

Optimal Monetary Policy with Endogenous Inattention

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April 2009

Abstract

This paper studies monetary policy in a sticky-information model where the frequency of information updating is endogenously determined. Relative to a setting with exogenous information updating, price stability becomes more important. In a stable price environment, price setters update their plans less frequently, which makes monetary policy more effective and efficient. By demonstrating the new channel of monetary policy, this paper offers an implication on the so-called “Great Moderation” in favor of the improved policy hypothesis.

1 Introduction

There is an active research frontier on the sticky-information model, which was originally proposed by Mankiw and Reis (2006). They introduced sluggish information updating as a replacement of Calvo style price adjustment. The model has become popular because it closely fits empirical regularities than the Calvo models.

Two substantial contributions has been made. One is on policy question, “What is the optimal monetary policy in the sticky-information model?” This question was studied by Ball, Mankiw and Reis (2005) (BMR hereafter). The other is on micro-foundations, “Where does information stickiness come from?” This question was studied by Reis (2006).

*I would like to thank George Evans, Ryo Kato, Nobu Kiyotaki, Ricardo Reis, Chris Sims, Lars Svensson, seminar participants at Princeton University, the Bank of Japan, and Tohoku University for their helpful comments. All errors are mine.

But this paper argues that an important economic relation has been neglected. An implication of BMR is that the frequency of information updating is an important determinant of optimal monetary policy. An implication of Reis (2006) is that policy is an important determinant of frequency of information updating. The interdependence is clear. We need a unified framework to better understand monetary policy in the sticky-information model.

This paper proposes a sticky-information model where the frequency of information updating is determined endogenously, and studies monetary policy in it. The model allows us to see the interaction between policy and inattentiveness. Two main findings are (1) a naive monetary policy—the optimal policy in BMR—is too accommodative compared to the sophisticated policy—the best policy in the new framework—and (2) price setters under a naive monetary policy are too attentive compared to price setters under the sophisticated policy.

The results have an important implication on the so-called “Great Moderation,” a substantial decline in the volatility of output and inflation over the past twenty years. Possible causes has been hotly debated. Some empirical studies question the quantitative importance of improved monetary policy and attribute the economic stability to exogenous changes (e.g., Stock and Watson (2003)), but Bernanke (2004) argued that:

First, monetary policies that brought down and stabilized inflation may have led to stabilizing changes in the structure of the economy as well, in line with the prediction of the famous Lucas (1976) critique.... Theories of “rational inattention” (Sims, 2003), according to which people vary the frequency with which they re-examine economic decisions according to the underlying economic environment, imply that the dynamic behavior of the economy would change—probably in the direction of greater stability and persistence—in a more stable pricing environment, in which people reconsider their economic decisions less frequently.¹

Notice that Bernanke clearly mentions the interaction between the monetary policy and the frequency of planning. By offering a framework with such a interaction, this paper examines Bernanke’s conjecture. In the model, a policy that stabilizes price also stabilizes output because in

¹Bernanke (2004) mentioned the rational inattention of Sims (2003), but the sticky-information model of Mankiw and Reis (2002) fits his notion better.

stable price environment, price setters update their plans less frequently. Hence, this paper gives a theoretical justification of the conjecture.

Branch, Carlson, Evans and McGough (2007a) is closely related to this paper. They endogenize the rate of information updating in the BMR model in a tractable way. But they do not rely on a micro-foundation in their framework. Moreover, their focus is on the relation between policymaker's preference and macroeconomic stability. They conduct comparative statics while maintaining the optimal policy in BMR model.² This paper endogenizes the length of inattention with the micro-foundation proposed by Reis (2006). My focus is on the optimal monetary policy in the new framework. The application on the Great moderation emerges in the comparison between the optimal policy in the new framework and the optimal policy in BMR model.

2 Economy with Exogenous Inattention

In this section, I study an inattentive economy where frequency of information updating is exogenously given. The economy has a representative consumer, a continuum of monopolistic producers living on a unit mass, and the monetary authority. The aggregate economy is reduced to two simple equations: the sticky-information Phillips curve that summarizes the supply side of the model and the monetary policy rule that summarizes the demand of the model.

2.1 The Supply Side

The representative consumer's instantaneous utility function is

$$\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \int \frac{L_{jt}^{1+\psi}}{1+\psi} dj \tag{1}$$

where C_t is a Dixit-Stiglitz aggregator

$$C_t = \left[\int_0^1 (C_{jt})^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}.$$

²Branch, Carlson, Evans and McGough (2007b) extends their own work by developing adaptive learning formulation.

C_{jt} is consumption of good j , and L_{jt} is labor supply to a producer j . The parameters measuring risk aversion (σ) and the marginal disutility of labor supply (ψ) are non-negative, while the elasticity of substitution between goods (γ) is larger than one.

The representative consumer maximizes an expected discounted utility stream subject to a standard budget constraint, which includes a government levied proportional sales tax τ_t assumed to follow a stationary process. The consumer's problem leads to a demand that, in log form, is given by

$$y_{it} = y_t - \gamma(p_{it} - p_t)$$

where p_t is the log of the usual price index.

