Fairness and externalities

Rodrigo A. Velez

Department of Economics
Texas A&M University

December, 2011

Fair allocation Theory

- What is a fair allocation?
  - Foley (1967)
- Are there fair allocations?
  - Varian (1975)...
- Is fairness compatible with efficiency?

Fair allocation Theory

- What is a fair allocation?
  - Foley (1967)
- Are there fair allocations?
  - Varian (1975)...
- Is fairness compatible with efficiency?

Consumption externalities

Outline

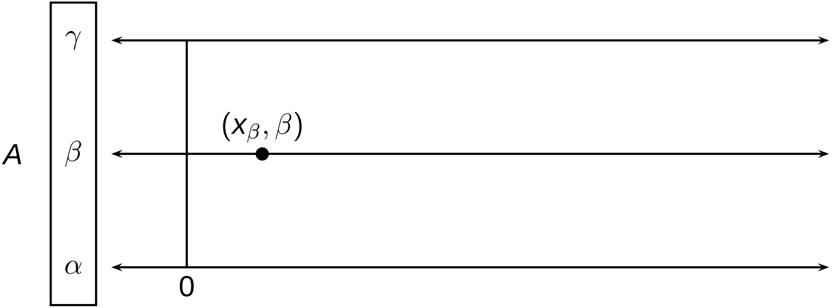
- Model
- Fairness
- Existence of fair allocations
- Compatibility of fairness and efficiency
- Applications

Model

- Resources: objects + money
- Bench mark case: no externalities
  - Svensson, Econometrica, 1983
  - Maskin, 1987
  - Alkan et. al., Econometrica, 1991
  - Aragones, SCW, 1995
  - Klijn, SCW, 2000

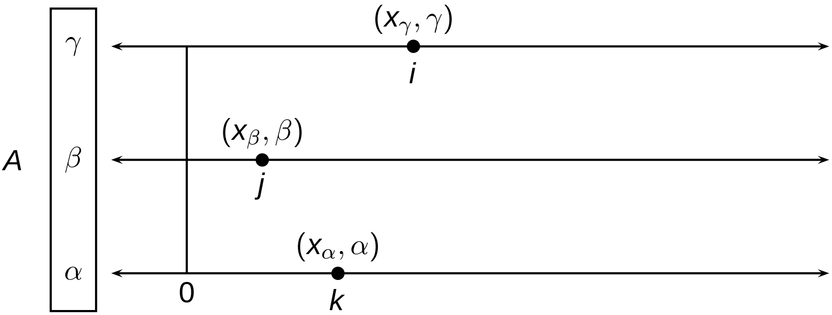
Model: consumption space

$N \equiv \{i, j, k\}$



Model: preference space

$N \equiv \{i, j, k\}$

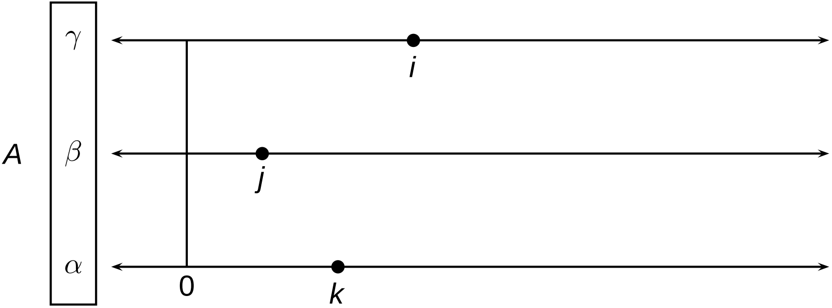


$Z \equiv (x, \mu)$ where $x = (x_\alpha, x_\beta, x_\gamma)$ and $\mu : N \to A$

