p_l (1 - q) + p_h q (1 - \pi_e q)] (V_h - \delta_2) \} - c \\ & = p_h [p_l + \pi_e (1 - q) p_h] (V_h - \delta_1). \end{aligned} \quad (9)$$

Remark 1 *Provided that the informative cost is negligible, i.e., $c = 0$, the information acquisition probability π_e reduces to 1, which is the same equilibrium outcome as that of the exogenous information acquisition (Proposition 1 in Fang and Morris, 2006), all parameters being equal.*

The type- V_l bidder always bids her valuation V_l . The type- (V_h, L) , type- (V_h, s_0) and type- (V_h, H) bidders play mixed strategy in equilibrium, and type- (V_h, L) 's support is lower than type- (V_h, s_0) 's support which is also lower than type- (V_h, H) 's. This is because that type- (V_h, L) bidder perceives her opponent to be more likely type- V_l , type- (V_h, H) bidder regards her opponent more likely as type- V_h bidder, and type- (V_h, s_0) bidder still takes her ex ante belief. Then their best responses would result in such a support ranking.

In this unique equilibrium of ex ante information acquisition, the object is always efficiently allocated. So the total ex ante expected social welfare is

$$TS^{EX} = p_l^2 V_l + (1 - p_l^2) V_h - 2\pi_e c, \quad (10)$$

which is total expected surplus minus total expected cost. Since bidder plays mixed strategy in information acquisition choice, we consider the bidder who does not acquire information. This bidder obtains positive surplus only when her type is (V_h, s_0) which occurs with probability p_h . If the type- (V_h, s_0) bidder submits a bid $b = \delta_1$, she wins against a type- V_l opponent which occurs with probability p_l and against a type- (V_h, L) opponent which occurs with probability $p_h \pi_e (1 - q)$. Then in equilibrium the bidder's total expected payoff is $p_h [p_l + p_h \pi_e (1 - q)] (V_h - \delta_1)$.

By substituting δ_1 into this formula of expected payoff, we have⁶

$$S^{EX}(q) = \frac{p_l q + p_h \pi_e q (1 - q)}{p_l q + p_h \pi_e (1 - q)^2} \cdot p_l p_h (V_h - V_l). \quad (11)$$

The following corollary tells that bidders' expected payoff has a monotone property.

Corollary 1 *In the presence of noisy signal, i.e. $q \in (0.5, 1)$, and for $c \leq p_l p_h^2 (V_h - V_l) (2q - 1)$, then bidder's net payoff S^{EX} is decreasing in informative cost c ; hence the net payoff S^{EX} also exceeds its counterpart from standard FPA without IA.*

In fact there would be no information acquisition in equilibrium if the informative cost is too high, namely, $c > p_l p_h^2 (V_h - V_l) (2q - 1)$. This together with the decreasing property of bidder's payoff in c , implies the competition between bidders is not so fierce as that of the counterpart standard auctions. This is because by knowing more about the opponents' valuations, bidders bid more conservatively. For instance, the type- (V_h, L) submits her bid very closely to V_l even if the signal is not completely accurate ($q < 1$).

The simple parametric model admits us to fully characterize the equilibrium behavior of bidders, a mapping from bidimensional type space to bidimensional strategy space.⁷ The multidimensional strategy space is one of the main characteristics of the model that differs from the current literature in auction models with information acquisition. Though this *ex ante* IA equilibrium outcome does not promise metric dimension equivalence between the type space and the strategy space as we demonstrate in Appendix A, it does not mean that the strategy space can be viewed as unidimensional. On one hand, the bidders' strategy has two parts: information acquisition action and bidding rule. On the other hand, the failure of one-to-one mapping is due to the assumption of *ex ante* information acquisition. See Appendix A for details.

If we relax the endogenous information acquisition assumption, i.e., permitting collecting information at *interim* stage, the results discussed in this section can even be intensified. This will be investigated in the next section.

⁶Note that here the revenue equivalence fails, since by (11) the expected surplus here exceeds the counterpart of standard FPA. It's true that allocations do not pin down the revenue in discrete type settings, which might contribute to the failure of revenue equivalence in standard auctions. But Fang and Morris (2006) already show that in standard auctions without information acquisition, the revenue equivalence holds in discrete setting. It follows that the realized asymmetric information structure contributes to the failure of the RET, that is, $0.5 < q < 1$; besides, the informative cost c also plays a role here. The situation is the same for interim IA in the next section.

⁷The definition of this mapping and its pertinent discussion can be seen in Appendix A.

4 Interim Information Acquisition

In the previous section we considered the auction economy in which the bidder can freely choose to collect information about her opponent's valuation *ex ante*, but it is still very restrictive in the sense that the type- V_l bidder in the interim stage would definitely ignore the signal that she receives. In other words, if the realized valuation is V_l , the bidder cannot benefit from the signal she received because she cannot do better than bidding her valuation V_l . In this section we consider the sealed-bid auction economy in which information acquisition choice is made after bidders' valuations are realized. This will give the bidders more freedom to formalize their strategy in the auction game.

The procedures of this auction game is as follows:

- Stage 1. Nature draws a valuation for each bidder and tells the bidder only what her own valuation is.
- Stage 2. Every bidder decides whether to incur a cost c to receive a signal s_i based on her valuation, and this decision as well as the signal received can not be observed by her opponent.
- Stage 3. Bidders submit their bids simultaneously based on their own valuations, and possibly the updated belief from the signal received according to (2).
- Stage 4. The bidder whose bid is the highest receives the object and pays what she bids.

Note that under the current specification, the bidders acquire information after knowing their own valuations. Then only the type- V_h bidder has incentives to receive costly informative signal. The unique equilibrium of the interim information acquisition is summarized as follows.

Proposition 2 *When the informative cost is sufficiently small, i.e. $c \leq p_l p_h (V_h - V_l)(2q - 1)$, if the bidder can acquire information at interim stage, then the unique equilibrium of the FPA is symmetric and is described as follows. For $i = 1, 2$*

1. *The type- V_l bidder does not incur any cost to receive any signal.*
2. *The type- V_h bidder, with probability π_i , chooses to incur a cost c to collect information on her opponent's private valuation; with probability $1 - \pi_i$ she does not incur any cost.*
3. $b_i(V_l, s_0) = V_l$.

4. Type- (V_h, s_0) bidder mixes over $[\beta_1, \beta_2]$ according to the cumulative distribution function (c.d.f.) $Q^{s_0}(\cdot)$ specified by

$$Q^{s_0}(b) = \frac{p_l + \pi_i(1-q)p_h}{p_h(1-\pi_i)} \cdot \frac{b - \beta_1}{V_h - b}, \quad b \in [\beta_1, \beta_2]. \quad (12)$$

5. Type- (V_h, L) bidder mixes over $[V_l, \beta_1]$ according to the c.d.f. $Q^l(\cdot)$ specified by

$$Q^l(b) = \frac{p_l q}{\pi_i p_h (1-q)^2} \cdot \frac{b - V_l}{V_h - b}, \quad b \in [V_l, \beta_1]. \quad (13)$$

6. Type- (V_h, H) bidder mixes over $[\beta_2, \beta_3]$ according to the c.d.f. $K^h(\cdot)$ specified by

$$Q^h(b) = \frac{p_l(1-q) + p_h q(1-\pi_i q)}{\pi_i p_h q^2} \cdot \frac{b - \beta_2}{V_h - b}, \quad b \in [\beta_2, \beta_3]. \quad (14)$$

Here

$$\beta_1 = \frac{p_l q V_l + \pi_i p_h (1-q)^2 V_h}{p_l q + \pi_i p_h (1-q)^2}, \quad (15)$$

$$\beta_2 = \frac{p_h (1-\pi_i) V_h + [p_l + \pi_i (1-q) p_h] \beta_1}{p_l + p_h (1-\pi_i q)}, \quad (16)$$

$$\beta_3 = \frac{\pi_i p_h q^2 V_h + [p_l(1-q) + p_h q(1-\pi_i q)] \beta_2}{p_l(1-q) + p_h q}, \quad (17)$$

and π_i is determined by the following condition

$$\begin{aligned} & p_l q (V_h - V_l) + [p_l(1-q) + p_h q(1-\pi_i q)] (V_h - \beta_2) - c \\ & = [p_l + \pi_i(1-q)p_h] (V_h - \beta_1). \end{aligned} \quad (18)$$

Remark 2 As in the unique equilibrium of ex ante information acquisition, when the informative cost is negligible, i.e., $c = 0$, the information acquisition probability π_e reduces to 1.