The producer j has access to the linear production function, $Y_{jt} = A_t L_{jt}$. A_t is an aggregate productivity shifter that follows an arbitrary stochastic process. The producer's pricing problem leads to an full-information optimal price of the form

$$p_{jt}^* = p_t + \alpha(y_t - y_t^n) + u_t, \tag{2}$$

where $\alpha = (\psi + \sigma) / (1 + \gamma\psi)$ and small letters denote the logarithm of the respective capital letters. y_t^n is the natural level of output that is defined as the level of output when prices are flexible (so $p_j = p_j^*$ for all j), which has a linear relation with the aggregate productivity

$$y_t^n = \frac{(1 + \psi) \log A_t - \log \left(\frac{\gamma}{(1-\gamma)(1-\bar{\tau})} \right)}{\psi + \sigma}.$$

u_t is mark-up shock that reflects random variation in sales taxes. The markup shock is the model's analogy of what practitioners often call supply shock or cost-push shock; the key feature of the shock is that they raise price level while depressing output. Hence, the shock make the monetary authority face a trade-off between price stability and output stability.

I introduce information stickiness following Ball et al. (2005). Each period, a fraction λ of producers, chosen randomly, receives complete information on the state of the economy. Prices are perfectly flexible in the sense that prices are adjusted by all producers in each period. However, prices are set based on the last information received by each producer. Taking a first order approximation, I measure the logarithm of aggregate price level p by the average of the individual

prices p_{jt}^* . I take a first order approximation to optimal price setting too, which yields certainty-equivalent behavior. Hence the aggregate price level is

$$p_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} [p_t + \alpha (y_t - y_t^n) + u_t]. \quad (3)$$

2.2 Monetary Authority

I close the model with the simplest possible demand side, the quantity equation:

$$y_t = m_t - p_t, \quad (4)$$

where m is the log of the money supply. The underlying assumption is a cash-in-advance constraint, which implies that nominal spending is proportional to the money supply. The monetary policy is a mapping from the current and past realizations of exogenous shocks, A_t and u_t , to the current money supply m_t . The monetary authority announces the monetary policy at the beginning of the history, and stick to it forever.

2.3 Equilibrium

An equilibrium in the economy with exogenous inattention λ is a pair of processes $\{y_t, p_t\}$ that satisfy (3) and (4) for all dates given the exogenous process u_t and the process m_t specified by the monetary authority.

2.4 Optimal Policy

I define welfare as the utility function of the representative consumer. I assume that the discount factor approaches one, so policymakers seek to minimize the unconditional expectation of welfare. A second order approximation of the utility function gives the following loss function:

$$Var [y_t - y_t^n] + \omega E [Var_i [p_{it} - p_t]]. \quad (5)$$

where $\omega = \gamma^2 (\psi + \gamma^{-1}) / (\psi + \sigma)$.³

³The approximation is taken around the point when all the shocks are at their means. It is important for the accuracy of the approximation that the approximation point is close to being efficient. To ensure that, I assume that

Let $u_t = \sum_{i=0}^{\infty} \rho_i \varepsilon_{t-i}$ be the MA(∞) representation of the markup shock. Under the optimal monetary policy, the aggregate price level follows

$$p_t = K_t + \sum_{i=0}^{\infty} \left(\frac{\rho_i}{\omega \alpha^2 + \frac{(1-\lambda)^{i+1}}{1-(1-\lambda)^{i+1}}} \right) \varepsilon_{t-i} \quad (6)$$

where K_t follows a deterministic path known at the beginning of time. The derivation is in the appendix. Therefore, the optimal policy is a flexible price level target.

Under the optimal policy, price is independent from the productivity shock a_t . It is clear from equation (3) that the output gap is independent from productivity shocks when the price is independent from productivity shocks. It is optimal to perfectly insulate the price and the output gap from productivity shocks.

The markup shock is treated differently. Because a positive markup shock raises price while depressing output, offsetting the effect either of them inevitably intensifies the effect on the other. The optimal policy is the best balance of the trade-off. That is, the monetary authority accommodates inflationary pressure to a certain degree to mitigate output slowdown.

2.5 Inattentive Period and the Welfare

The following lemma is derived when I hypothetically compare the two economies that only differs in the frequency of information updating.

Proposition 1 *Given the monetary policy is optimally conducted, the welfare improves as the economy becomes more inattentive.*

The proof is in the appendix. Increase of the inattentive length improves welfare given that the monetary policy is optimally conducted.⁴ This is because longer inattention is favorable for monetary policy in two ways. First, because it takes longer time until a shock is known by all the producers, response to the shock has real effect for longer time. Second, the monetary authority can affect output with a smaller change in price level because fewer producers observe a shock at a given point in time after its occurrence.

the average level of tax $\bar{\tau}$ satisfies $1 - \bar{\tau} = \gamma / (\gamma - 1)$, so that the expected markup is 1.

⁴Recall that the natural rate of output is defined as output level when producers are always attentive and the markup shock is at its average level. Hence, the level of the monetary authority's loss is not necessarily zero even if all the producers are attentive, i.e., $D = 0$.