Set of allocations: $Z \subset \mathbb{R}^A \times A^N$

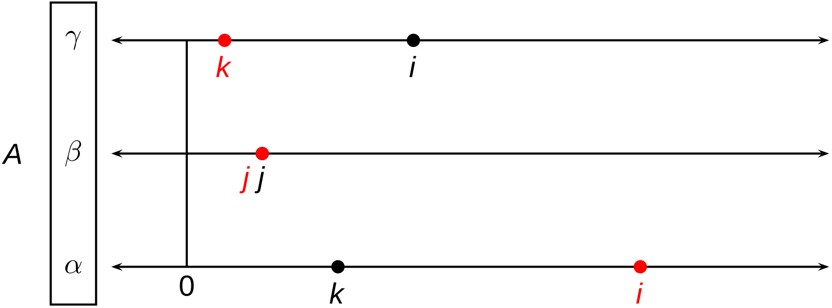
Model: preference space

Agents' preferences are complete and transitive binary relations on Z



Model: preference space

Agents' preferences are complete and transitive binary relations on Z



Preference notation: $z\ R_i\ z'$, $z\ P_i\ z'$, and $z\ I_i\ z'$

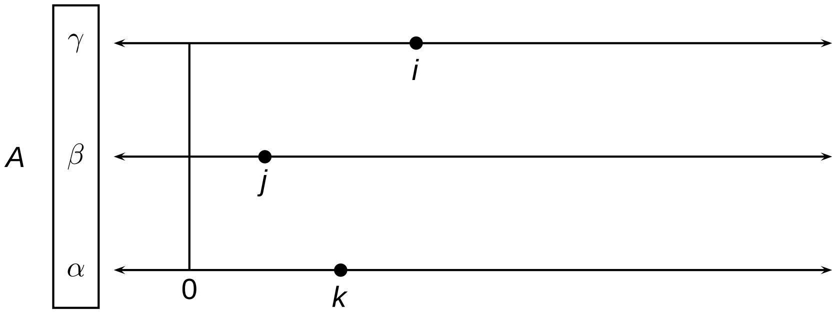
Axioms on preferences: I

Continuity

Weak upper and weak lower contour sets are closed

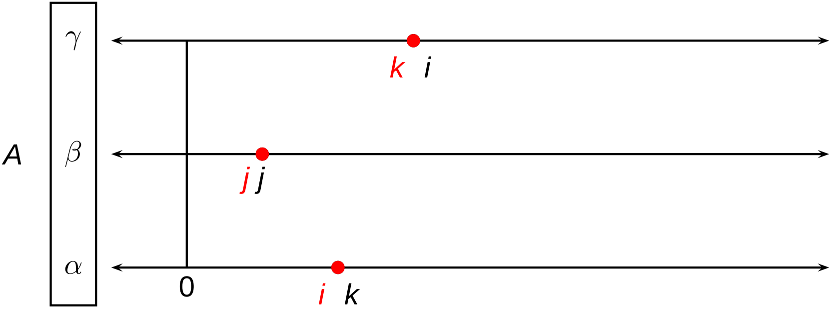
Axioms on preferences: II

Anonymity of externalities



Axioms on preferences: II

Anonymity of externalities



$z' \; I_i \; z$

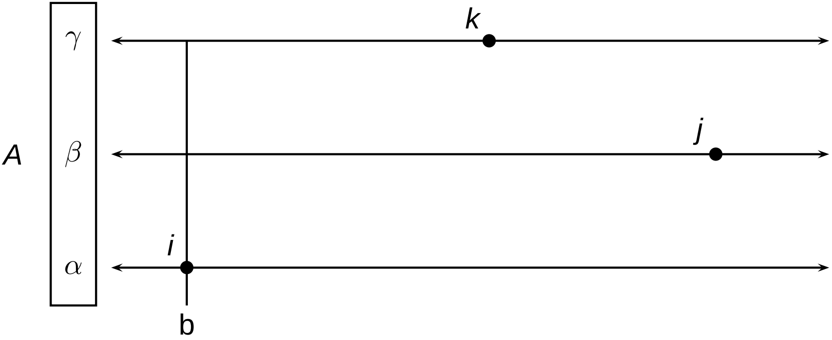
Axioms on preferences: III

equal-budget compensation assumption (agent i)

