Let them cheat!

Rodrigo Velez
joint with William Thomson
Department of Economics
University of Rochester

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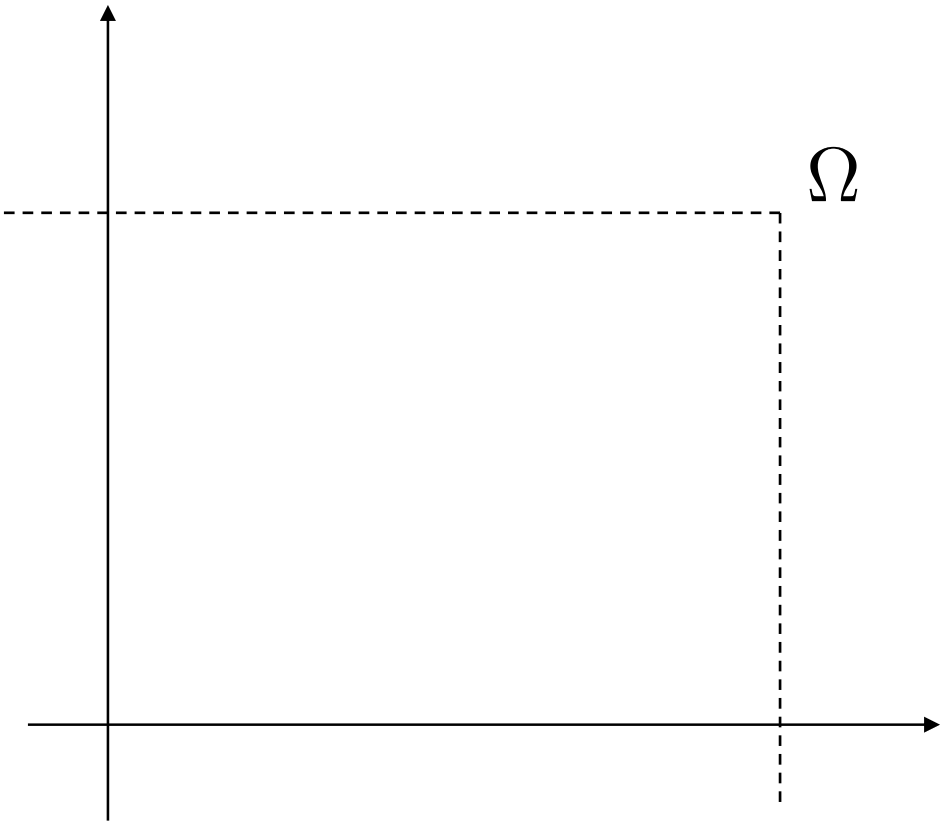
General objectives:

- Propose a solution for the fair allocation of a social endowment of goods: the equal-sacrifice solution.

- Analyze incentives properties of the proposed solution.