3 Endogenous Inattention

3.1 Individual Producer's Planning

Suppose that producers have to pay costs of planning, κ , when they update their plans. In the continuous time version of the inattentive economy, Reis (2006) and Jinnai (2007) find an analytical solution that approximates optimal inattentiveness. In this paper, I employ a discrete time analogue of the approximate solution: a producer is being inattentive for

$$d = \sqrt{\frac{2\kappa}{\bar{\lambda} \sum_{t=0}^{\infty} (1 - \bar{\lambda})^t \left(-\frac{\pi_{pp}}{2}\right) E_0 \left[\left(p_{j,t}^* - E_0 \left[p_{j,t}^* \right] \right)^2 \right]}} \quad (7)$$

where π_{pp} is the second order derivative of the producer's period profit and $\bar{\lambda}$ is the frequency of information updating in the economy.

3.2 Endogenous Inattention

Reis shows a sufficiency condition with which the distribution of inattentiveness follows a Poisson process with parameter $1/d$. The consistency requires

$$\frac{1}{\bar{\lambda}} = \sqrt{\frac{2\kappa}{\sum_{i=0}^{\infty} \bar{\lambda} (1 - \bar{\lambda})^i \left(-\frac{\pi_{pp}}{2}\right) \left(p_{j,i}^* - E_0 \left[p_{j,i}^* \right] \right)^2}}.$$

Proposition 2 *The endogenous inattention exists and is unique under any monetary policy.*

3.3 Welfare Measure

Producers pay costs of planning, but to whom? How to model it is important for welfare measure. It may be natural to model it as labor costs to collect, absorb, and interpret information. In this case, increase of the length of inattention directly improves welfare. Such a cost is new in this framework.

But I do not take this path because my focus in this paper is the indirect effect of inattention, i.e., the welfare impact though aiding the conduct of monetary policy. So I assume that the planning costs are wealth transfer. There are several ways to theoretically formalize it: assuming

that producers consult the central bank at costs K is the simplest way. Then the derivation of the loss function (5) is valid, and I maintain it as the welfare measure in the model with endogenous inattention too.⁵

3.4 Naive Equilibrium and Sophisticated Equilibrium

I define two equilibrium concepts.

Definition 1 (D^{nai}, p_t^{nai}) is the naive equilibrium if (i) D^{nai} is the endogenous inattention given the policy p_t^{nai} , and (ii) p_t^{nai} is the best response policy given D^{nai} , i.e., p_t^{nai} is a member of stochastic process (6) given D^{nai} .

The policy p_t^{nai} is the same as the optimal policy in the economy with exogenous inattention D^{nai} . So one can interpret that the monetary authority in the naive equilibrium does not take into account the endogeneity of inattention, but takes the best policy as if D^{nai} is exogenously given. This is the reason why I call it the naive equilibrium.

Definition 2 (D^{sop}, p_t^{sop}) is the sophisticated equilibrium if (i) D^{sop} is the endogenous inattention given the policy p_t^{sop} , and (ii) p_t^{sop} is the best policy in Ω .

The monetary authority in the sophisticated equilibrium chooses the best policy among the set Ω . So the monetary authority correctly takes into account the endogeneity of inattention. This is why I call it the sophisticated equilibrium.

4 Main Results

4.1 Qualitative Results

I compare the naive equilibrium and the sophisticated equilibrium. The first result is about the length of endogenous inattention.

Proposition 3 *The endogenous inattention in the sophisticated equilibrium is longer than, or as long as, the endogenous inattention in any naive equilibrium, i.e., $D^{nai} \leq D^{sop}$.*

⁵Because both the direct and the indirect effects favor longer inattention, incorporating the direct effect in the welfare measure will reinforce the result.

Producers in the naive equilibrium are, in general, too attentive. The proof appears in the appendix, but the basic argument is a simple application of Lemma 1. Pick any naive equilibrium (D^{nai}, p_t^{nai}) , and consider an economy with inattention \hat{D} that is smaller than D^{nai} ; $\hat{D} < D^{nai}$. Lemma 1 implies that the level of loss L in the economy with \hat{D} must be bigger than the level of loss L in the naive equilibrium (remember p_t^{nai} is the best response to D^{nai}). Because L in the sophisticated equilibrium cannot be lower than L in the naive equilibrium, D^{sop} cannot be shorter than D^{nai} .

The next two results are about policies. I show in the appendix that the sophisticated policy is a flexible price level target $p_t^{sop} = \bar{P}_{t-D^{sop}} + \sum_{i=0}^{D^{sop}-1} \phi_i^{sop} \varepsilon_{t-i}$, where $\bar{P}_{t-D^{sop}}$ is a target price level that is announced before $t - D^{sop}$. Remember that a naive policy is $p_t^{nai} = \bar{P}_{t-D^{nai}} + \sum_{i=0}^{D^{nai}-1} \phi_i^{nai} \varepsilon_{t-i}$, where ϕ_i^{nai} is defined in (6) given D^{nai} . Given these policy forms, the next proposition is a corollary of proposition 1.

Corollary 1 *The sophisticated policy has to announce longer-range price level target, and responds larger number of shocks than any naive policy does.*

These differences appear because information disseminates more slowly in the sophisticated equilibrium. Because of that, the monetary authority has to announce longer-range price level target to make it known by all the producers. The monetary authority responds more shocks because policy is effective for longer periods. The next proposition shows that there is an important difference in the degree of accommodation too.