In the unique equilibrium if interim information acquisition specified by Proposition 2, type- V_l bidder has no incentive to acquire information and bids her valuation V_l , type- V_h bidder plays a mixed strategy in information acquisition choice. The type- (V_h, L) , type- (V_h, s_0) and type- (V_h, H) bidders randomly draw their bids in equilibrium, and type- (V_h, L) 's support is lower than type- (V_h, s_0) 's, the latter is also lower than type- (V_h, H) 's. This is similar to the bidding rule of ex ante information acquisition, so is the reason behind.

Note that only type- V_h bidder acquires information, and the type- V_h bidders receiving different signals draw bids from disparate distributions. Apparently, it is a one-to-one mapping from bidder's type space to strategy space. Therefore, the equilibrium strategy passes the metric

examination as we have shown in Appendix A. This suggests that the metric dimension of the strategy space is at least as large as the type space, which we call the *real* multidimensionality.

In this unique equilibrium of interim information acquisition, the object is efficiently allocated, hence the total ex ante expected social welfare is

$$TS^{INT} = p_l^2 V_l + (1 - p_l^2) V_h - 2\pi_i c, \quad (19)$$

which equates to the total expected surplus minus total expected informative cost. Now that the type V_l bidder bids her valuation and gets zero surplus, we just need to consider the type- V_h bidder. Since bidder with valuation V_h randomizes in information acquisition choice, she must obtain the same expected surplus whatever the choice is. Conditional on that the bidder's type is (V_h, s_0) , she mixes over the interval $[\beta_1, \beta_2]$ and any bid drawn from this interval gives her the same payoff. Suppose she submits a bid $b = \beta_1$, she wins against a type- V_l bidder which occurs with probability p_l , and against a type- (V_h, L) bidder which occurs with probability $p_h \pi_i (1 - q)$. Then in equilibrium the bidder's total expected payoff is $p_h [p_l + p_h \pi_i (1 - q)] (V_h - \beta_1)$. By substituting β_1 into this formula of expected payoff, we have

$$S^{INT}(q) = \frac{p_l q + p_h \pi_i q (1 - q)}{p_l q + p_h \pi_i (1 - q)^2} \cdot p_l p_h (V_h - V_l). \quad (20)$$

Hence we have conclusions in the following corollary.

Corollary 2 *In the presence of noisy signal, i.e. $q \in (0.5, 1)$, and for $c \leq p_l p_h (V_h - V_l) (2q - 1)$, we have*

1. *Bidder's net payoff S^{INT} is decreasing in informative cost c , and hence exceeds its counterpart payoff from standard FPA without IA;*
2. *It also exceeds the payoff in the symmetric ex ante IA equilibrium, i.e., $S^{EX} \leq S^{INT}$, other conditions being equal.*

Obviously the type V_l bidder always get zero surplus and has no incentive to acquire information. In this section, such a bidder needs not to pay any cost, which is impossible for her in the ex ante information acquisition environment. This leads to a higher probability to acquire information for type- V_h bidder, see proof of Corollary 2. As a result bidders' ex ante expected surplus here turns to be higher than its counterpart with ex ante information acquisition.

5 Discussion: Implications of Information Acquisition

5.1 Comparison Across Auction Formats

It is obvious that the object is always efficiently allocated since bidders have binary values in our first price auction model (FPA) with information acquisition. As we have discussed in the very beginning, information acquisition plays no role in formulating bidders' strategies in second price auction (SPA) since each bidder has weakly dominant strategy to bid truthfully, but it remains worth making comparisons across FPA and SPA formats because the revenue equivalence still holds in these auction in standard settings without any type of information acquisition.⁸

Given that the FPA and SPA both are permitted in our basic setting with information acquisition, the total surplus then is ranked according to the expected total informative cost across auction formats. Note that bidders' strategies do not change in SPA, and remain the same as the standard case, which says that the allocation efficiency is always achieved in SPA. This leads to the following proposition

Proposition 3 *Assume $0 < c < p_l p_h^2 (V_h - V_l)(2q - 1)$ and that all parameters are equal, both FPA and SPA are feasible in our basic settings. Then the ex ante expected total surplus in SPA exceeds the counterpart in FPA regardless of the stage at which the IA option is permitted.*

The conclusion in the proposition is straightforward because information acquisition is socially costly in our specifications and it does not have any influence on allocation efficiency at the same time.

From the perspective of sellers, they might mainly concern how much revenue the selling mechanism can get for them. On the other hand, bidders may care about their surplus in the auction. In SPA, the seller gets V_h if and only if both bidders have values V_h , which occurs with probability p_h^2 , otherwise the seller receives V_l . Therefore in SPA, seller's expected surplus, denoted by χ^{SPA} , would be

$$\chi^{\text{SPA}} = (1 - p_h^2)V_l + p_h^2 V_h. \quad (21)$$

On the other hand, each bidder obtains positive surplus, i.e. $(V_h - V_l)$ if and only if her own value is V_h and her opponent has value V_l , which occurs with probability $p_l p_h$. Then bidders' expected surplus from SPA, denoted by S^{SPA} , is

$$S^{\text{SPA}} = p_l p_h (V_h - V_l). \quad (22)$$

This yields the following results:

⁸This can be seen in the Appendix of Fang and Morris (2006)

Proposition 4 *Suppose $0 < c < p_l p_h^2 (V_h - V_l)(2q - 1)$, $q \in (0.5, 1)$ and all parameters being equal, then we have*

1. *Bidder's expected surplus from FPA is strictly higher than that from SPA, regardless of the stage at which the IA choice is made, i.e. $S^{EX} > S^{SPA}$ ($S^{INT} > S^{SPA}$).*
2. *Seller gets strictly less expected revenue from FPA than that from SPA, no matter at what stage bidder can acquire information, i.e. $\chi^{EX} < \chi^{SPA}$ ($\chi^{INT} < \chi^{SPA}$).*

The first item could follow from corollaries 1 and 2, since the revenue equivalence still holds with discrete values across standard FPA and SPA (without information acquisition), and IA plays no role in formulating strategies in SPA.

It is obvious that bidders have strong incentives to acquire information about their opponents' valuations intentionally in FPA if it is permitted, but not SPA. The other fact is that the auctioneer is worse-off in FPA, if such an information acquisition occurs, which differs from the results that revenue increases associated with traditional information acquisition. A seller might not prefer to use FPA to sell his asset if bidders' private values are independent. But as we have said when stating the reason to focus on first price auctions, there are many other considerations that needs taking into account to decide which auction format is the best, and this IA influence might be weaker than others.

5.2 Endogenous Information Acquisition

One typical characteristic in our model specification is the notion of endogenous information acquisition. The setting in this paper differs from Fang and Morris' model mainly in this notion, namely whether information acquisition is a choice variable of the buyer or not. Fang and Morris analyze the model given that both bidder have already acquired information which result in irrational behavior. Since information acquisition is generally costly and bidder does not necessarily need information about opponent's valuations to submit bids, rational bidder should estimate the benefit and cost of acquiring information, and then decide whether to take such an action. In other words, as a rational person, information acquisition should be part of the bidder's strategy or one of his choice variable, instead of being given exogenously. In this sense, the exogenous IA model in Fang and Morris' paper is an irrational self-restricted equilibrium analysis. If information acquisition becomes buyer's choice variable, the outcome of the model would be a global equilibrium since now both bidders' strategy space and type space now turn to be bidimensional as we have stated.