Model

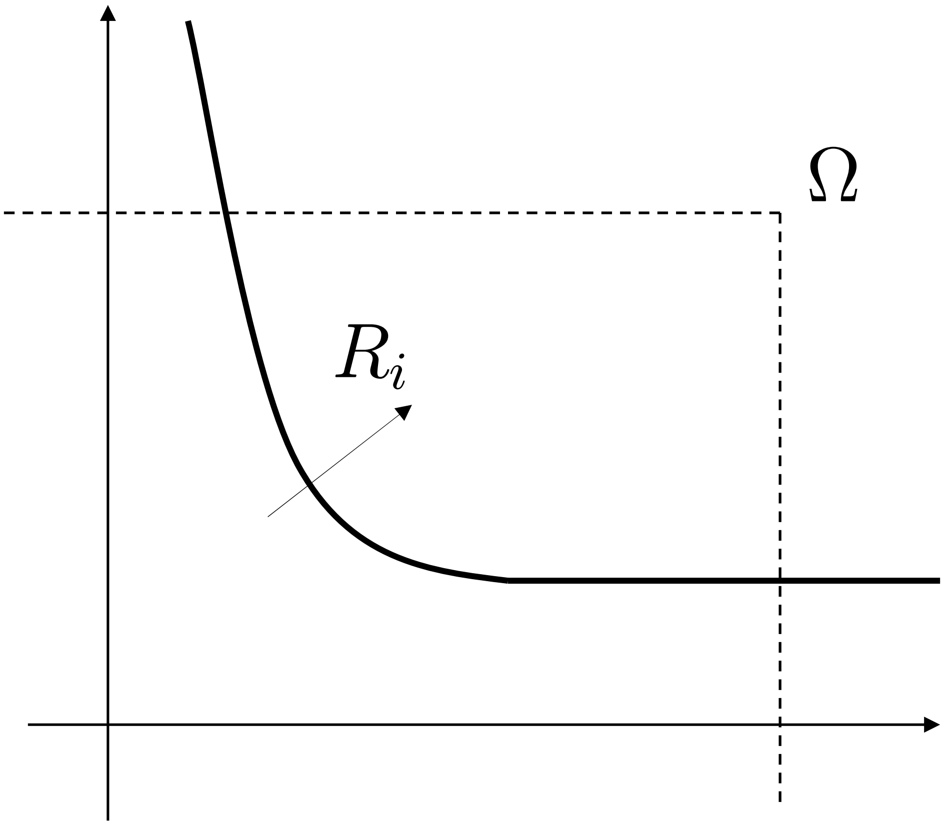
Set of agents N



Model

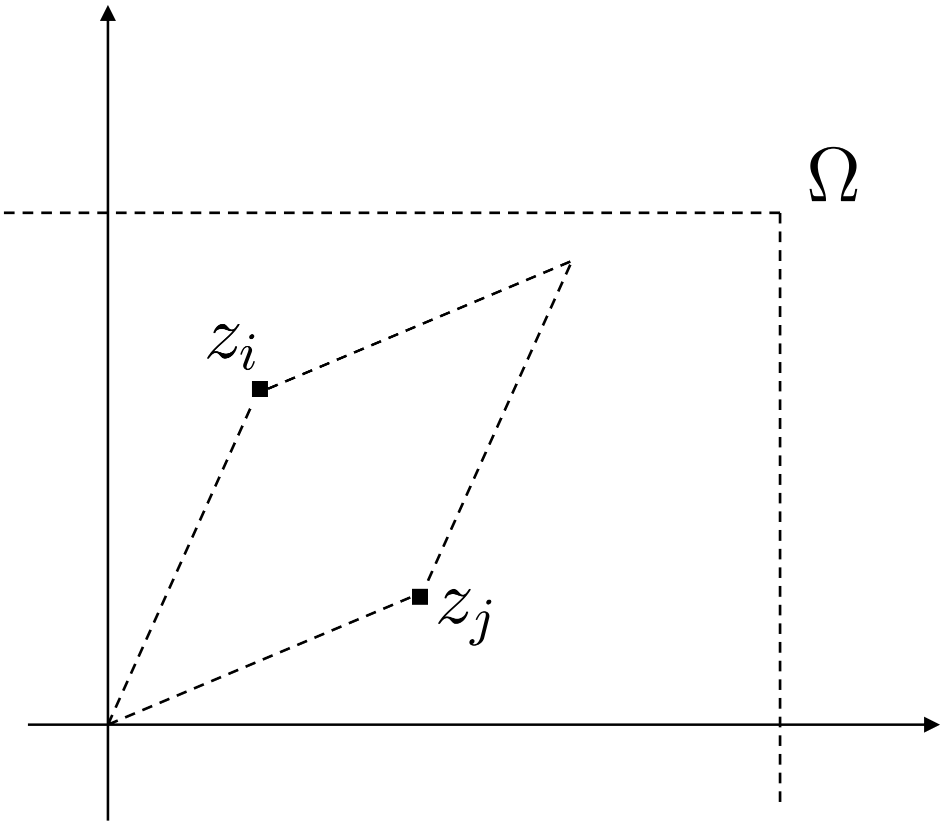
Set of agents N

Preference domain: $\mathcal{U}$



Feasible allocations

$$Z \equiv \{z \equiv (z_i)_{i \in N} : \sum_{i \in N} z_i \leq \Omega\}$$



Equal-sacrifice correspondence

Social choice correspondence: $F : \mathcal{U}^N \rightrightarrows Z$

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- Measure an agent's "sacrifice" at a proposed allocation.