Proposition 4 *The naive policy is more accommodative than the sophisticated policy is, i.e., $0 \leq \phi_i^{sop} \leq \phi_i^{nai}$ for $0 \leq i \leq D^{nai} - 1$.*

Hence, the monetary authority in the sophisticated equilibrium responds to more shocks, but responds less accomodative to the same shock. Proposition 1 and 2 are closely related. The naive monetary authority accommodates markup shocks more than the sophisticated monetary authority does. Such a policy may succeed in short-run output stabilization, but it makes producers more attentive. The sophisticated monetary authority is less accomodative in the short-run. Such a policy may sacrifice the short-run output stability, but it makes producers less attentive. Because longer inattention favors the conduct of monetary policy, the sophisticated policy improves welfare.

4.2 Numerical Example

I present a numerical example of the naive equilibrium and the sophisticated equilibrium.⁶ The length of inattention in the naive equilibrium is 2. The length of inattention in the sophisticated equilibrium is 6.

The top panel of Figure ?? shows the degree of accommodations (ϕ_i) of the sophisticated policy (solid line) and the naive policy (dotted line). The sophisticated policy responds to a markup innovation for longer periods, but less for the first 2 periods than the naive policy does. The middle panel of Figure 1 shows output responses to a positive markup innovation. The less accommodative responses in the first 2 periods results in severer output decline, but the sophisticated policy succeeds to stabilize output in the middle-run because the policy is effective for longer periods. The bottom panel of Figure 1 shows responses of cross-sectional price variability. Because it is soft and gradual, the sophisticated policy disturbs relative prices less than the naive policy does.

In this calculation, the volatility of output gap, price level, and cross-sectional price variability are all smaller in the sophisticated equilibrium than in the naive equilibrium. With the assumption that the two policies share the same price level target, say $\bar{P}_t = 0$ for all t , the volatility of inflation is also smaller in the sophisticated equilibrium.

5 Uncontrollable Shocks

Imagine that there is a shock that the monetary authority cannot control. In the standard analysis, the optimal policy is studied by laying the shock aside because it is uncontrollable by definition. Hence, uncontrollable shock is irrelevant to the policy. But in this paper, the story is different. Monetary policy affects length of inattention, and the length of inattention applies all the shocks. By doing so, even uncontrollable shocks are relevant to the policy.

To see this point, I introduce a policy control error. Suppose that the quantity of money at time t is determined by

$$m_t = \hat{m}_t + e_t, \tag{8}$$

⁶I assume the markup shocks follow an AR(1) process with a coefficient $\rho = 0.8$. I set parameters as $(\theta, \sigma, \psi) = (10, 1, 6.7)$. I set $\xi K/\sigma_\varepsilon^2$ so that $D = 2$ is the endogenous inattention of the naive equilibrium. The solid line may not be the sophisticated equilibrium, in fact, it plots a (D, p_t) pair which gives better welfare than the naive equilibrium.

where \hat{m}_t is money supply target set by the monetary authority at period t and e_t is a policy control error. The control error is a serially uncorrelated random shock which is realized after the monetary authority sets the target. Responses of price level and output gap to the control error, which I call ϕ_e and φ_e respectively, are

$$\phi_e = \frac{\alpha}{D + \alpha}, \quad \varphi_e = \frac{D}{D + \alpha}. \quad (9)$$

If the length of inattention is exogenous, these responses are exogenous too. Hence the dichotomy holds: the existence of control error does not change the optimal policy. But if the length of inattention is endogenous, the dichotomy breaks. Even though policy cannot influence the control error directly, policy can affect these responses through the length of inattention.

How does the control error change the optimal policy? The answer depends on parameter specifications and the nature of the shocks. But unfortunately, it is difficult to measure them, especially the latter. We need a serious empirical study to quantify them, and I leave it for future research.⁷

6 Conclusion

This paper studies monetary policy in a sticky-information model where the frequency of information updating is endogenously determined. The interaction between policy and inattentiveness is a novel feature of the model. I define two equilibrium concepts; the naive equilibrium in which the monetary authority does not take into account the endogeneity of inattention, and the sophisticated equilibrium in which the monetary authority does take into account the endogeneity of inattention. Policies in the both equilibria are flexible price level targets, but they are qualitatively different. The sophisticated policy sets longer-range price level target, and is less accommodative to shocks in the short-run. Under the policy, price setters update their plans less frequently, which favors the conduct of monetary policy, and hence improves the welfare.

⁷Ireland (2004) estimated relative importance of cost-push shocks and technology shocks in a new Keynesian model. We need a similar empirical study in a sticky information model.

References

- Ball, Laurence, N. Gregory Mankiw, and Ricardo Reis**, “Monetary Policy for Inattentive Economies,” *Journal of Monetary Economics*, 2005, *52*, 703–725.
- Bernanke, Ben S.**, “The Great Moderation,” 2004. Remarks by Governor Ben S. Bernanke, At the meetings of the Eastern Economic Association, Washington, DC, February 20, 2004.
- Branch, William A., John Carlson, George W. Evans, and Bruce McGough**, “Adaptive Learning, Endogenous Inattention, and Changes in Monetary Policy,” 2007. mimeo.
- , – , – , and – , “Monetary Policy, Endogenous Inattention, and the Volatility Trade-Off,” 2007. forthcoming in *Economic Journal*.
- Ireland, Peter N.**, “A Method for Taking Models to the Data,” *Journal of Economic Dynamics and Control*, 2004, *28*, 1205–1226.
- Jinnai, Ryo**, “Optimal Inattentive Length in Macroeconomic Models,” *Economics Letters*, 2007, *94*, 174–179.
- Lucas, Robert E., Jr.**, “Econometric policy evaluation: A critique,” *Carnegie-Rochester Conference Series on Public Policy*, 1976, *1*, 19 – 46.
- Mankiw, N. Gregory and Ricardo Reis**, “Pervasive Stickiness,” *The American Economic Review*, 2006, *96*, 164–169.
- Reis, Ricardo**, “Inattentive Producers,” *Review of Economic Studies*, 2006, *73*, 793–821.
- Sims, Christopher A.**, “Implications of rational inattention,” *Journal of Monetary Economics*, 2003, *50* (3), 665 – 690.
- Stock, James H. and Mark W. Watson**, “Has the Business Cycle Changed? Evidence and Explanations,” 2003. forthcoming FRB Kansas City symposium, Jackson Hole, Wyoming, August 28-30, 2003.