One then may naturally ask a question, like what is the significant difference between the results of exogenous IA and endogenous IA models? Well from the perspective of bidders, there

indeed exists difference in the amount of expected surplus. Since the ex ante IA model and interim IA model give bidders similar form of expected surplus, see equations (11) and (20), we just consider interim IA. Exogenous IA is equivalent to set $\pi_i = 1$, that is to acquire information with probability one. From equation (20), bidders' payoff would be $\frac{p_l q + p_h q(1-q)}{p_l q + p_h(1-q)^2} \cdot p_l p_h (V_h - V_l)$ regardless of the cost. Since information acquisition is costly, then the net payoff of bidders from exogenous IA model, denoted by S^C , would be

$$S^C = \frac{p_l q + p_h q(1-q)}{p_l q + p_h(1-q)^2} \cdot p_l p_h (V_h - V_l) - c. \quad (23)$$

Figure 1 demonstrates how bidder's interim IA expected payoff and exogenous IA expected payoff evolve as the informative cost ranges over $[0, c_0]$ where $c_0 = p_l p_h (V_h - V_l)(2q - 1)$. Note that $\pi_i = 1$ when informative cost is zero from Remark 2, which implies that $S^C = S^{\text{INT}}$ at $c = 0$. We know that S^C and S^{INT} both are decreasing in informative cost, and S^C is a linear function of c , and also S^{INT} is concave in c , then figure (a), (b) and (c) are the possible three patterns.⁹ Obviously bidder gets more expected payoff from interim IA than from exogenous IA in cases (a) and (b), and this is also true in case (a) if the informative cost is smaller than some critical point. Those patterns in the figure are the same when the interim IA payoff is replaced by the ex ante IA payoff S^{EX} , just by redefining the unit cost (now the cost $c_0 = p_l p_h^2 (V_h - V_l)(2q - 1)$ by proposition 1).

Thought it is not guaranteed that bidder can achieve more expected payoff from endogenous IA than from exogenous IA without further assumptions, we have seen that in most cases bidder obtains higher payoff in our exogenous settings. This provides another notion for incorporating the information acquisition choice into bidders' strategy space.

6 Some Variants

This paper consider covertly information acquisition on opponents' valuation, it is crucial to maintain the assumption that choice of acquiring information is unobservable to the opponent. It will result in two problems without this assumption. First, the signal structure is not inherently consistent. This is a model with binary valuations, hence only the type- V_h bidder i has incentive to acquire information. If this action is public information, then it is a perfect signal that tells

⁹The concavity of S^{INT} in c mainly comes from the fact that S^{INT} is concave in π_i , and it is not an immediate property since π_i is a complex function of c , hence we do not formally state it in Propositions 1 and 2. The concavity says the marginal benefit of one additional unit decrease in informative cost is decreasing, because the marginal benefit of IA probability is decreasing and this probability goes up when informative cost reduces. Theoretically, (a) is corresponding to the case $p_h > \frac{1}{2-q}$, (b) is corresponding to the case $p_h = \frac{1}{2-q}$ and (c) is corresponding to the case $p_h < \frac{1}{2-q}$. We choose different values of p_h and fix the accuracy of q because otherwise there might be no pattern (a) for any $q \in (0.5, 1)$ if we fix some p_h , i.e. $p_h = 0.5$.

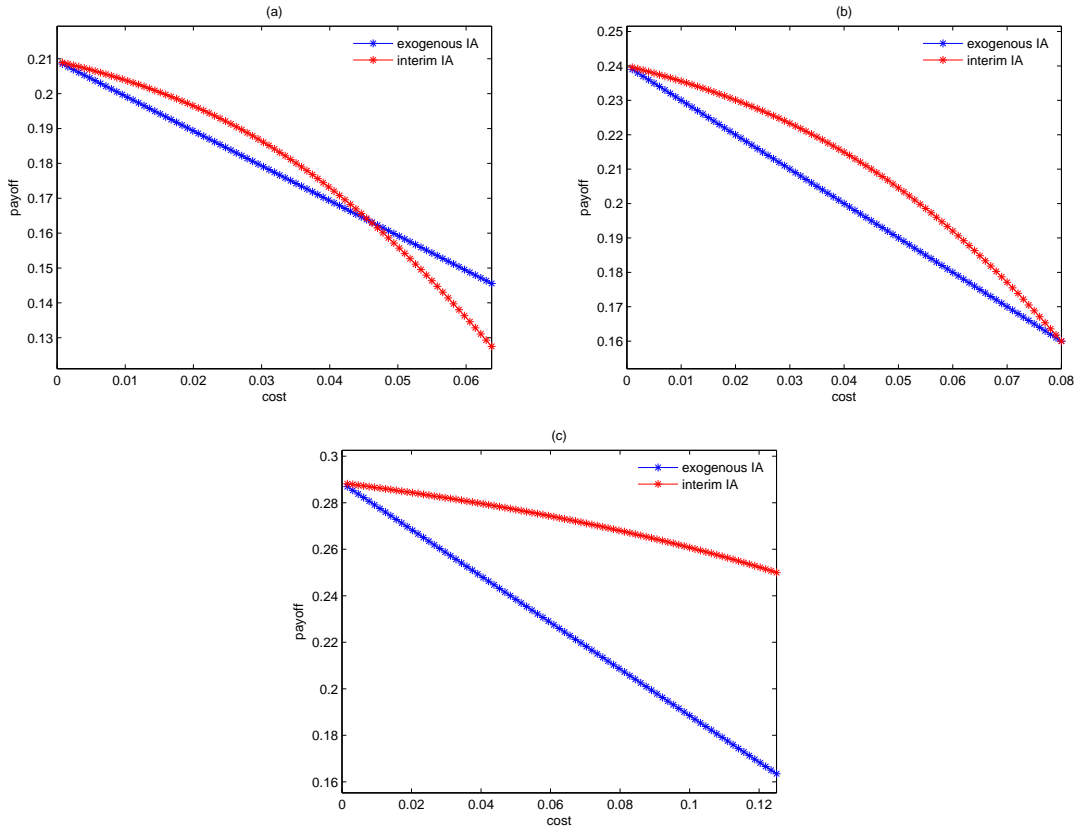


Figure 1: Bidder's Payoff from Interim IA and Exogenous IA

Numerically $q = 0.75, V_h - V_l = 1$, these parameters are the same in (a), (b) and (c); and $p_h = 0.85$ in (a), $p_h = 0.8$ in (b), and $p_h = 0.5$ in (c). (a), (b) and (c) illustrate three possible patterns of the interim IA equilibrium payoff and exogenous IA equilibrium payoff of bidders.

bidder j that i 's valuation is V_h . Also bidder j could be a type- V_h bidder who already acquires information and receives a signal s_j , then s_j is meaningless for the bidder, then why should the bidder incur a cost to acquire information? Second, there could be no equilibrium in some subcases. We consider public interim information acquisition. Suppose bidder i has valuation V_h and acquires information and bidder j does not, and this is common knowledge. In this case, no equilibrium bidding strategy exists with fair tie breaking rule.¹⁰ It is similar for ex ante public information acquisition. Hence observable information acquisition is undesirable for our modeling.

Besides the binary valuations, more points of value support could be added in. As Fang and Morris (2006) have observed: it does not admit any equilibrium for exogenous information

¹⁰Fair tie breaking rule means when several bidders submit the same highest bid, each of them obtains the object with equal probability. This example is the same case as the Example 4.4 on pp.53 of Krishna's book, which is another version of the example in Maskin and Riley (2000), the equilibrium could be restored but we need Vickrey tie breaking rule. In our settings the same equilibrium still exists (the strategy differs only in one point of the randomization support) if we use the fair tie-breaking rule.

acquisition with generic parameters even in binary valuation support model. So for more points, the equilibrium analysis would be also problematic. As for continuous valuation support, the formation of bidding rule with multiple arguments turns to be a rather involved problem, which is the essential question with multidimension. This is an open question for future research.

Admittedly, we specify an independent private value setting, which is restrictive in some sense. However, we believe the finding that buyers get higher expected payoff from FPA with permissible IA than that from standard FPA or SPA is generally true. For example, this seems also true in interdependent values with discrete settings, which is the environment we haven't considered here but are working on.

7 Conclusion

This paper mainly investigates bidders' covertly information acquisition on opponents' valuations in first price seal-bid auctions with independent private values, which leads to real multidimensional auction environment—bidimensional strategy space and bidimensional type space. This multi-dimensionality leads to a global equilibrium analysis of bidders' behavior in auctions compared with exogenous information acquisition. We present two different specifications: ex ante information acquisition and interim information acquisition. We find that bidder's expected payoff exceeds that of the counterpart standard FPA when informative cost is sufficiently small and the signal is informative and noisy, it also exceeds bidder's payoff from the exogenous IA model when informative cost is small. Due to the settings of binary values, the auctioned object is always efficiently allocated. In all cases, the auctioneer's expected revenue is lower than that of the standard auction, which contradicts result in the current information acquisition literature.