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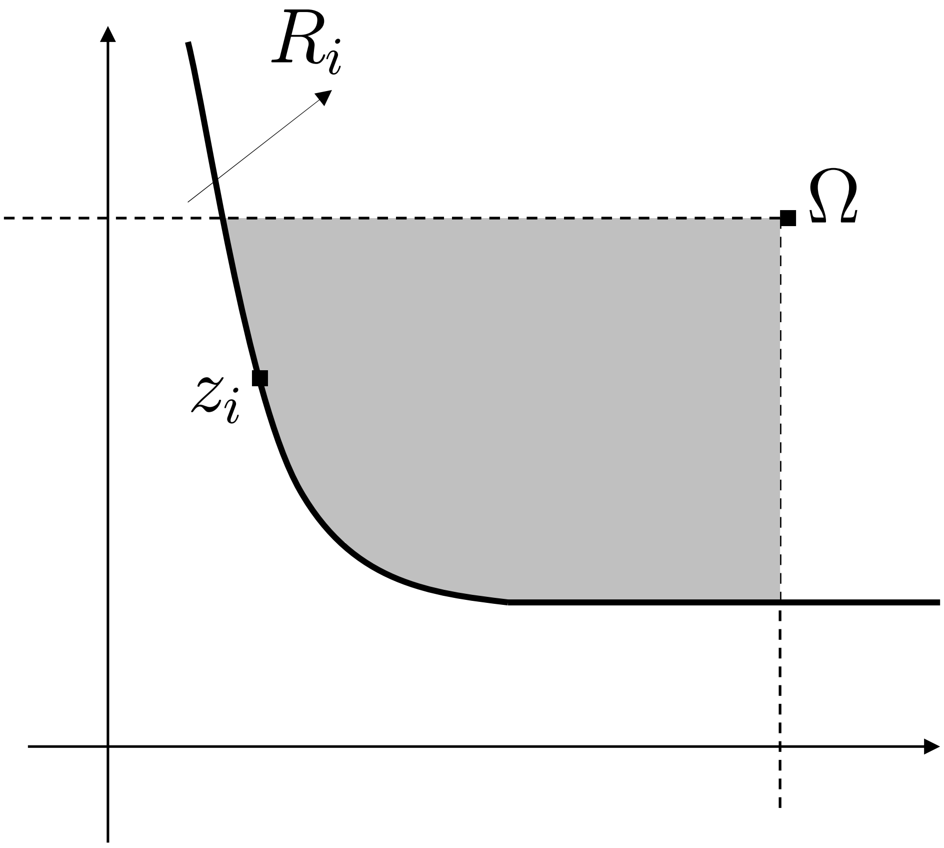
- Select the allocations at which sacrifices are equal across agents and this common sacrifice is minimal.

