Optimal Inattentive Length in Macroeconomic Models*

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March 2006

Abstract

This paper proposes an approximate solution of firm's optimal inattentive length in a standard macroeconomic model. The approximate solution conceptually fits a sticky-information model better, and predicts empirically more plausible inattentive length than the one proposed by Reis (2006).

Keywords: inattentiveness; sticky information; asymmetric information; approximate solution
JEL classification: E31; E39

1 Introduction

The sticky-information model proposed by Mankiw and Reis (2002) has received much attention. A micro-foundation of the model was recently given by Reis (2006); he introduced costs of planning and derived sporadic-information updating as a firm's optimal behavior. Using perturbation methods, he also found a general approximate solution of the optimal inattentive length. But when he applied his general approximate solution to a standard macroeconomic model, he assumed that firms have symmetric information. The assumption is undesirable because asymmetric information due to staggering-information updating is at the heart of the sticky-information model.

This paper presents an approximate solution of the optimal inattentive length in the standard macroeconomic model without the symmetric-information assumption. The approximate solution conceptually fits the sticky-information model better, and predicts an empirically more plausible inattentive length than the one proposed by Reis (2006).

*I would like to thank Fabio Araujo, Carlos Carvalho, Marco Gonzalez-Navarro, Ricardo Reis, and Nicola Zaniboni for their valuable comments and discussions.

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2 The Proposition

The economy consists of many identical firms indexed by j, each a monopolist setting the price of her own good, $P(j)$, facing a state of the economy composed of the level of aggregate prices, P, the level of aggregate demand, Y, and shocks to productivity, A. The profit function is $\pi(p(j)-p,y,a)$, where small letters denote the logarithms of the respective capital letters. Firms have to pay non-negative costs of planning, K, when they update their information set. Firms decide when they update their information set, or in other words, how long they are inattentive. Note that firms have different information if they update their information set at different points in time. The natural level of output, y^n, is defined as the output level if the costs of planning are zero so all the producers are attentive. In other words, the natural level of output when productivity is a is implicitly defined by $\pi_p(0,y^n(a),a)=0$. Let $p(j)^*$ denote the optimal price level with current information: $p(j)^*$ satisfies $\pi_p\left(p(j)^*-p,y,a\right)=0$. The following proposition holds in this economy.

Proposition 1 *When the costs of planning, K, are close to zero, a producer is inattentive approximately for*

$$\sqrt{\frac{4K}{-\pi_{pp}\left(\alpha^2 Var\left[y-y^n\right]+2\alpha Cov\left[p,y-y^n\right]+Var\left[p\right]\right)}} \tag{1}$$

where $\alpha=-\pi_{py}/\pi_{pp}$, or equivalently

$$\sqrt{\frac{4K}{-\pi_{pp}Var\left[p(j)^*\right]}}. \tag{2}$$

Proof. Let $G((p_0,y_0,a_0),t)$ denote the expected difference between profits earned with full information and profits earned while following a pre-chosen plan:

$$G((p_0,y_0,a_0),t)=E_0\left[\pi\left(p_t(j)^*-p_t,y_t,a_t\right)-\pi\left(\hat{p}_t(j)-p_t,y_t,a_t\right)\right] \tag{3}$$

where time 0 is the normalized last planning time, and $\hat{p}_t(j)$ is the optimal price level at time t with the information at time 0: $\hat{p}_t(j)$ satisfies $E_0\left[\pi_p\left(\hat{p}_t(j)-p_t,y_t,a_t\right)\right]=0$. A second order Taylor approximation around $(0,y^n(E_0[a_t]),E_0[a_t])$ gives

$$\begin{aligned}&\pi\left(p(j)^*-p,y,a\right)-\pi\left(\hat{p}(j)-p,y,a\right)\\ =\;&\frac{1}{2}\pi_{pp}\left[\left(p(j)^*-p\right)^2-\left(\hat{p}(j)-p\right)^2\right]\\ &+\left(p(j)^*-\hat{p}(j)\right)\left(\pi_{py}\left(y-y^n\left(E[a]\right)\right)+\pi_{pa}\left(a-E[a]\right)\right).\end{aligned} \tag{4}$$

For brevity, I suppress time subscripts, and suppress evaluation points as long as functions are evaluated at $(0,y^n(E_0[a_t]),E_0[a_t])$. A log-linear approximation of $\pi_p(0,y^n(a),a)=0$ gives

$$\pi_{py}\left(y^n(a)-y^n\left(E[a]\right)\right)=-\pi_{pa}\left(a-E[a]\right). \tag{5}$$