Our paper is the first step to explore the properties of information acquisition on opponents' valuations in the context of independent private value auctions. We do find bidders have incentives to take this IA action. To see the robustness of the results, continuous support necessitates exploration. It seems that the difficulty rests with the multidimensional bidding rules. Moreover, the probability of acquiring information is probably a function of her valuation. However, we believe that in equilibrium, the bidder would most likely mix in acquiring information.

Appendix A: A Note on Multidimensional Strategy Space

We know that the proposition for preservation of metric dimension says that if $f : X \rightarrow Y$ is a one-to-one mapping and its inverse function f^{-1} satisfies Lipschitz condition, i.e. there exists some positive constant κ , such that $d_X(x, x') \leq \kappa \cdot d_Y(f(x), f(x'))$ for all x, x' in X , then $\dim Y \geq \dim X$.

To see our strategy space is a *real* multidimensional one, we first redefine some notations, and then give a brief proof. Let

$$\pi : \mathcal{V} \rightarrow [0, 1],$$

be the bidder's information acquisition choice, and

$$b : \mathcal{V} \circ \mathcal{S}[\pi(\mathcal{V})] \rightarrow M^*,$$

be the bidding rule. Here $\mathcal{V} \circ \mathcal{S}[\pi(\mathcal{V})]$ is not a Cartesian product of \mathcal{V} and $\mathcal{S}[\pi(\mathcal{V})]$, instead it is defined as $\mathcal{V} \circ \mathcal{S}[\pi(\mathcal{V})] = \bigcup_{v_i \in \mathcal{V}} \{v_i\} \times \mathcal{S}[\pi(v_i)]$, and $\mathcal{S}[\pi(v_i)]$ represents all the possible signals received by the type- v_i bidder who adopts the mixed strategy $\pi(v_i)$ in IA choice; and the set $M = [0, m]$ for some positive real number m , is the support of bids. M^* then represents all the possible distributions on M including non-atomless and discrete ones. Now let $f = (\pi, b)$, $\Omega_f = \mathcal{V} \circ \mathcal{S}[\pi(\mathcal{V})]$, then the bidder's strategy can be integrated as

$$f : \Omega_f \rightarrow [0, 1] \times M^*,$$

which is a mapping that has bidimensional domain and value. Suppose $(v_1, s_1), (v_2, s_2)$ are two elements in Ω_f , the metric on Ω_f is defined by

$$d_{\Omega_f}((v_1, s_1), (v_2, s_2)) = \sqrt{(v_1 - v_2)^2 + (s_1 - s_2)^2},$$

where each signal $s \in \bar{\mathcal{S}}$ corresponds to a real value, i.e. s_0, L , and H corresponding to 0,1,2 respectively. Providing that $(\pi_1, b_1), (\pi_2, b_2) \in [0, 1] \times M^*$, the metric on $[0, 1] \times M^*$ is given by

$$d_{[0,1] \times M^*}((\pi_1, b_1), (\pi_2, b_2)) = \sqrt{(\pi_1 - \pi_2)^2 + E(b_1 - b_2)^2}.$$

Note that the bid of bidder might be randomly drawn from some distribution on M^* , and is a random variable, hence we use the expectation operator E in the metric.

In our model, the valuation set \mathcal{V} and the signal collection $\bar{\mathcal{S}}$ are finite, and the bids are bounded (hence the Lipschitz condition is trivially satisfied given that the inverse function

exists), then if we can show that the strategy mapping f for each bidder is a one-to-one mapping in equilibrium, it means that the metric dimension of strategy is actually preserved and cannot be reduced by redefinition according to the preservation of metric dimension proposition.

As for the ex ante information acquisition model, in equilibrium,

$$\Omega_f = \{(V_l, s_0), (V_l, L), (V_l, H), (V_h, s_0), (V_h, L), (V_h, H)\},$$

and $f(v, s) = (\pi_e, V_l)$, for $(v, s) \in \{(V_l, s_0), (V_l, L), (V_l, H)\}$, $f(V_h, s_0) = (\pi_e, J^{s_0}(b))$, $f(V_h, s_l) = (\pi_e, J^l(b))$, $f(V_h, H) = (\pi_e, J^h(b))$ where $J^{s_0}(b)$, $J^l(b)$, and $J^h(b)$ are defined in Proposition 1. The mapping f equivalent to this ex ante equilibrium is not a one-to-one mapping, then we cannot apply the theorem to justify the dimension of the strategy space. This is because the assumption of ex ante information acquisition imposes the restriction on the mapping, i.e. the first element of image under f for all profile arguments is the same, say, π_e .

Instead in interim information acquisition model, the so-defined mapping f equivalent to the equilibrium outcome is a one-to-one mapping. Note that now

$$\Omega_f = \{(V_l, s_0), (V_h, s_0), (V_h, L), (V_h, H)\},$$

and $f(V_l, s_0) = (0, V_l)$, $f(V_h, s_0) = (\pi_i, Q^{s_0}(b))$, $f(V_h, L) = (\pi_i, Q^l(b))$, $f(V_h, H) = (\pi_i, Q^h(b))$, where $Q^{s_0}(b)$, $Q^l(b)$, and $Q^h(b)$ are defined in Proposition 2. Then we can come into a conclusion that the metric dimension of bidders' strategy space in interim IA equilibrium is at least as large as bidders' type space.

Appendix B: Proofs

We will need the result of this proof frequently. We present it here for convenience.

Lemma A. In any equilibrium of the FPA, type- (V_l, s) bidders bid V_l in pure strategies for $s \in \{H, L, s_0\}$. \square

The claim in this lemma is evident, and the proof can be found in the Appendix A of Fang and Morris (2006).

Proof of Proposition 1: We take several steps to complete this argument. First we verify that the strategies actually constitute an equilibrium. Then we show that there is no other symmetric equilibrium. Last we argue that there is no asymmetric equilibrium either. The argument follows several intermediate claims:

Claim 1. *Given that the bidder's opponent follows the provided strategy in the proposition, the bidder herself will choose the same strategy in the proposition.*

Note that by lemma A, it is only necessary to check the bidder with valuation V_h . We just need to verify that the bidder gets exactly the same expected payoff if she plays any strategy in the support of provided mixed strategy, and that the bidder cannot obtain more expected payoff by deviation. In other words, the provided strategy is an equilibrium. Since it can be done by direct calculations, the verification procedure is omitted.

Claim 2. *There exists no other types of symmetric equilibria.*

The argument takes 4 steps.

Step 1: We show that in any symmetric equilibrium, both bidders would choose to acquire information with positive probability. In other words, we only need to show that the case where no bidder chooses to acquire information is not an equilibrium. Suppose this is the case. Then it reduces to the standard FPA equilibrium strategy; that is,

- None of the bidders incurs any cost to receive any signal.
- $b_i(V_l) = V_l$.
- Type- V_h bidder mixes over $[V_l, \bar{\gamma}]$ according to cumulative distribution function (c.d.f.) $F(\cdot)$ specified by

$$F(b) = \frac{p_l}{p_h} \cdot \frac{b - V_l}{V_h - b}, \quad b \in [V_l, \bar{\gamma}],$$

where

$$\bar{\gamma} = p_l V_l + p_h V_h. \tag{24}$$

If two bidders follow this strategy, then their surplus under this strategy profile is $p_l p_h (V_h - V_l)$. Now suppose bidder 1 follows the above strategy. Bidder 2 acquires information and adopts the strategy as follows:

- She acquires information in the first stage.
- $b_i(V_l) = V_l$, $b(V_h, L) = V_l$ and $b(V_h, H) = \bar{\gamma}$.

Her payoff will be given by

$$\begin{aligned} & p_h [p_l q (V_h - V_l) + (p_l (1 - q) + p_h q) \cdot (V_h - \bar{\gamma})] - c \\ & = p_l p_h (V_h - V_l) [1 + (2q - 1) p_h] - c. \end{aligned} \quad (25)$$

Since c is sufficiently small, then the term $p_l p_h (V_h - V_l) [1 + (2q - 1) p_h] - c$ would exceed $p_l p_h (V_h - V_l)$, so bidder A would deviate. Then this is not an equilibrium. Therefore there exists no other type of symmetric equilibria under this ex ante IA specification.

Step 2: We show that in any symmetric equilibria type- (V_h, s_0) , type- (V_h, L) , and type- (V_h, H) bidders should bid in mixed strategies. For instance, by way of contradiction, say that a type- (V_h, s) $s \in \{s_0, L, H\}$ bidder A bids a fixed amount $b^* < V_h$. Then the same type- (V_h, s) bidder j ($j \neq A$) can deviate to make more surplus by bidding $b^* + \epsilon$, where ϵ is arbitrarily small. This deviation will lead to a positive jump in type- (V_h, s) bidder j 's winning probability, and hence is profitable.