Equal-sacrifice correspondence



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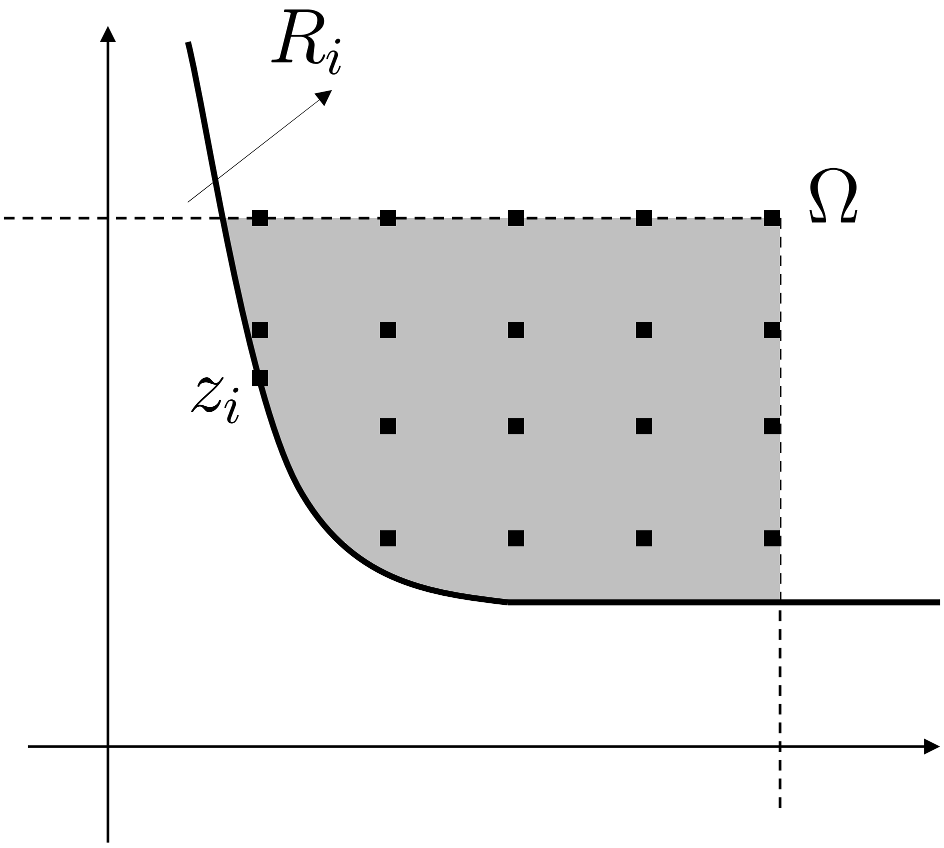

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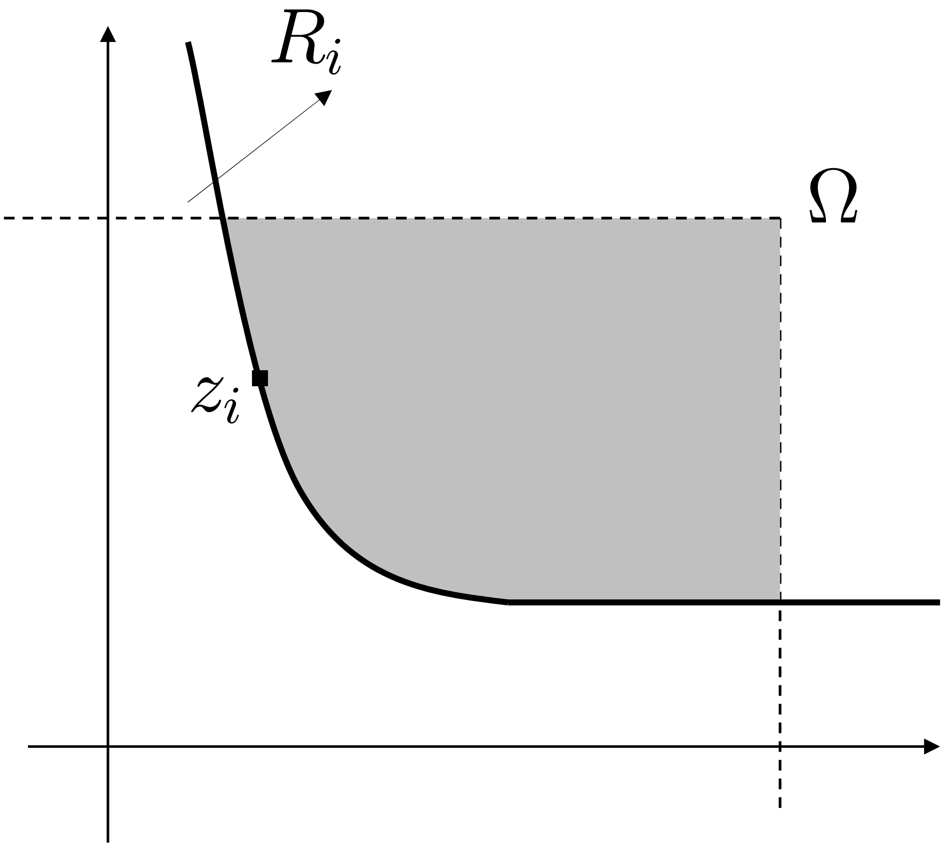