No object is infinitely better than the other

Axioms on preferences: III

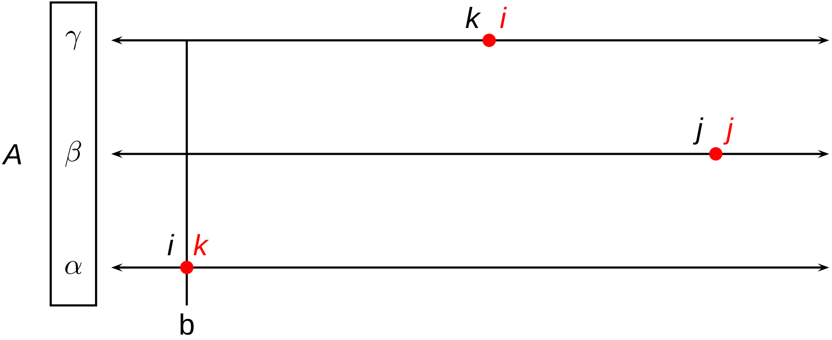
equal-budget compensation assumption (agent *i*)



If M is large enough, then agent i would prefer to swap her consumption with an agent whose consumption of money is greater than b.

Axioms on preferences: III

equal-budget compensation assumption (agent *i*)



If M is large enough, then agent i would prefer to swap her consumption with an agent whose consumption of money is greater than b.

Axioms on preferences: III

R_i satisfies the equal-budget compensation assumption if:

$\forall b \in \mathbb{R}$, $\exists M(R_i, b) \in \mathbb{R}$ s.t.,

(i) $M(R_i, b) \geq nb$;

(ii) for each $M > M(R_i, b)$ and each $(x, \mu) \in Z(M)$ such that for each $\alpha \in A$, $x_\alpha \geq b$, if $x_{\mu(i)} = b$, then there is a bijection $\mu' \in A^N$ such that $x_{\mu'(i)} > b$ and $(x, \mu')\, R_i\, (x, \mu)$; and

(iii) as $b \to -\infty$, $M(R_i, b) \to -\infty$.

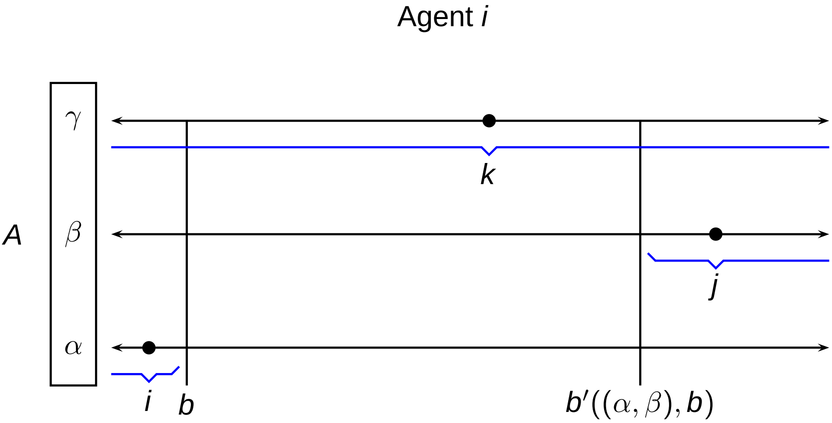
Axioms on preferences: III-A

Ceteris paribus compensation assumption (agent *i*)

