Theory of Public Goods

A *public good* is often defined to be a good that is both *nonrivalrous* and *nonexcludable* in consumption.

The *nonrivalrous* property holds when use of a unit of the good by one consumer does not preclude or diminish the benefit from another consumer using the same unit of the good. Thus there is jointness in consumption of the good—one unit of the good produced generates multiple units of consumption. Nonrivalrous implies that the opportunity cost of the marginal user is zero. Some classic examples are a radio broadcast—my listening to the broadcast does not diminish your ability to benefit from the same broadcast; air quality—if a power plant reduces its sulfur dioxide emissions, all consumers in the area exposed to the plant emissions enjoy an increase in air quality; charitable giving—when one person donates to a charity and the activities of the charity expand, everyone in the community can benefit.

The *nonexcludable* property holds when it is impossible to prevent others from jointly consuming a unit of the good once it is produced. This is fundamentally a question of establishing effective property rights. Whether a good is excludable or nonexcludable is a function of the cost of setting up and enforcing a private property right to the good. Classic examples of nonexcludable goods are air and water quality over certain sites, fish in a certain part of the
ocean, and a section of unfenced land. Note that the
excludability decision is an inherently nonconvex problem.
Rivalrousness is a more continuous concept. The degree of
rivalrousness can be defined by the size of the marginal
opportunity cost of an additional user. Zero is one extreme.
As argued by Starret, the other extreme can defined by the
condition that the marginal opportunity cost is at least as
large as the average provision cost. He defines such a good
as fully rivalrous.

Pure private goods can be defined as goods that are both
excludable and fully rivalrous. Impure public goods are
defined as goods that are nonexcludable and partially
rivalrous (these are sometimes termed congestible public
goods). Examples here would include a swimming pool, a
park, a highway.
A Simple Introductory Model

A community consists of $n$ consumers indexed by $i = 1, \ldots, n$.

Assume there are two goods: a numeraire private good $x$ and a public good $g$.

Each consumer $i$ has quasi-linear preferences represented by utility function

$$u_i = x_i + v_i(g_i)$$

where $v_i(\cdot)$ is increasing, strictly concave and $v_i(0) = 0$.

Each consumer has some strictly positive endowment of the numeraire $w_i$.

Assume that the cost of supplying a unit of public good (measured in units of the numeraire) is a constant $c$.

An allocation for the community is a vector of consumption bundles $((x_1, g_1), \ldots, (x_n, g_n))$.

An allocation is feasible if (i) for all $i$, $g_i$ in $[0, g]$ and $g_i \geq 0$ and (ii) $\sum x_i \leq \sum w_i - cg$. Note the difference from the case of a two private goods economy.

An allocation is Pareto efficient if (i) it is feasible and (ii) there exists no alternative feasible allocation which Pareto dominates it.
In this formulation $g$ is the aggregate quantity of the public good produced. Individual consumption is constrained to be less than or equal to the total quantity supplied.

If the public good is also assumed to be non-excludable, then 

$$ g_i = g \quad \text{for all } i. $$

**Proposition 1:** An allocation $(x^e, g^e)$ such that $x^e_i > 0$ for all $i$ is Pareto efficient if and only if

(i) for all $i$, $g^e_i = g^e$;

(ii) $\sum v'_i (g^e) \leq c \quad (= \text{if } g^e > 0)$.

(iii) $\sum x^e_i = \sum w_i - cg^e$.

Condition (i) requires that all consumers get to consume all the public good provided (an implication of nonrivalrousness). If the public good is non-excludable, then (i) will necessarily be satisfied.

Condition (iii) is a resource balance/nonwastefulness condition that requires that all of the endowment be used for private goods or public goods consumption.

Condition (ii) is one the most important results in theoretical public economics. The Pareto Optimal level of the public good is such that the sum of the marginal benefits must equal the marginal cost. This is known in the public economics literature as the *Samuelson Rule*.

In our simple quasi-linear preference environment, the efficient level of the public good is independent of the allocation of the numeraire private good among consumers. This is not a general property of public goods economies.
Public Goods and Market Failure

(i) Non-excludable public goods

Consider our simple economy and assume that the public good is provided by the market mechanism. There is no collective action by the consumers. Competition among suppliers would yield a price per unit for the public good of \( p = c \).