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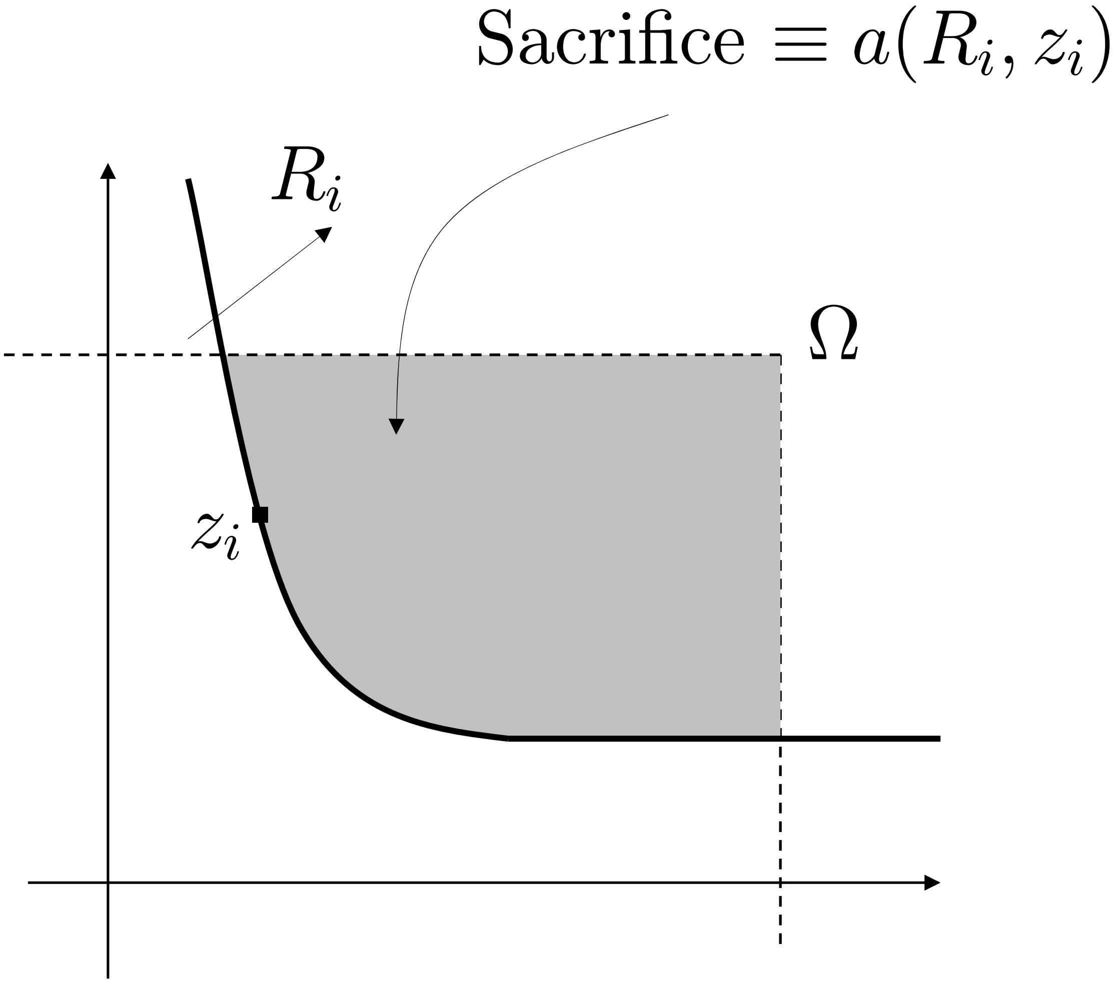

Equal-sacrifice correspondence

$$E(R) \equiv \left\{ z \in Z : \forall i \in N,\ a(R_i, z_i) = \min_{z' \in Z} \left\{ \max_{j \in N} a(R_j, z'_j) \right\} \right\}$$

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Proposition: Domain $\mathcal{U}$