Appendix

A Attentive Economy

In each period, the economy experiences one of finitely many events s_t . I denote by $s^t = (\dots, s_{-1}, s_0, s_1, \dots, s_t)$ the history of events up through and including period t . The probability, as of period 0, of any particular history s^t is $g(s^t)$. The initial realization s^0 is given.

A.1 Representative consumer

Consumer's preference is given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t g(s^t) \left[\frac{C(s^t)^{1-\sigma}}{1-\sigma} - \int \frac{L_i(s^t)^{1+\psi}}{1+\psi} di \right].$$

The consumer chooses $C(s^t)$, $L_i(s^t)$, $M(s^t)$, and $B(s^t)$. He is facing flow budget constraint and the cash-in-advance constraint,

$$P(s^t) C(s^t) + M(s^t) + \frac{B(s^t)}{R(s^t)} = \int W_i(s^t) L_i(s^t) di + M(s^{t-1}) + B(s^{t-1}) + T(s^t) + \Pi(s^t)$$

$$P(s^t) C(s^t) \leq M(s^t)$$

and a no-Ponzi game condition.

First order conditions are

$$L_i(s^t)^\psi = C(s^t)^{-\sigma} \frac{W_i(s^t)}{P(s^t)} \tag{10}$$

$$\frac{1}{R(s^t)} = \beta \sum_{s^{t+1}} g(s^{t+1}|s^t) \frac{P(s^t) C(s^{t+1})^{-\sigma}}{P(s^{t+1}) C(s^t)^{-\sigma}}. \tag{11}$$

As long as $R(s^t)$ is greater than one,

$$P(s^t) C(s^t) = M(s^t). \tag{12}$$

A.2 Intermediate good producers

Intermediate good producer i is to choose $P_i(s^t)$ to maximize

$$[(1 - \tau(s^t)) P_i(s^t) - W_i(s^t)] Y_i^d(s^t)$$

subject to the demand function for the product and the inverse labor-demand function:

$$Y_i^d(s^t) = \left[\frac{P_i(s^t)}{P(s^t)} \right]^{-\gamma} Y(s^t)$$

$$\frac{W_i(s^t)}{P(s^t)} = Y_i^d(s^t)^\psi Y(s^t)^\sigma$$

Substituting the constraints into the objective and deviding by $P(s^t)$,

$$(1 - \tau(s^t)) \left[\frac{P_i(s^t)}{P(s^t)} \right]^{1-\gamma} Y(s^t) - \left[\frac{P_i(s^t)}{P(s^t)} \right]^{(1+\psi)(-\gamma)} Y(s^t)^{1+\psi+\sigma}$$

The optimal price is

$$\frac{P_i(s^t)}{P(s^t)} = \frac{\gamma(1+\psi)}{(\gamma-1)(1-\tau(s^t))} \frac{W_i(s^t)}{P(s^t)}. \quad (13)$$

A.3 Final good producer

Final good producers behave competitively. In each period t , they choose inputs $Y_i(s^t)$, for all $i \in [0, 1]$, to maximize profits given by

$$P(s^t) Y(s^t) - \int P_i(s^t) Y_i(s^t) di$$

subject to

$$Y(s^t) = \left[\int Y_i(s^t)^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}}.$$

Solving this problem gives the input demand function

$$Y_i(s^t) = \left[\frac{P_i(s^t)}{P(s^t)} \right]^{-\gamma} Y(s^t).$$

Zero profit condition implies

$$P(s^t) = \left[\int P_i(s^t)^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}. \quad (14)$$

A.4 Monetary and fiscal authorities

Monetary authority decides the nominal money supply $M(s^t)$. Fiscal authority decides lump-sum transfer $T(s^t)$ and nominal bond supply $B(s^t)$, and is responsible for the government budget constraint

$$\frac{B(s^t)}{R(s^t)} - B(s^{t-1}) + M(s^t) - M(s^{t-1}) + \tau(s^t) P(s^t) Y(s^t) = T(s^t).$$