Even with competitive price-taking behavior on the part of consumers, the basic consumer choice problem is not a decision-theoretic problem. The optimal quantity demanded by an individual consumer depends upon what the expected quantity demanded by the other consumers. So we have a strategic choice problem.

Let \( s_i \) denote the amount of public good that is purchased by citizen \( i \). Adopting a Nash behavioral approach, the solution to the individual choice problem can be characterized by

\[
    s_i^* = \arg \max_{s_i \in [0, w_i/p]} w_i - cs_i + v_i \left( \sum_{j \neq i} s_j^* + s_i \right)
\]

An allocation \((x_i^*, g_i^*)\) is a market equilibrium if there exists an equilibrium vector of public good subscriptions (purchases) \( s^* \) such that for all \( i \) (i) \( g_i^* = \sum_j s_j^* \) and (ii) \( x_i^* = w_i - cs_i^* \).

**Proposition 2:** Suppose that \( g^e > 0 \). Then if \((x_i^*, g_i^*), i = 1,..n\), is a market equilibrium it is not efficient.

Efficiency conditions (i) and (iii) are satisfied, but the Samuelson condition is not.
In market equilibrium, for all consumers $i$

$$v_i'(\sum g_j^*) \leq c \quad \text{(with } = \text{ if } g_i^* > 0)$$

If $g^e > 0$, then the market will under-provide the public good. In the case that $g_i^* = 0$ for all $i$ the result follows immediately. If $g_i^* > 0$ for some $i$, then

$$\sum_i v_i'(\sum_i g_j^*) > c$$

which implies that $\sum_i g_j^* < g^e$.

The underprovision result here is often referred to as the Free Rider Problem in public goods provision.
(ii) Excludable Public Goods

If free riding is the fundamental problem, then if the public good were excludable, then market provision might be more likely to be efficient. Modeling market provision of an excludable public good is, however, not a simple exercise. We still have the jointness feature—one unit of the good supplied can be consumed, and thus sold, to multiple consumers,

One famous approach to modeling a market for an excludable public good is the concept of a Lindahl equilibrium.

An allocation \((x_i^*, g_i^*), i = 1, n\), is a Lindahl equilibrium if there exists an equilibrium vector of personalized public good prices \((p_i^*), i = 1, n\), such that (i) for all \(i\) \((x_i^*, g_i^*)\) solves the problem

\[
\text{Max } x_i + v_i(g_i) \quad \text{s.t. } x_i + p_i^*g_i = w_i
\]

(ii) for all \(i\) and \(j\), \(g_i^* = g_j^*\); and (iii) \(\sum p_i^* = c\).

So each consumer is assumed to choose her own optimal private and public good consumption level given her income and her own personalized price. The equilibrium prices induce each consumer to choose a common optimal value for the public good. Firms producing the public good receive the sum of the prices paid by all buyers for each unit they produce, and the full price across consumers equals marginal (equals average cost here) at equilibrium by condition (iii).
Proposition 3: A Lindahl equilibrium allocation is Pareto efficient.

For all \( i \)

\[
v'_i(g_i^*) = p_i^*,
\]

and \( g_i^* = g^* \) with \( \sum p_i^* = c \). Then

\[
\sum_i v'_i(g^*) = c.
\]

The problem here is that it is hard to envision how the system of personalized/discriminating prices is established. If there is no price discrimination across consumers, then a common price across buyers must hold for each unit of the public good. What is that common price? Is it the same price across units of the public good supplied? Won’t inefficiency obtain?

We can illustrate some of the fundamental issues here by assuming a discrete public good model, i.e. \( g \) is \( \{0,1\} \).

Assume that \( \sum v_i(1) \geq c \), i.e. the efficient level of provision of the public good is 1.

Lindahl pricing here would be such that for all \( i \), \( p_i^* \) in \( [0, v_i(1)] \) and \( \sum p_i^* = c \).

Suppose instead we assume that there is a common consumer price

Let \( D(p) = \#\{i: v_i(1) \geq p\} \) for all \( p \).

If \( pD(p) < c \) for all \( p \), then the market will not provide the public good.
If $pD(p) \geq c$ for some $p$, then the market would provide the public good at the price such that $pD(p) = c$.

But individuals for whom $v_i(1) < p$ would be excluded—which is an inefficiency under nonrivalrousness.