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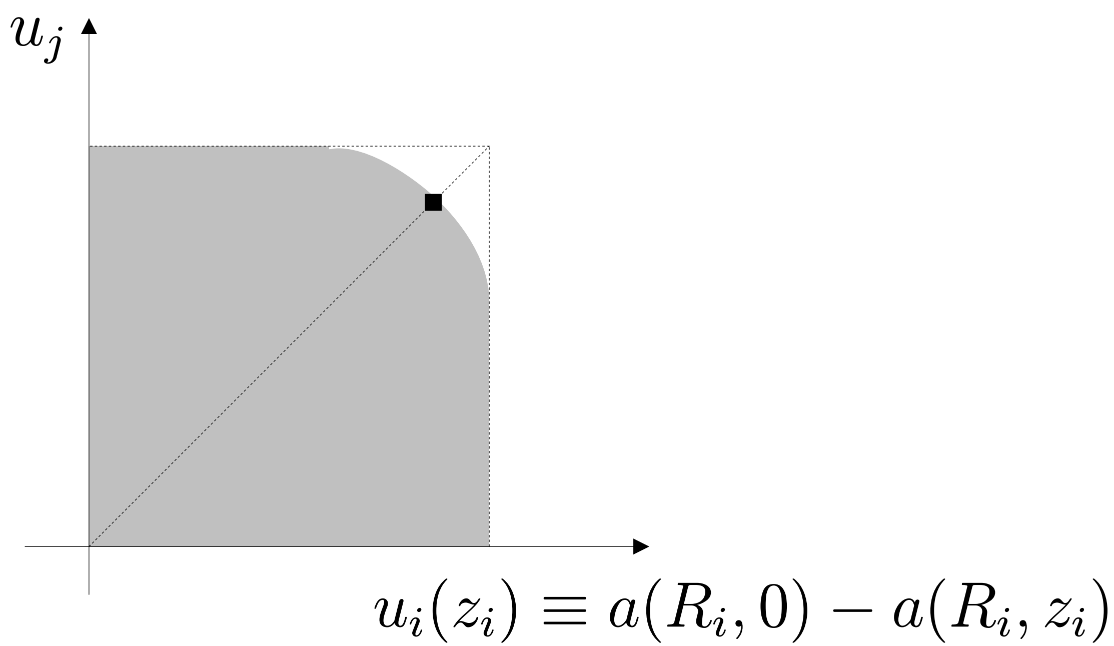

Efficiency

Proposition: Domain $\mathcal{U}$

1. $E \subseteq \text{Weak}P$
2. $E \subseteq P$ whenever
   - There are only two-agents
   - Preferences are strictly-increasing in the interior of the consumption space.

Manipulation of Equal-sacrifice correspondence

'…no one should find it profitable to "cheat," where cheating is defined as behavior that can be made look "legal" by misrepresentation of the participant's preferences…, with the proviso that the fictitious preferences should be within certain "plausible" limits.' (Hurwicz, 1972)

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Theorem: (Hurwicz, 1972; Serizawa, JET 2002),

Domain (Strict-mon. interior + homothetic + smooth)

$$\text{no } f \text{ satisfies ete} + P + \text{Strat-proofness}$$

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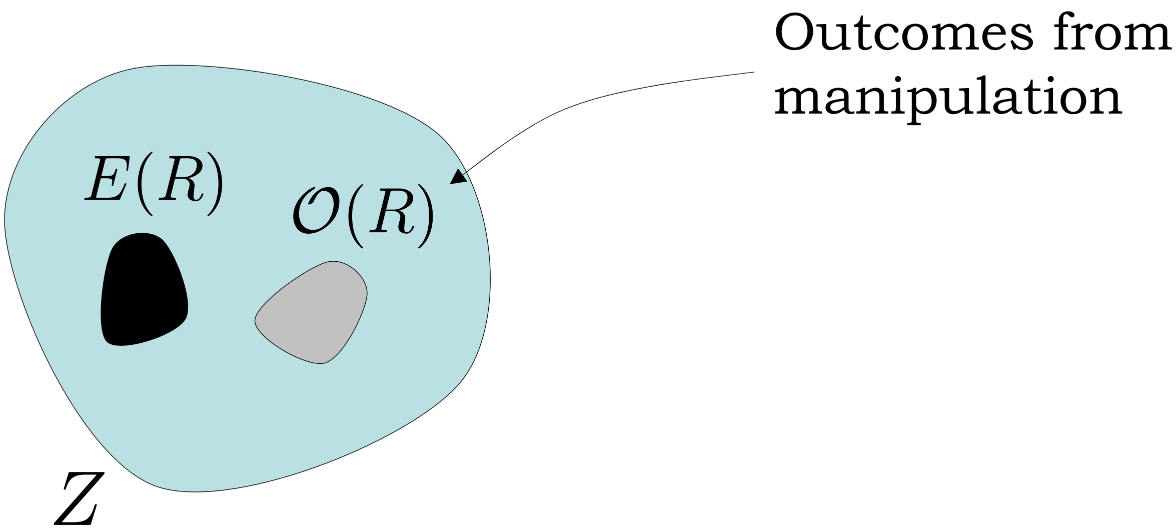