A.5 Competitive Equilibrium

Symmetry implies

$$L_i(s^t) = Y(s^t)$$

$$P_i(s^t) = P(s^t)$$

$$W_i(s^t) = W(s^t).$$

Market clearing implies

$$C(s^t) = Y(s^t).$$

Substituting them into (10), (11), (12), and (13),

$$Y(s^t)^\psi = Y(s^t)^{-\sigma} \frac{W(s^t)}{P(s^t)} \quad (15)$$

$$\frac{1}{R(s^t)} = \beta \sum_{s^{t+1}} g(s^{t+1}|s^t) \frac{P(s^t) Y(s^{t+1})^{-\sigma}}{P(s^{t+1}) Y(s^t)^{-\sigma}} \quad (16)$$

$$P(s^t) Y(s^t) = M(s^t) \quad (17)$$

$$1 = \frac{\gamma(1+\psi)}{(\gamma-1)(1-\tau(s^t))} \frac{W(s^t)}{P(s^t)}. \quad (18)$$

A competitive equilibrium in the attentive economy is a collection of allocation and prices $\{Y(s^t), R(s^t), P(s^t), W(s^t), M(s^t)\}$ that satisfies the following conditions: (i) consumer opti-

mality, namely (15), (16), and (17) for all s^t and (ii) intermediate good producer optimality, namely, (18) for all s^t .

A.6 Finding the equilibrium

From (15) and (18),

$$Y(s^t)^\psi = \frac{(\gamma - 1)(1 - \tau(s^t))}{\gamma(1 + \psi)} Y(s^t)^{-\sigma}. \quad (19)$$

Because the left-hand side is the marginal disutility of leisure and $Y_t^{-\sigma}$ is the marginal utility from consumption, the efficiency is achieved when

$$1 - \tau(s^t) \equiv 1 - \bar{\tau} = \frac{\gamma(1 + \psi)}{\gamma - 1}$$

which means $\bar{\tau}$ is negative, i.e., the sales subsidy. Rearranging (19),

$$y(s^t) = \frac{1}{\psi + \sigma} \log \left(\frac{(\gamma - 1)(1 - \tau(s^t))}{\gamma(1 + \psi)} \right).$$

I define the natural rate of output as the output in the attentive economy in which tax rate $\tau(s^t) = \bar{\tau}$ for all dates; i.e.,

$$y^N(s^t) = 0.$$

B Exogenous Inattention economy

The consumer, final good producers, and monetary and fiscal authorities are the same as before. A fraction λ of intermediate good producer, chosen randomly, receives complete information on the state of the economy, and makes price plan that he follows until he has a new opportunity to update information. Because expected period- t profit based on period $t - k$ information is

$$\sum_{s^t} g(s^t | s^{t-k}) \left[(1 - \tau(s^t)) \left[\frac{P_i(s^{t-k})}{P(s^t)} \right]^{1-\gamma} Y(s^t) - \left[\frac{P_i(s^{t-k})}{P(s^t)} \right]^{(1+\psi)(-\gamma)} Y(s^t)^{1+\psi+\sigma} \right].$$

Hence, the optimal period- t price for the intermediate producer who has period $t - k$ information, $P^k(s^{t-k})$, satisfies

$$\sum_{s^t} g(s^t | s^{t-k}) \left[(1 - \tau(s^t)) \left(\frac{P^k(s^{t-k})}{P(s^t)} \right)^{-\gamma} Y(s^t) - \frac{\gamma(1 + \psi)}{\gamma - 1} \left(\frac{P^k(s^{t-k})}{P(s^t)} \right)^{(1 + \psi)(-\gamma) - 1} Y(s^t)^{1 + \sigma + \psi} \right] = 0. \quad (20)$$

B.1 Competitive Equilibrium

(14) implies

$$P(s^t) = \left[\lambda \sum_{k=0}^{\infty} (1 - \lambda)^k \left(P^k(s^{t-k}) \right)^{1 - \gamma} \right]^{\frac{1}{1 - \gamma}}. \quad (21)$$

A competitive equilibrium in the exogenous inattention economy is a collection of allocation and prices $\{Y(s^t), P(s^t), \{P^k(s^{t-k})\}_{k=0,1,\dots}, M(s^t)\}$ that satisfies the following conditions: (i) consumer optimality, namely, (17) for all s^t , (ii) producers' optimality, namely, (20) and (21) for all s^{t-k} .

B.2 Log linearize

First order Taylor approximation of (20) around define $p_t^k = p_t, y_t = y_t^N = 0, \log(1 - \tau_t) = \log(1 - \bar{\tau})$ is

$$\begin{aligned} p_t^k &= E_{t-k} \left[p_t + \underbrace{\frac{\psi + \sigma}{1 + \gamma\psi} y_t}_{\alpha} + \underbrace{\frac{1}{1 + \gamma\psi} \log\left(\frac{1 - \bar{\tau}}{1 - \tau_t}\right)}_{u_t} \right] \\ &= E_{t-k} [p_t + \alpha y_t + u_t]. \end{aligned}$$

First order Taylor approximation of (21) around $p_t^0 = p_t^1 = \dots = p_t$ is

$$p_t = \lambda \sum_{k=0}^{\infty} (1 - \lambda)^k p_t^k.$$

Combining the two,

$$p_t = \lambda \sum_{k=0}^{\infty} (1 - \lambda)^k E_{t-k} [p_t + \alpha y_t + u_t]. \quad (22)$$

Taking logarithm of (17),

$$p_t + y_t = m_t. \quad (23)$$

Equations (22) and (23), given the stochastic processes u_t and m_t , characterize the competitive equilibrium.