Stronger form of the equal-budget compensation assumption

Axioms on preferences: III-A

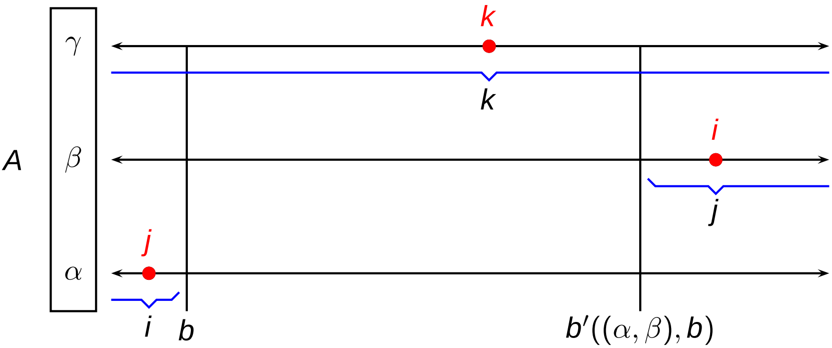
Ceteris paribus compensation assumption



Axioms on preferences: III-A

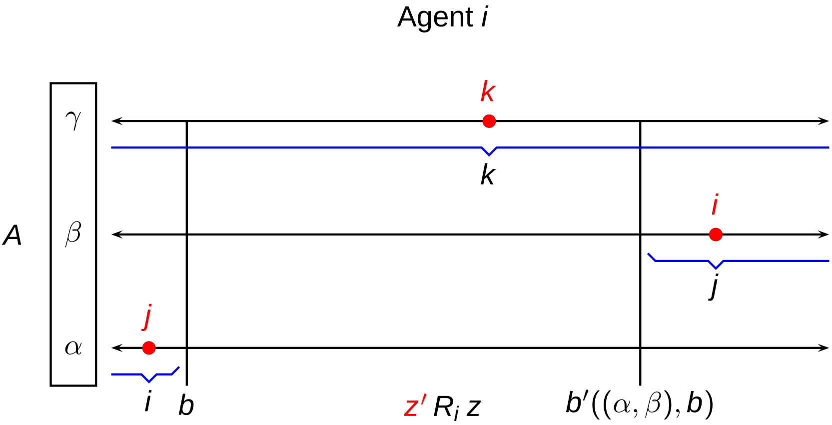
Ceteris paribus compensation assumption

Agent i



Axioms on preferences: III-A

Ceteris paribus compensation assumption



Axioms on preferences: III-A

R_i satisfies the ceteris paribus compensation assumption if:
$\forall(\alpha, \beta)$ s.t. $\alpha \neq \beta$, $\forall b \in \mathbb{R}$, $\exists b' \in \mathbb{R}$ s.t.,
$\forall(x, \mu) \in Z$ satisfying

- $x_\alpha \leq b$
- $x_\beta \geq b'$
- $\mu(i) = \alpha$

we have that $(x, \mu') \, R_i \, (x, \mu)$ where μ' is obtained from μ by swapping the consumption of agent i with the consumption of the agent who receives β at μ

Axioms on preferences: summary

Basic domain $\mathcal{B}$

- continuity
- anonymity of externalities
- equal-budget compensation assumption

Environment

$$N, A, M \in \mathbb{R}$$

$$|A| = |N|$$

$$R \equiv (R_i)_{i \in N} \in \mathcal{B}^N$$

$$e \equiv (R, M) \in \mathcal{E}$$

$$Z(e) \equiv \{(x, \mu) \in Z : \sum x_\alpha \leq M\}$$

$$Z(M) \equiv \{(x, \mu) \in Z : \sum x_\alpha = M\}$$

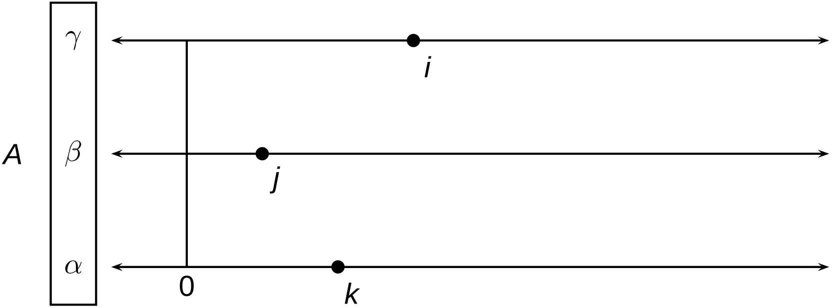
What's fair?