Step 3: We prove that in any symmetric mixed strategy equilibria, the supports of $J^l(\cdot)$ and $J^{s_0}(\cdot)$ and the supports of $J^{s_0}(\cdot)$ and $J^h(\cdot)$ are contiguous and non-overlapping. Note that the contiguity of supports follows from the same ϵ -deviation argument as that in part 1 to rule out pure strategies for bidders with valuation V_h .

Now suppose that the supports of $J^l(\cdot)$ and $J^{s_0}(\cdot)$ overlap over an interval $[b_1, b_2]$ where $b_1 < b_2$. Mixed strategy requires that the expected surplus is constant for both type- (V_h, L) and type- (V_h, s_0) bidders from any bid $b \in [b_1, b_2]$. Hence we have that the following equations hold for some constants $C(V_h, s_0)$ and $C(V_h, l)$:

$$(V_h - b) [p_l + p_h \pi_e (1 - q) J^l(b) + p_h (1 - \pi_e) J^{s_0}(b)] = C(V_h, s_0), \quad (26)$$

$$(V_h - b) \left[\frac{p_l q}{p_l q + p_h (1 - q)} + \frac{p_h (1 - q) \cdot \pi_e (1 - q) J^l(b)}{p_l q + p_h (1 - q)} + \frac{p_h (1 - q) (1 - \pi_e) J^{s_0}(b)}{p_l q + p_h (1 - q)} \right] = C(V_h, l). \quad (27)$$

Multiplying equation (26) by $(1 - q)$ and equation (27) by $-[p_l q + p_h (1 - q)]$, and summing

up, we have

$$(V_h - b)p_l(1 - 2q) = (1 - q)C(V_h, s_0) - [p_lq + p_h(1 - q)]C(V_h, l). \quad (28)$$

Since the left hand side of equation (28) is a constant, this equation cannot hold for any value of bid $b \in [b_1, b_2]$ for $q > \frac{1}{2}$. Therefore, the supports for $J^{s_0}(\cdot)$ and $J^l(\cdot)$ are non-overlapping in the symmetric equilibrium.

By the same method, if the supports of $J^{s_0}(\cdot)$ and $J^h(\cdot)$ overlap over an interval $[b'_1, b'_2]$ where $b'_1 < b'_2$, then for some constants $\hat{C}(V_h, s_0)$ and $\hat{C}(V_h, H)$, we have

$$(V_h - b)[p_l + p_h\pi_e(1 - q) + p_h(1 - \pi_e)J^{s_0}(b) + p_h\pi_eqJ^h(b)] = \hat{C}(V_h, s_0), \quad (29)$$

$$(V_h - b)\left[\frac{p_l(1 - q)}{p_l(1 - q) + p_hq} + \frac{p_hq \cdot \pi_e(1 - q)}{p_l(1 - q) + p_hq} + \frac{p_hq(1 - \pi_e)J^{s_0}(b)}{p_l(1 - q) + p_hq} + \frac{p_hq\pi_eJ^h(b)}{p_l(1 - q) + p_hq}\right] = \hat{C}(V_h, h). \quad (30)$$

Multiplying equation (29) by q and equation (30) by $-[p_l(1 - q) + p_hq]$, and summing up, we have

$$(V_h - b)p_l(2q - 1) = q\hat{C}(V_h, s_0) - [p_l(1 - q) + p_hq]\hat{C}(V_h, h). \quad (31)$$

Again the left hand side of equation (31) is a constant. This equation cannot hold for any value of bid $b \in [b'_1, b'_2]$ for $q > \frac{1}{2}$. Therefore, the supports for $J^{s_0}(\cdot)$ and $J^h(\cdot)$ are non-overlapping in the symmetric equilibrium.

Step 4: Now we show that the support of the type- (V_h, L) bidder should be lower than that of the type- (V_h, s_0) bidder, and the support of the type- (V_h, s_0) bidder should be lower than that of the type- (V_h, h) bidder. Since we have three possible type bidders that use mixed strategies, there exist six possible rank permutations for the three corresponding supports. We should rule out all the other five cases¹¹. Now let $[V_l, \tilde{b}_1]$ be the support of the type- (V_h, s_0) bidder's mixed strategy with cumulative distribution function (cdf) $\tilde{J}^{s_0}(\cdot)$. Let $[\tilde{b}_1, \tilde{b}_2]$ be the support of the type- (V_h, l) bidder's mixed strategy with cdf $\tilde{J}^l(\cdot)$, and let $[\tilde{b}_2, \tilde{b}_3]$ be the support of the type- (V_h, h) bidder's mixed strategy with cdf $\tilde{J}^h(\cdot)$. Since type- (V_h, l) bidders randomize on the interval $[\tilde{b}_1, \tilde{b}_2]$, their expected surpluses should be constants for any bids drawn from the

¹¹Denote $S(V_h, s)$ as the mixed strategy support of type- (V_h, s) bidder where $s \in \{H, L, s_0\}$ and \ll is interpreted, "lower than." The five cases are $S(V_h, s_0) \ll S(V_h, l) \ll S(V_h, h)$, $S(V_h, s_0) \ll S(V_h, h) \ll S(V_h, l)$, $S(V_h, l) \ll S(V_h, h) \ll S(V_h, s_0)$, $S(V_h, h) \ll S(V_h, l) \ll S(V_h, s_0)$, $S(V_h, h) \ll S(V_h, s_0) \ll S(V_h, l)$.

interval; that is

$$\begin{aligned} (V_h - b) & \left[\frac{plq}{plq + p_h(1-q)} + \frac{p_h(1-q)(1-\tilde{\pi}_e)}{plq + p_h(1-q)} + \frac{p_h(1-q)\tilde{\pi}_e(1-q)}{plq + p_h(1-q)} \tilde{J}^l(b) \right] \\ & = \frac{plq + p_h(1-q)(1-\tilde{\pi}_e)}{plq + p_h(1-q)} (V_h - \tilde{b}_1). \end{aligned}$$

From this equation we can solve for $\tilde{J}^l(b)$:

$$\tilde{J}^l(b) = \frac{plq + p_h(1-q)(1-\tilde{\pi}_e)}{p_h(1-q)^2\tilde{\pi}_e} \frac{b - \tilde{b}_1}{V_h - b}. \quad (32)$$

Assume bidder 1 follows the provided strategy. That is, the type- (V_h, l) bidder 1 randomizes over $[\tilde{b}_1, \tilde{b}_2]$ according to cdf $\tilde{J}^l(\cdot)$, the type- (V_h, s_0) bidder 1 randomizes over $[V_l, \tilde{b}_1]$ according to cdf $\tilde{J}^{s_0}(\cdot)$, and the type- (V_h, h) bidder 1 randomizes over $[\tilde{b}_2, \tilde{b}_3]$ according to cdf $\tilde{J}^h(\cdot)$. By submitting bid $b \in [\tilde{b}_1, \tilde{b}_2]$, the expected surplus for type- (V_h, s_0) bidder 2 is given by

$$\begin{aligned} & (V_h - b) \left[pl + p_h(1-\tilde{\pi}_e) + p_h\tilde{\pi}_e(1-q) \tilde{J}^l(b) \right] \\ & = \frac{(2q-1)pl}{1-q} \cdot b + \left[(pl + p_h(1-\tilde{\pi}_e))V_h - \frac{plq + p_h(1-q)(1-\tilde{\pi}_e)\tilde{b}_1}{1-q} \right], \end{aligned}$$

which is strictly increasing in b for $b \in [\tilde{b}_1, \tilde{b}_2]$. Hence the type- (V_H, s_0) bidder has incentive to deviate while her opponent follows the prescribed strategy. This is a contradiction. This means the supposed case cannot be an equilibrium.

The other four cases can be ruled out using the same method.