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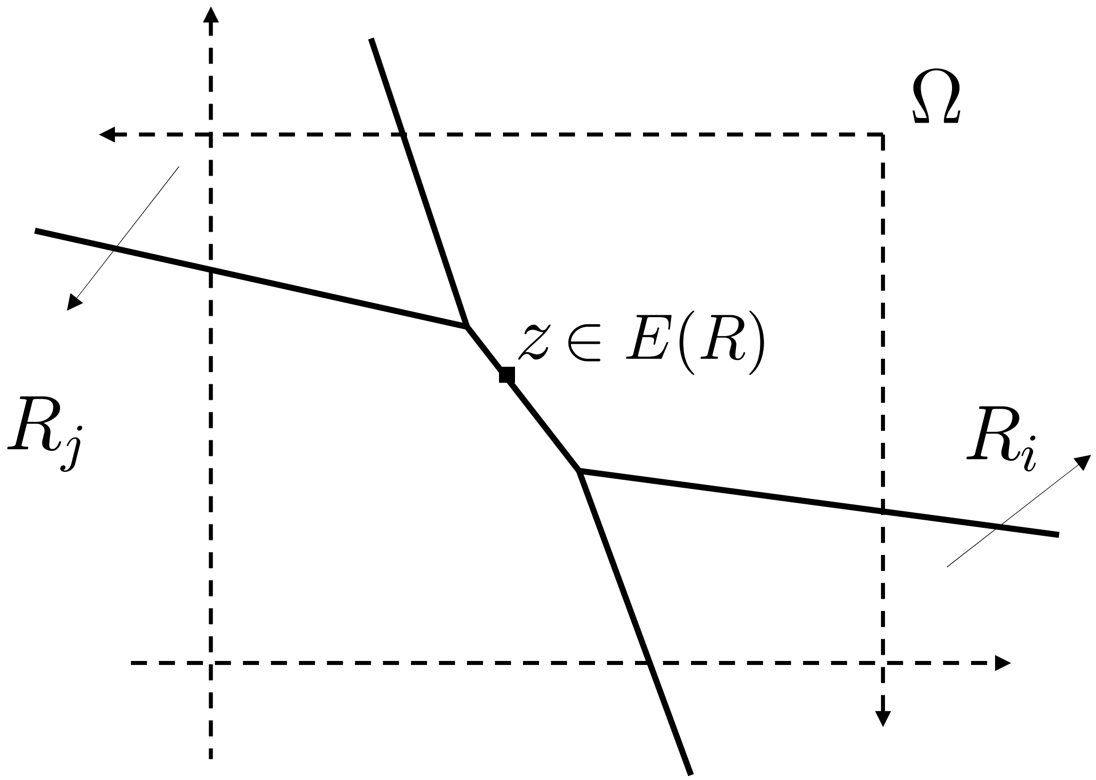