What's fair?

An allocation is non-contestable on fairness grounds if no agent prefers the allocation obtained by swapping her consumption with that of any other agent.

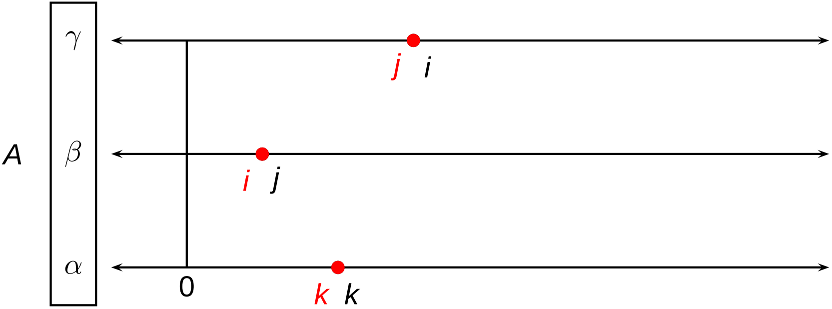
What's fair?

An allocation is non-contestable on fairness grounds if no agent prefers the allocation obtained by swapping her consumption with that of any other agent.



What's fair?

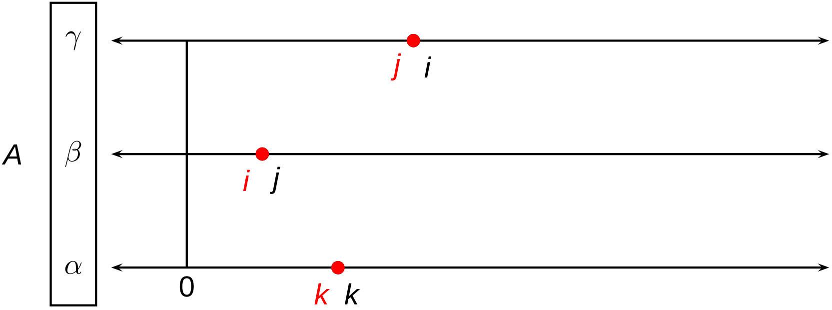
An allocation is non-contestable on fairness grounds if no agent prefers the allocation obtained by swapping her consumption with that of any other agent.



$z \, R_i \, z'$

What's fair?

An allocation is non-contestable on fairness grounds if no agent prefers the allocation obtained by swapping her consumption with that of any other agent.



Non-contestable allocations for e: $Nc(e)$

Efficiency

$e \in \mathcal{E}$ and $z \in Z(e)$ $z \in P(e)$ if there is no $z' \in Z(e)$ s.t. $\forall i$,

$z' \, R_i \, z$ and for some j, $z' \, P_j \, z$

General possibility result

Theorem: for all $e \equiv (R, M) \in \mathcal{E}$,

$$Nc(e) \cap Z(M) \neq \emptyset$$

Generalizes Alkan, Demange, and Gale (1991) Econometrica

Minimum consumption of money requirement

- $e \equiv (R, M) \in \mathcal{E}$, $b \in \mathbb{R}$.

  $B_b(e)$: satisfy minimum individual consumption b

Minimum consumption of money requirement

- $e \equiv (R, M) \in \mathcal{E}$, $b \in \mathbb{R}$.

  $B_b(e)$: satisfy minimum individual consumption b
- No externalities: fix b and R; if M is large enough then $Nc(R, M) \cap B_b(R, M) \neq \emptyset$

Minimum consumption of money requirement

- $e \equiv (R, M) \in \mathcal{E}$, $b \in \mathbb{R}$.