C Formal Planning Problem

Reis (2006) and Jinnai (2007) show that in the continuous time version of the model, an individual producer chooses to be inattentive for

$$d = \sqrt{\frac{4\kappa}{\int_0^\infty \rho e^{-\rho t} \left(-\pi_{pp} E_0 \left[\left(p_{j,t}^* - E_0 p_{j,t}^* \right)^2 \right] \right) dt}}$$

where 0 is the normalized last planning time.⁸

D Optimal Policy with Exogenous Inattention

Notice that we can think of the aggregate price p_t as policy instrument because p_t is a linear function of m_t and the exogenous disturbances. It is clear from (3) that if p_t is independent from the productivity shock A_t , the output gap is also independent from A_t . Because A_t is independent from the markup shock u_t , perfectly insulating the price level and the output gap from A_t in this way clearly improves the welfare. Hence, the aggregate price level and output gap under the optimal policy are functions of markup shocks alone.

I denote $p_t := \sum_{i=0}^\infty \phi_i \varepsilon_{t-i}$ and $y_t - y_t^n := \sum_{i=0}^\infty \varphi_i \varepsilon_{t-i}$. Plugging these expressions into (3) and equating the coefficients of ε_{t-j} ,

$$\left(1 - \left(\lambda \sum_{i=0}^j (1-\lambda)^i \right) \right) \phi_j = \left(\lambda \sum_{i=0}^j (1-\lambda)^i \right) (\alpha \varphi_j + \rho_j). \quad (24)$$

⁸This approximation is accurate when the cost of information acquisition κ is close to zero, and the rate parameter ρ is very large.

These relations imply

$$\text{Var}(y_t - y_t^n) = \left\{ \frac{1}{\alpha^2} \left(\sum_{i=0}^{D-1} \left(\frac{D-i}{1+i} \phi_i - \rho_i \right)^2 + \sum_{i=D}^{\infty} \rho_i^2 \right) \right\} \sigma_\varepsilon^2, \quad (25)$$

$$E[\text{Var}_i(p_{it} - p_t)] = E \left[\frac{1}{D+1} \sum_{i=0}^D (E_{t-i}[p_t + \alpha y_t + u_t] - p_t)^2 \right] \quad (26)$$

$$= \left\{ \sum_{i=0}^{D-1} \frac{D-i}{1+i} \phi_i^2 \right\} \sigma_\varepsilon^2. \quad (27)$$

Hence the loss function of the monetary authority is

$$L = \left\{ \sum_{i=0}^{D-1} \left(\frac{1}{\alpha^2} \left(\frac{D-i}{1+i} \phi_i - \rho_i \right)^2 + \omega \frac{D-i}{1+i} \phi_i^2 \right) + \frac{1}{\alpha^2} \sum_{i=D}^{\infty} \rho_i^2 \right\} \sigma_\varepsilon^2. \quad (28)$$

The remaining task is finding the optimal coefficients ϕ_i . (28) is the loss function because p_t is a function of markup shocks. A first order condition is

$$\phi_i^* = \frac{\rho_i}{\frac{D-i}{1+i} + \omega \alpha^2} \text{ for } 0 \leq i \leq D-1. \quad (29)$$

ϕ_i for $i \geq D$ is irrelevant for the welfare. Because $\sum_{i=D}^{\infty} \phi_i \varepsilon_{t-i}$ is nothing more than a price level target \bar{P}_{t-D} , the proof is completed.

E Proof of Lemma 1

Substituting (29) in (28), we find the minimized loss in an economy where the inattentive period is D :

$$\tilde{L}(D) = \left[\sum_{i=0}^{D-1} \left(\frac{\omega}{A_i^D + \omega \alpha^2} \right) \rho_i^2 + \frac{1}{\alpha^2} \sum_{i=D}^{\infty} \rho_i^2 \right] \sigma_\varepsilon^2 \quad (30)$$

where $A_i^D = \frac{D-i}{1+i}$. Then,

$$\begin{aligned} \frac{\tilde{L}(D) - \tilde{L}(D+1)}{\sigma_\varepsilon^2} = & \\ \sum_{i=0}^{D-1} \left(\frac{\omega}{A_i^D + \omega\alpha^2} - \frac{\omega}{A_i^{D+1} + \omega\alpha^2} \right) \rho_i^2 & \\ + \left(\frac{1}{\alpha^2} - \frac{\omega}{A_D^{D+1} + \omega\alpha^2} \right) \rho_D^2. & \end{aligned}$$

Since A_i^D is monotone increasing in D ,

$$\sum_{i=0}^{D-1} \left(\frac{\omega}{A_i^D + \omega\alpha^2} - \frac{\omega}{A_i^{D+1} + \omega\alpha^2} \right) \rho_i^2 > 0. \quad (31)$$

Since $A_D^{D+1} > 0$,

$$\left(\frac{1}{\alpha^2} - \frac{\omega}{A_D^{D+1} + \omega\alpha^2} \right) \rho_D^2 \geq 0. \quad (32)$$

Henceforth,

$$\tilde{L}(D) > \tilde{L}(D+1). \quad (33)$$

F Proof of Proposition 1

Remember that $\tilde{L}(D)$ is the minimized loss in the economy with exogenous inattention D . Lemma 1 implies that $\tilde{L}(D)$ is monotone decreasing in D as depicted in Figure 1. Since the naive policy p_t^{nai} is the best response to D^{nai} , we have $L(p_t^{nai}, D^{nai}) = \tilde{L}(D^{nai})$ where $L(p_t^{nai}, D^{nai})$ is the loss in the naive equilibrium. Because the loss in the sophisticated equilibrium, $L(p_t^{sop}, D^{sop})$, is not larger than $L(p_t^{sop}, D^{sop})$, we must have $L(p_t^{nai}, D^{nai}) \geq L(p_t^{sop}, D^{sop}) \geq \tilde{L}(D^{sop})$. This means that D^{sop} cannot be shorter than D^{nai} .