Claim 3: *There exists no asymmetric equilibrium in which bidder 1 and bidder 2 acquire information with disparate probabilities.*

Suppose there exist such an equilibrium. Denote $\hat{\pi}_{1e}$ and $\hat{\pi}_{2e}$ as bidder 1 and bidder 2's information acquisition probability respectively. Without loss of generality let $\hat{\pi}_{1e} > \hat{\pi}_{2e}$. The following several steps show that the distribution functions and the parameters satisfy some necessary conditions imposed by the mixed strategy. Remember that similar arguments can be used to show that, in an asymmetric equilibrium, the support of type- (V_h, L) bidder i must be lower than, non-overlapping, and contiguous with that of type- (V_h, s_0) bidder i . The support of type- (V_h, s_0) bidder i must be lower than, non-overlapping, and contiguous with that of type- (V_h, H) bidder i , for $i = 1, 2$ (refer to Steps 3 and 4 in Claim 2). Then bidder 1 and bidder 2's strategies should be as follows:

- Bidder 1 acquires information with probability $\hat{\pi}_{1e}$. The type- (V_h, L) bidder 1 randomizes over the interval $[V_l, \hat{b}_{11}]$ according to the cdf $\hat{J}_1^l(\cdot)$, the type- (V_h, s_0) bidder 1 randomizes

over the interval $[\hat{b}_{11}, \hat{b}_{12}]$ according to the cdf $\hat{J}_1^{s_0}(\cdot)$, and the type- (V_h, H) bidder 1 randomizes over the interval $[\hat{b}_{12}, \hat{b}_{13}]$ according to the cdf $\hat{J}_1^h(\cdot)$, where $V_l < \hat{b}_{11} < \hat{b}_{12} < \hat{b}_{13} < V_h$.

- Bidder 2 acquires information with probability $\hat{\pi}_{2e}$. The type- (V_h, L) bidder 2 randomizes over the interval $[V_l, \hat{b}_{21}]$ according to the cdf $\hat{J}_2^l(\cdot)$, the type- (V_h, s_0) bidder 2 randomizes over the interval $[\hat{b}_{21}, \hat{b}_{22}]$ according to the cdf $\hat{J}_2^{s_0}(\cdot)$, the type- (V_h, H) bidder 2 randomizes over the interval $[\hat{b}_{22}, \hat{b}_{23}]$ according to the cdf $\hat{J}_2^h(\cdot)$, where $V_l < \hat{b}_{12} < \hat{b}_{22} < \hat{b}_{23} < V_h$.

Note that the upper bound of the support for type- (V_h, H) bidder 1 and type- (V_h, H) bidder 2 should be the same; that is, $\hat{b}_{13} = \hat{b}_{23} = \hat{b}_3$. Then we derive the necessary conditions that pin down the cumulative distribution functions of supports and parameters.

Step 1: To begin, we first need to show that $\hat{b}_{11} > \hat{b}_{12}$ and $\hat{b}_{12} < \hat{b}_{22}$. Suppose that $\hat{b}_{11} \leq \hat{b}_{12}$. Then both type- (V_h, L) bidder 1 and type- (V_h, L) bidder 2 should be indifferent between any bids that belong to the interval $[V_l, \hat{b}_{11}]$. By drawing a bid from $[V_l, \hat{b}_{11}]$, type- (V_h, L) bidder 1's expected surplus is given by

$$\begin{aligned} & (V_h - b) \left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{2e}(1-q)}{p_l q + p_h(1-q)} \hat{J}_2^l(b) \right] \\ = & (V_h - V_l) \frac{p_l q}{p_l q + p_h(1-q)} & b = V_l \\ = & (V_h - \hat{b}_{11}) \left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{2e}(1-q)}{p_l q + p_h(1-q)} \hat{J}_2^l(\hat{b}_{11}) \right] & b = \hat{b}_{11}, \end{aligned}$$

and type- (V_h, L) bidder 2's expected surplus is given by

$$\begin{aligned} & (V_h - b) \left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{1e}(1-q)}{p_l q + p_h(1-q)} \hat{J}_1^l(b) \right] \\ = & (V_h - V_l) \frac{p_l q}{p_l q + p_h(1-q)} & b = V_l \\ = & (V_h - \hat{b}_{11}) \left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{1e}(1-q)}{p_l q + p_h(1-q)} \right] & b = \hat{b}_{11}. \end{aligned}$$

From these two equations we get $\hat{\pi}_{1e} = \hat{\pi}_{2e} \hat{J}_2^l(\hat{b}_{11})$, contradict to the assumption $\hat{\pi}_{1e} > \hat{\pi}_{2e}$.

Now suppose $\hat{b}_{12} \geq \hat{b}_{22}$. Then both type- (V_h, H) bidder 1 and type- (V_h, H) bidder 2 should be indifferent between any bids that belong to the interval $[\hat{b}_{12}, \hat{b}_3]$. By drawing a bid from $[\hat{b}_{12}, \hat{b}_3]$, type- (V_h, H) bidder 1's expected surplus is given by

$$\begin{aligned} & (V_h - b) \left[\frac{p_l(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q \hat{\pi}_{2e}(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q(1-\hat{\pi}_{2e})}{p_l(1-q) + p_h q} + \frac{p_h q \hat{\pi}_{2e} q \hat{J}_2^h(b)}{p_l(1-q) + p_h q} \right] \\ = & (V_h - \hat{b}_{12}) \left[\frac{p_l(1-q) + p_h q(1-q\hat{\pi}_{2e})}{p_l(1-q) + p_h q} + \frac{p_h q \hat{\pi}_{2e} q \hat{J}_2^h(\hat{b}_{12})}{p_l(1-q) + p_h q} \right] & b = \hat{b}_{12} \\ = & V_h - \hat{b}_3 & b = \hat{b}_3, \end{aligned}$$

and the type- (V_h, H) bidder 2's expected surplus is given by

$$\begin{aligned}
& (V_h - b) \left[\frac{p_l(1-q)}{p_l(1-q) + p_hq} + \frac{p_hq\hat{\pi}_{1e}(1-q)}{p_l(1-q) + p_hq} + \frac{p_hq(1-\hat{\pi}_{1e})}{p_l(1-q) + p_hq} + \frac{p_hq\hat{\pi}_{1e}q\hat{J}_1^h(b)}{p_l(1-q) + p_hq} \right] \\
&= (V_h - \hat{b}_{12}) \left[\frac{p_l(1-q) + p_hq(1-q\hat{\pi}_{1e})}{p_l(1-q) + p_hq} \right] & b = \hat{b}_{12} \\
&= V_h - \hat{b}_3 & b = \hat{b}_3.
\end{aligned}$$

From these two equations we obtain $\hat{\pi}_{2e} - \hat{\pi}_{1e} = \hat{\pi}_{1e}\hat{J}_2^h(\hat{b}_{12}) > 0$, which contradicts to the fact that $\hat{\pi}_{2e} < \hat{\pi}_{1e}$.

Now we have the rank for the extreme points of the supports as $V_l < \hat{b}_{21} < \hat{b}_{11} < \hat{b}_{12} < \hat{b}_{22}\hat{b}_3 < V_h$.

Step 2: Now we consider type- (V_h, L) bidder 1's surplus. Since she randomizes over the interval $[V_l, \hat{b}_{11}]$, her expected surplus is given by

$$\begin{aligned}
& (V_h - b) \left[\frac{p_lq}{p_lq + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{2e}(1-q)}{p_lq + p_h(1-q)} \hat{J}_2^l(b) + \frac{p_h(1-q)(1-\hat{\pi}_{2e})}{p_lq + p_h(1-q)} \hat{J}_2^{s_0}(b) \right] \\
&= (V_h - V_l) \frac{p_lq}{p_lq + p_h(1-q)} & b = V_l \\
&= (V_h - \hat{b}_{21}) \left[\frac{p_lq + p_h(1-q)\hat{\pi}_{2e}(1-q)}{p_lq + p_h(1-q)} \right] & b = \hat{b}_{21} \\
&= (V_h - \hat{b}_{11}) \left[\frac{p_lq + p_h(1-q)\hat{\pi}_{2e}(1-q) + p_h(1-q)(1-\hat{\pi}_{2e})\hat{J}_2^{s_0}(\hat{b}_{11})}{p_lq + p_h(1-q)} \right] & b = \hat{b}_{11},
\end{aligned}$$

and should be a constant for all bids in this interval. By this indifference condition we get

$$\hat{b}_{21} = \frac{p_lqV_l + p_h(1-q)^2\hat{\pi}_{2e}V_h}{p_lq + p_h(1-q)^2\hat{\pi}_{2e}}; \tag{33}$$

$$\hat{J}_2^l(b) = \frac{p_lq}{p_h(1-q)^2\hat{\pi}_{2e}} \cdot \frac{b - V_l}{V_h - b}, \quad b \in [V_l, \hat{b}_{21}]; \tag{34}$$

$$\hat{J}_2^{s_0}(b) = \frac{p_lq + p_h(1-q)^2\hat{\pi}_{2e}}{p_h(1-q)(1-\hat{\pi}_{2e})} \cdot \frac{b - \hat{b}_{21}}{V_h - b}, \quad b \in [\hat{b}_{21}, \hat{b}_{11}]. \tag{35}$$