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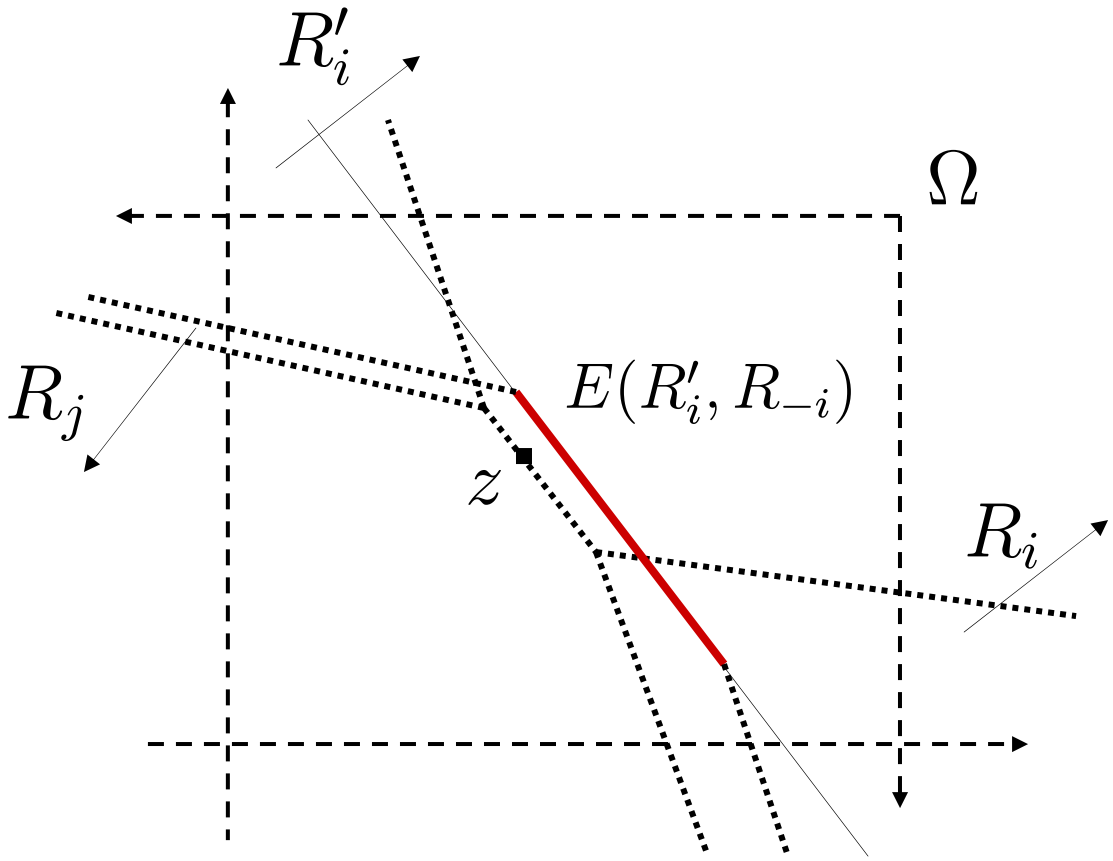

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- We propose to complete the description of the allocation process suggested by the solution.

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$$O\langle \mathcal{D}, E, f \rangle (R, z) \equiv \begin{cases} z \equiv (z_i)_{i \in N} & \text{if } z \in E(R) \\ f(R) & \text{otherwise} \end{cases}$$

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Nash Equilibria: $\mathcal{N}\langle \mathcal{D}, E, f, R^0 \rangle$

Nash Equilibrium outcomes: $\mathcal{O}\langle \mathcal{D}, E, f, R^0 \rangle$

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$$\forall f \text{ and } g,\ \mathcal{O}\langle \mathcal{D}, E, f, R^0 \rangle = \mathcal{O}\langle \mathcal{D}, E, g, R^0 \rangle$$

$$\text{Common set} \equiv \mathcal{O}\langle \mathcal{D}, E, R^0 \rangle$$

Main result

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Theorem: $R^0 \in \mathcal{I}^N$ (str.inc.interior + all goods normal)

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$$\mathcal{O}\langle \mathcal{I}, E, R^0 \rangle = W^c_{ed}(R^0)$$

- $\mathcal{O}\langle \mathcal{I}, E, R^0 \rangle \subseteq \text{Envy-free}(R^0) \cap B_{ed}(R^0)$
- $\mathcal{O}\langle \mathcal{I}, E, R^0 \rangle \subseteq P(R^0)$
- Implementation implications.

Conclusion

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Thanks for your attention