  $B_b(e)$: satisfy minimum individual consumption b
- No externalities: fix b and R; if M is large enough then $Nc(R, M) \cap B_b(R, M) \neq \emptyset$
- What if there are externalities?

A basic theorem for fair allocation

Theorem: $e \equiv (R, M) \in \mathcal{E}$; if $M \geq m_{(n-1)}(R, b)$,

$$Nc(e) \cap B_b(e) \cap Z(M) \neq \emptyset$$

Generalizes:

- Svensson (1983) Econometrica
- Maskin (1985)
- Alkan, Demange, and Gale (1991) Econometrica

Compatibility of fairness and efficiency

Theorem (Svensson, 1983): $e \equiv (R, M) \in \mathcal{E}$. If there are no externalities

$$Nc(e) \cap Z(M) \subseteq P(e)$$

- Does not generalize to domain with externalities
- Can we identify sufficient conditions that restore the inclusion?

Compatibility of fairness and efficiency

$e \in \mathcal{E}$

- $z \equiv (x, \mu) \in Z(e)$ is **non-wasteful** for e if for each $z' \equiv (x', \mu')$ that weakly Pareto dominates z, it must be the case that $x' \geq x$.

Compatibility of fairness and efficiency

$e \in \mathcal{E}$

- $z \equiv (x, \mu) \in Z(e)$ is **non-wasteful** for e if for each $z' \equiv (x', \mu')$ that weakly Pareto dominates z, it must be the case that $x' \geq x$.

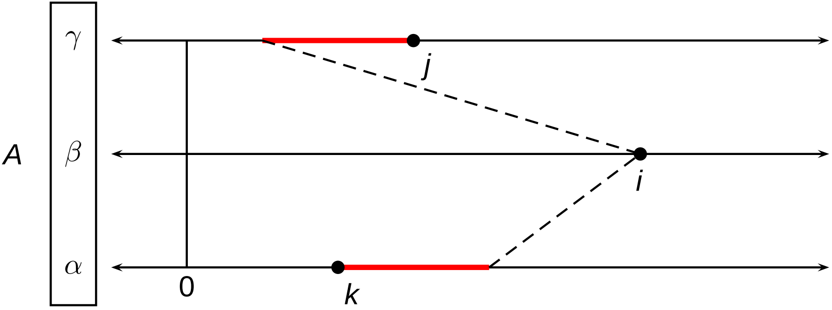
Theorem: $e \equiv (R, M) \in \mathcal{E}$, $z \in Z(e)$

- $z \in Nc(e) \cap z(M)$
- z is non-wasteful for e

then

$$z \in P(e)$$

Inequity-averse preferences (Fehr-Schmidt)



$$u_i(z) \equiv \quad v_{\mu(i)}(\mu(i)) + x_{\mu(i)}$$

$$-\frac{a}{n-1}\sum_{\alpha\in A}\max\{v_{\mu(i)}(\alpha) + x_\alpha - [v_{\mu(i)}(\mu(i)) + x_{\mu(i)}], 0\}$$

$$-\frac{c}{n-1}\sum_{\alpha\in A}\max\{v_{\mu(i)}(\mu(i)) + x_{\mu(i)} - [v_{\mu(i)}(\alpha) + x_\alpha], 0\}$$

where $c \in [0,1)$ and $c \leq a$

Summary of applications

Domain	Compatibility Fairness and Eff.
Inequity averse (F-S)	Yes if $\forall i, c_i < \frac{n-1}{n}$
Keeping up with the Joneses	Yes
Surplus-motivated	Yes (Interest overlap property)

Conclusions

- Model: general preference domain.
- Provided an operative test of fairness.
- General possibility results.
- Sufficient conditions that guarantee compatibility of fairness and efficiency.
- Theory delivers results for ample domains of other-regarding preferences.

Thank you for your attention.