We turn to the type- (V_h, L) bidder 2. Since she randomizes over the interval $[V_l, \hat{b}_{12}]$, her expected surplus is given by

$$\begin{aligned}
& (V_h - b) \left[\frac{p_lq}{p_lq + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{1e}(1-q)}{p_lq + p_h(1-q)} \hat{J}_1^l(b) \right] \\
&= (V_h - V_l) \frac{p_lq}{p_lq + p_h(1-q)} & b = V_l \\
&= (V_h - \hat{b}_{21}) \left[\frac{p_lq + p_h(1-q)\hat{\pi}_{1e}(1-q)\hat{J}_1^l(\hat{b}_{21})}{p_lq + p_h(1-q)} \right] & b = \hat{b}_{21},
\end{aligned}$$

and should be a constant for all bids in this interval $[V_l, \hat{b}_{12}]$. Since type- (V_h, L) bidder 1 and type- (V_h, L) bidder 2 have the same payoff according to the above indifference conditions, we have relation

$$\hat{\pi}_{2e} = \hat{\pi}_{1e} \hat{J}_1^l(\hat{b}_{21}). \quad (36)$$

Now we turn to the type- (V_h, s_0) bidder 2. Since this type bidder 2 randomizes over the interval $[\hat{b}_{21}, \hat{b}_{22}]$, her expected surplus is given by

$$\begin{aligned} & (V_h - b) [p_l + p_h \hat{\pi}_{1e} (1 - q) \hat{J}_1^l(b) + p_h (1 - \hat{\pi}_{1e}) \hat{J}_1^{s_0}(b) + p_h \hat{\pi}_{1e} q \hat{J}_1^h(b)] \\ = & (V_h - \hat{b}_{21}) [p_l + p_h \hat{\pi}_{1e} (1 - q) \hat{J}_1^l(\hat{b}_{21})] & b = \hat{b}_{21} \\ = & (V_h - \hat{b}_{11}) [p_l + p_h \hat{\pi}_{1e} (1 - q)] & b = \hat{b}_{11}. \end{aligned}$$

and should be a constant for all bids in $[\hat{b}_{21}, \hat{b}_{22}]$. Then we have

$$\hat{b}_{11} = \frac{[p_l + p_h (1 - q) \hat{\pi}_{2e}] \hat{b}_{21} + p_h (1 - q) (\hat{\pi}_{1e} - \hat{\pi}_{2e}) V_h}{p_l + p_h (1 - q) \hat{\pi}_{1e}}. \quad (37)$$

Step 3: Now we give the conditions that determine the key parameters $\hat{\pi}_{1e}$ and $\hat{\pi}_{2e}$. Note that when bidder 1 or bidder 2 uses a mixed strategy to acquire information, then the surplus should be the same between acquiring information and not acquiring information. Suppose bidder 1 acquires information. With probability $p_h [p_l q + p_h (1 - q)]$, bidder 1's type is (V_h, L) and the corresponding surplus is $\frac{p_l q (V_h - V_l)}{p_l q + p_h (1 - q)}$. With probability $p_h [p_l (1 - q) + p_h q]$ bidder 1's type is (V_h, H) and the corresponding surplus is $(V_h - \hat{b}_3)$. The type- V_l bidder has zero surplus. Then the total surplus is $p_h [(V_h - V_l) p_l q + (p_l (1 - q) + p_h q) (V_h - \hat{b}_3)] - c$. Now suppose bidder 1 does not acquire information. With probability p_h her type is (V_h, s_0) and the corresponding surplus is $(V_h - \hat{b}_{11}) [p_l + p_h \hat{\pi}_{2e} (1 - q) + p_h (1 - \hat{\pi}_{2e}) \hat{J}_2^{s_0}(\hat{b}_{11})]$. With probability p_l her valuation is V_l and the corresponding surplus is zero. Then bidder 1's total non-IA surplus is $p_h (V_h - \hat{b}_{11}) [p_l + p_h \hat{\pi}_{2e} (1 - q) + p_h (1 - \hat{\pi}_{2e}) \hat{J}_2^{s_0}(\hat{b}_{11})]$. In equilibrium, the bidder should be indifferent between acquiring information and not acquiring information; that is

$$\begin{aligned} & p_h [(V_h - V_l) p_l q + (p_l (1 - q) + p_h q) (V_h - \hat{b}_3)] - c \\ & = p_h (V_h - \hat{b}_{11}) [p_l + p_h \hat{\pi}_{2e} (1 - q) + p_h (1 - \hat{\pi}_{2e}) \hat{J}_2^{s_0}(\hat{b}_{11})]. \end{aligned} \quad (38)$$

By the same analysis, we have a similar condition for bidder 2:

$$\begin{aligned} & p_h [(V_h - V_l) p_l q + (p_l (1 - q) + p_h q) (V_h - \hat{b}_3)] - c \\ & = p_h (V_h - \hat{b}_{21}) [p_l + p_h \hat{\pi}_{1e} (1 - q) \hat{J}_1^l(\hat{b}_{21})]. \end{aligned} \quad (39)$$

Note that the left hand side of the two bidders' indifference conditions are the same and $\hat{\pi}_{2e} = \hat{\pi}_{1e} \hat{J}_1^l(\hat{b}_{21})$. These two conditions also impose the necessary conditions for $\hat{\pi}_{1e}$ and $\hat{\pi}_{2e}$:

$$\hat{\pi}_{1e}(1-q) = \hat{\pi}_{2e}(1-q) + (1-\hat{\pi}_{2e})\hat{J}_2^{s_0}(\hat{b}_{11}). \quad (40)$$

Substituting \hat{b}_{11} and $\hat{J}_2^{s_0}(\cdot)$ (defined in step 2) into the equation yields

$$(\hat{\pi}_{1e} - \hat{\pi}_{2e})(1-q) = \frac{(\hat{\pi}_{1e} - \hat{\pi}_{2e})[p_l q + p_h(1-q)^2 \hat{\pi}_{2e}]}{p_l + p_h(1-q)\hat{\pi}_{2e}}, \quad (41)$$

which holds only when $q = \frac{1}{2}$. This contradicts the fact $q > \frac{1}{2}$. Then there exists no such equilibrium that bidder 1 and bidder 2 acquire information with different probabilities.

Claim 4: *It cannot be an equilibrium that one bidder acquires information and the other bidder does not acquire information.*

By way of contradiction, suppose we can construct such an equilibrium. Without loss of generality, let bidder 1 choose not to acquire information. Then she will bid V_l if her valuation is V_l , and she will mix over the interval $[V_l, \tilde{\gamma}]$ for some constant $\tilde{\gamma}$ according to some cdf $\tilde{H}(\cdot)$. Assume bidder 2 acquires information with probability $\tilde{\pi}$. Then in equilibrium the type- V_l bidder 2 will bid V_l , the type- (V_h, s_0) bidder 2 will mix over the same interval $[V_l, \tilde{\gamma}]$ according to some cdf $\tilde{H}_2^{s_0}$, the type- (V_h, l) bidder 2 will mix over the same interval $[V_l, \tilde{\gamma}]$ according to some cdf \tilde{H}_2^l , and the type- (V_h, h) bidder 2 will mix over the same interval $[V_l, \tilde{\gamma}]$ according to some cdf \tilde{H}_2^h . Note that by lemma A, the bidder with valuation V_l bids V_l in any equilibrium, and by ϵ -deviation argument, the type- V_h bidder should randomize to submit bids over some interval. Since bidder 2 knows that the type- V_h bidder 1 never bids more than $\tilde{\gamma}$, the type- V_h bidder 2 also randomizes over the same interval.