**Conclusion:**

Market mechanisms are unlikely to implement efficient allocations in public good economies. The institutional response could be to turn to government or collective action to provide such goods. The question then is whether government institutions can establish mechanisms that lead to efficient allocations in public good economies.
**Classical Theory of Clubs**

What if the public good is both excludable and partially rivalrous/congestible?

At least 3 important dimensions to thinking about modeling this type of good: (i) population homogeneous or heterogeneous (ii) population is partitioned into clubs or not partitioned (iii) is the public good provided by the club fixed or variable in use.

**Simple utility-maximizing representative club model**

Club is assumed to organize so as to solve the following problem:

\[
\begin{align*}
\text{Max} & \quad U(g, \eta, n\eta, c) \\
g, \eta, n, c
\end{align*}
\]

subject to

\[
nc + C(g) = 0.
\]

where \( g \) is nonexcludable (within club) public good (size and/or type of facility), \( \eta \) is individual use of the excludable element of the public good (visits), \( n \) is the number of club members, and \( c = x - w = \) net consumption of the numeraire private good.

Necessary conditions for a utility-maximizing club:

i) \( \frac{nU_g}{U_c} = C_g \), 

ii) \( \frac{U_\eta}{U_c} = - \frac{nU_G}{U_c} \), 

iii) \( - \frac{nGU_G}{U_c} = C \),
where \( G = n \eta \) = total utilization of excludable element.

Implications:

Condition i) is the Samuelson condition—the nonexcludable element of the club is chosen such that the sum of the marginal benefits across club members equals marginal cost.

Condition ii) is also a form of the Samuelson condition since congestion takes the form of a nonexcludable public “bad” within the club.

Condition iii) is the group size condition. Dividing by \( G \), this condition says that the marginal opportunity cost of an additional user equals average provision cost. Thus at the optimal club size the nonexcludable public good (facility) is fully rivalrous when shared by the group. This result makes intuitive sense. If club members are treated symmetrically, then new members will be expected to contribute the average cost of the collective good. If that contribution is greater than the marginal opportunity cost to group of adding this member, then she should be included. An efficient club adds members until the average and marginal costs are equal.

Note that, from condition ii), an optimal pseudoprice or Pigovian tax on use (trips) should be set at the sum of the marginal congestion damage inflicted upon other members of the club.

Note also that combining conditions ii) and iii), pseudopricing at marginal opportunity cost (marginal congestion cost) would yield shadow revenue that exactly covers the total costs of the optimal facility.
Clubs and Competitive Equilibrium

Some remarks based upon Scotchmer (1994, in the Quigley and Smolensky volume on the Reading list)

In the simplest version of the standard models in this literature the utility function is of the form $U^i[g,n,x_i]$. The cost of the public good is of the form $C(g,n)$. In the simplest settings, $n$ is simply the number of individuals with whom $i$ shares in the consumption of public good of amount $g$. In more complex settings, $n$ is a vector of integers indicating the number of each type of individual with whom agent $i$ shares in the consumption of public good in amount $g$. The consumption of the public good within a club is nonexcludable, but admission to the club is excludable. A club is characterized by a two-tuple $(g,n)$ and $\pi(g,n)$ is the admission price to a club that provides services in amount $g$ and has membership of size $n$ (in more complex models, admission prices can vary by type).

The most common concept of equilibrium used in the club theory literature is a “utility-taking” equilibrium: firms observe preferences and choose prices; consumers choose whether to join a real or hypothetical group, given the price system. In equilibrium, no real or hypothetical group could attract any consumer (unilaterally).

Scotchmer (1985) equilibrium approach: entrepreneur $j$ chooses facility size $g_j$ and price of admission $p_j$, and consumers sort among clubs, so that, conditional on the strategies $(p_j,g_j)$, no consumer could increase utility by switching. Modeled as two stage game. In first stage, clubs simultaneously choose $(p_j,g_j)$ in the second stage the consumers distribute across offerings. Clubs anticipate the distribution of consumers in the second stage when choosing their strategies in the first stage.
Scotchmer shows that admission prices in equilibrium are typically higher the “competitive” prices in a finite economy, but that facilities will be efficiently provided to members. Firms earn rents (since demand is not “infinitely elastic” due to endogeneity of congestion quality to prices) so excessive incentive to enter, and number of clubs in equilibrium is typically too large. As economy grows via replication, equilibrium prices and club sizes converge to competitive ones. For a sufficiently large economy, equilibrium in pure strategies exists.