G Proofs of Corollary 1 and Proposition 2

I first show that the sophisticated policy has the form $p_t^{sop} = \bar{P}_{t-D^{sop}} + \sum_{i=0}^{D^{sop}-1} \phi_i^{sop} \varepsilon_{t-i}$. Notice again if p_t does not depend on productivity shocks a_t , output gap $y_t - y_t^n$ does not depend on a_t either. Making price level and output gap independent from productivity shocks directly improves

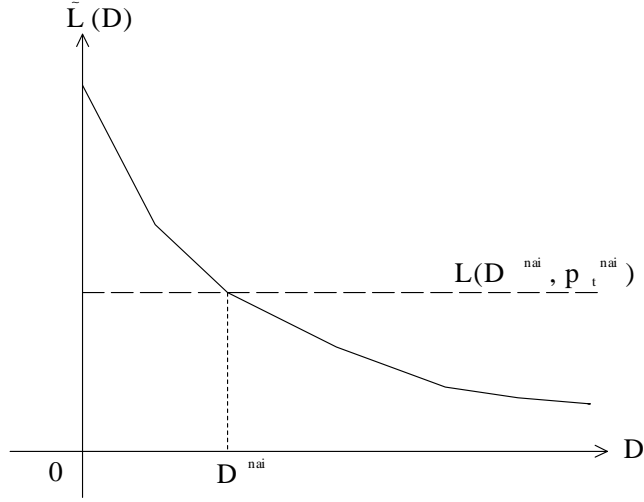


Figure 1: $\tilde{L}(D)$, the minimized loss in the economy with exogenous inattention D .

welfare, and also induces producers to choose longer inattention, which is favorable for monetary policy. Hence, the best commitment policy has the form $p_t^{sop} = \bar{P}_{t-D^{sop}} + \sum_{i=0}^{D^{sop}-1} \phi_i^{sop} \varepsilon_{t-i}$.

Next, I show that $\phi_i^{nai} \geq \phi_i^{sop} \geq 0$. Notice that the condition $|f(n)| \leq |f(n+1)|$ is equivalent to the condition $f(n) + f(n+1) \geq 0$ for any increasing function $f(\cdot)$. Hence, D^{sop} becomes endogenous inattention if

$$\left(D^{sop} + \frac{1}{2}\right) \left\{ \sum_{i=0}^{D^{sop}-1} \left(\frac{D^{sop}+1}{1+i} \phi_i\right)^2 \right\} + \frac{D^{sop}+1}{2} (\phi_{D^{sop}})^2 \geq \frac{\xi K}{\sigma_\varepsilon^2}, \quad (34)$$

$$\left(D^{sop} - \frac{1}{2}\right) \left\{ \sum_{i=0}^{D^{sop}-2} \left(\frac{D^{sop}+1}{1+i} \phi_i\right)^2 \right\} + \frac{D^{sop}}{2} \left(\frac{D^{sop}+1}{D^{sop}} \phi_{D^{sop}-1}\right)^2 \leq \frac{\xi K}{\sigma_\varepsilon^2}. \quad (35)$$

The sophisticated policy minimizes L subject to (34) and (35). Because $\phi_{D^{sop}}$ is irrelevant for welfare in the economy with inattention D^{sop} , and (34) is satisfied for sufficiently large $\phi_{D^{sop}}$, (34) does not restrict $\{\phi_i\}_{i=0}^{D^{sop}-1}$. Hence, optimal $\{\phi_i\}_{i=0}^{D^{sop}-1}$ can be found by minimizing L subject to (35). A first order condition is

$$\begin{aligned} \phi_i &= \frac{\rho_i}{\frac{D^{sop}-i}{1+i} + \omega\alpha^2 + \mu\alpha^2 \left(D^{sop} - \frac{1}{2}\right) \left(\frac{D^{sop}+1}{1+i}\right)^2 \left(\frac{1+i}{D^{sop}-i}\right)} \text{ for } 0 \leq i \leq D^{sop} - 2, \\ \phi_{D^{sop}-1} &= \frac{\rho_{D^{sop}-1}}{\frac{1}{D^{sop}} + \omega\alpha^2 + \mu\alpha^2 \frac{(D^{sop}+1)^2}{2}} \end{aligned}$$

where μ is Lagrange multiplier of (35). Because $\mu \geq 0$, and $D^{nai} \leq D^{sop}$ from Proposition 1,

$$\phi_i^{nai} = \frac{\rho_i}{\frac{D^{nai}-i}{1+i} + \omega\alpha^2} \geq \frac{\rho_i}{\frac{D^{sop}-i}{1+i} + \omega\alpha^2} \geq \phi_i^{sop}$$

for $i \leq D^{nai} - 1$.