By the requirement of mixed strategy, the expected surplus for the type- (V_h, L) and the type- (V_h, h) bidder must be constant. Hence for some constant \tilde{C}_l and \tilde{C}_2 , the following equations should hold, which is impossible unless $q = \frac{1}{2}$:

$$\begin{aligned} \left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)}{p_l q + p_h(1-q)} \cdot \tilde{H}_1(b) \right] (V_h - b) &= \tilde{C}_l, \\ \left[\frac{p_l(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q}{p_l(1-q) + p_h q} \cdot \tilde{H}_1(b) \right] (V_h - b) &= \tilde{C}_2. \end{aligned}$$

Thus we have a contradiction.

Claim 5: *There is no asymmetric equilibrium in which two bidders acquire information with the same probability and submit bids according to different strategies.*

Proof. First similar arguments can be used to show that in an asymmetric equilibrium, the

support of a type- (V_h, L) bidder must be lower than that of a type- (V_h, s_0) bidder, and the support of a type- (V_h, s_0) bidder must be lower than that of a type- (V_h, H) bidder (see Step 4 in Claim 2).

Let $\tilde{\pi}$ be the probability that bidders 1 and 2 acquire information about their opponents' valuations. Now suppose that the type- (V_h, L) bidder 1 plays a mixed strategy over the support of $[V_l, \tilde{b}_1^l]$ with c.d.f $\tilde{T}_1^l(\cdot)$, and the type- (V_h, L) bidder 2 plays a mixed strategy over the support of $[V_l, \tilde{b}_2^l]$ with c.d.f $\tilde{T}_2^l(\cdot)$. Without loss of generality, assume that $\tilde{b}_1^l > \tilde{b}_2^l$. Note that the type- (V_h, L) bidder 1 is indifferent among any bids in $(V_l, \tilde{b}_2^l]$. Then $\tilde{T}_2^l(\cdot)$ should satisfy

$$\begin{aligned} & (V_h - b) \left[\frac{p_l q}{p_l q + p_h(1 - q)} + \frac{p_h(1 - q)\tilde{\pi}(1 - q)\tilde{T}_2^l(b)}{p_l q + p_h(1 - q)} \right] \\ &= (V_h - V_l) \frac{p_l q}{p_l q + p_h(1 - q)}, \end{aligned} \quad (42)$$

from which we obtain

$$\tilde{T}_2^l(b) = \frac{p_l q}{p_h(1 - q)^2 \tilde{\pi}} \cdot \frac{b - V_l}{V_h - b}, \quad (43)$$

and $\tilde{T}_2^l(\tilde{b}_2^l) = 1$. Since bidder 2 is also indifferent among any bids in $[V_l, \tilde{b}_2^l]$, $\tilde{T}_1^l(\cdot)$ should also satisfy

$$\begin{aligned} & (V_h - b) \left[\frac{p_l q}{p_l q + p_h(1 - q)} + \frac{p_h(1 - q)\tilde{\pi}(1 - q)\tilde{T}_1^l(b)}{p_l q + p_h(1 - q)} \right] \\ &= (V_h - V_l) \frac{p_l q}{p_l q + p_h(1 - q)}. \end{aligned} \quad (44)$$

Then we have

$$\tilde{T}_1^l(b) = \frac{p_l q}{p_h(1 - q)^2 \tilde{\pi}} \cdot \frac{b - V_l}{V_h - b}, \quad (45)$$

which is exactly the same as $\tilde{T}_2^l(b)$. Then $\tilde{T}_2^l(\tilde{b}_2^l) = 1$. This contradicts the fact that $\tilde{T}_2^l(\tilde{b}_2^l) < 1$ for bidder 1 mixes over the interval $[V_l, \tilde{b}_1^l]$. Hence it should be the case that type- (V_h, L) bidder 1 and 2 both mix in the same interval $[V_l, \tilde{b}^l]$.

Now suppose that the type- (V_h, s_0) bidder 1 mixes over the interval $[\tilde{b}^l, \tilde{b}_1^{s_0}]$ according to some c.d.f. $\tilde{T}_1^{s_0}(\cdot)$, and the type- (V_h, s_0) bidder 2 mixes over the interval $[\tilde{b}^l, \tilde{b}_2^{s_0}]$ according to some c.d.f. $\tilde{T}_2^{s_0}(\cdot)$. Without loss of generality, assume $\tilde{b}_1^{s_0} > \tilde{b}_2^{s_0}$. Then by applying the same analysis above, we will obtain $\tilde{b}_1^{s_0} = \tilde{b}_2^{s_0}$ and $\tilde{T}_1^{s_0}(\cdot) = \tilde{T}_2^{s_0}(\cdot)$. This will in turn imply that the type- (V_h, H) bidders 1 and 2 will use exactly the same mixed strategies. This completes the claim.

Claim 6: π_e is uniquely determined by (9).

Proof. The informative cost can be expressed as

$$c = \frac{qp_l p_h (V_h - V_l)}{p_l q + p_h (1 - q)^2 \pi_e} \cdot \frac{p_l p_h (2q - 1)(1 - \pi_e)}{p_l + p_h (1 - q \pi_e)}, \quad (46)$$

which is reduced from (9). The righthand side is strictly decreasing in π_e , then π_e is unique as claimed.

Now the proof of the Proposition 1 is complete. \square

Proof of Corollary 1: The righthand side of (46) is strictly decreasing in π_e , so π_e is decreasing in c . Since S^{EX} is increasing in π_e , the former part of the result follows. Note that bidders' expected payoff in standard FPA can be obtained by setting $\pi_e = 0$, then the result is immediate.

Proof of Proposition 2: All procedures of the proof are exactly parallel to proposition 1. By lemma A, the bidder with valuation V_l always bids her valuation V_l and gets zero surplus. Therefore, the type V_l bidder has no incentives to acquire information in equilibrium. Then the ex ante information acquisition and interim information acquisition do make the type- V_h bidder confront the same situations except that the ex ante expected cost is less than the latter case. This means the indifference condition by mixed strategy in information acquisition choice now is at the interim stage and only for the type- V_h bidder. \square

Proof of Corollary 2. For Item 1, the informative cost can be expressed as

$$c = \frac{qp_l p_h (V_h - V_l)}{p_l q + p_h (1 - q)^2 \pi_i} \cdot \frac{p_l (2q - 1)(1 - \pi_i)}{p_l + p_h (1 - q \pi_i)}, \quad (47)$$

which is reduced from (18). The righthand side is strictly decreasing in π_i , then π_i is decreasing in c . Since S^{INT} is increasing in π_i , the former part of item 1 follows. Note that bidders' expected payoff in standard FPA is $p_l p_h (V_h - V_l)$ as we have stated in the proof of corollary 1, then the second part of item 1 follows.

For item 2, note that S^{EX} and S^{INT} only differ in π . To show that $S^{INT}(q)$ is larger than $S^{EX}(q)$, we only need to show that $\pi_i > \pi_e$. By Propositions 1 and 2, all the counterpart parameters and distribution functions have the same form, or they are distinguished only due to the different value of mixing probability π . For instance, if $\pi_e = \pi_i$, then $\delta_k = \beta_k$ for $k = 1, 2, 3$ and $J^s(\cdot) = Q^s(\cdot)$ for $s \in \bar{S}$. Now we regard δ_k (and hence β_k) as the function of mixing probability π in IA choice. Then $\delta_k(\cdot) = \beta_k(\cdot)$ is increasing in π for $k = 1, 2, 3$. Recall π_e and π_i

defined in equations (9) and (18). We define a function of π by

$$\begin{aligned} \phi(\pi) = & \{p_l q(V_h - V_l) + [p_l(1 - q) + p_h q(1 - \pi q)](V_h - \delta_2(\pi))\} \\ & - [p_l + \pi(1 - q)p_h](V_h - \delta_1(\pi)). \end{aligned} \quad (48)$$

Note that if $[p_l + \pi(1 - q)p_h](V_h - \delta_1(\pi)) = \frac{p_l(V_h - V_l)[p_l q + p_h q(1 - q)\pi]}{p_l q + p_h(1 - q)^2 \pi}$ is increasing in π , then the function $\phi(\cdot)$ is decreasing in π . Now by our notations π_e and π_i can be redefined as

$$\phi(\pi_e) = \frac{c}{p_h} > c = \phi(\pi_i);$$

hence $\pi_e < \pi_i$. This completes the proof. \square

Proof of Proposition 4: For item 1, it follows from equations (11) and (20) since $q \in (0.5, 1)$.

For item 2, the result follows immediately from item 1 and Proposition 3 because the total surplus is equal to bidders' payoff plus seller's revenue. \square

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