Using (5),

$$\begin{aligned}
& \pi\left(p^*(j)-p, y, a\right)-\pi\left(\hat{p}(j)-p, y, a\right) \\
= & \frac{1}{2}\pi_{pp}\left[\left(p^*(j)-p\right)^2-\left(\hat{p}(j)-p\right)^2\right]+\left(p^*(j)-\hat{p}(j)\right)\pi_{py}x \qquad (6)
\end{aligned}$$

where $x = y - y^n(a)$. A log-linear approximation to $\pi_p\left(p^*(j)-p, y, a\right)=0$ together with (5) gives $p^*(j) = p + \alpha x$ where $\alpha = -\pi_{py}/\pi_{pp}$. A log-linear approximation to $E\left[\pi_p\left(\hat{p}(j)-p, y, a\right)\right]=0$ together with (5) gives $\hat{p}(j) = E[p+\alpha x]$. Using these relations,

$$\begin{aligned}
& \frac{1}{2}\pi_{pp}\left[\left(p^*(j)-p\right)^2-\left(\hat{p}(j)-p\right)^2\right]+\pi_{py}\left(p^*(j)-\hat{p}(j)\right)x \\
= & \frac{1}{2}\pi_{pp}\left[\alpha^2\left(x^2-\left(E[x]\right)^2\right)-\left(p-E[p]\right)^2+2\alpha E[x]\left(p-E[p]\right)\right] \qquad (7) \\
& +\pi_{py}\left[\alpha x\left(x-E[x]\right)+x\left(p-E[p]\right)\right].
\end{aligned}$$

Taking expectations and rearranging give

$$G\left(\left(p_0, y_0, a_0\right), t\right) = -\frac{\pi_{pp}}{2}\left(\alpha^2 Var(x) + Var(p) + 2\alpha Cov(p, x)\right) \qquad (8)$$

or equivalently,

$$G\left(\left(p_0, y_0, a_0\right), t\right) = -\frac{\pi_{pp}}{2} Var(p^*(j)). \qquad (9)$$

Applying proposition 4, the general approximate solution, in Reis (2006) completes the proof. ■

3 Qualitative Comparison

Reis (2006) finds that the optimal inattentive length for a firm in the above economy is approximated by

$$\sqrt{\frac{4K}{-\pi_{pp}\alpha^2 Var\left[y-y^n\right]}}. \qquad (10)$$

In (10), only the variance of the output gap, $Var\left[y-y^n\right]$, matters. In (1), however, not only the variance of the output gap, $Var\left[y-y^n\right]$, but also the variance of the aggregate price level, $Var[p]$, and the covariance of them, $Cov\left[p, y-y^n\right]$, matter, because all three terms affect the variance of their full-information-optimal price $Var\left[p(j)^*\right]$.

The difference comes from the definition of $G\left(\left(p_0, y_0, a_0\right), t\right)$ function. Recall that $G(\cdot)$ is the expected difference between profits earned with full information and profits earned while following a pre-chosen plan. Reis defines it as

$$G\left(\left(p_0, y_0, a_0\right), t\right) = E_0\left[\pi\left(p_t^*(j)-p_t, y_t, a_t\right)-\pi\left(0, y_t, a_t\right)\right] \qquad (11)$$

by assuming all the firms are attentive at time 0. That is, Reis approximates around the point where all the firms are attentive. But if all the firms are attentive at time 0, they have no uncertainty about future path of the aggregate price level: every firm knows the future path because it is the same as her own price plan. This is the reason for the absence of the aggregate price level p in (10).

But solving the individual firm's problem under symmetric information among firms is undesirable because the objective of Reis (2006) is giving a micro-foundation to the sticky-information model à la Mankiw and Reis (2002). Asymmetric information due to staggering-information updating is the heart of the model. My definition of $G(\cdot)$ embraces asymmetric information among firms, and hence, the approximate solution proposed in this paper conceptually fits the sticky-information model better than the one proposed by Reis.

4 Optimal Expected Length of Inattentiveness: Quantitative Comparison

Reis (2006) assesses predictions of the approximation (10) with reasonable parameter values. I do the same exercise with the approximation (1) to analyze quantitative differences.

The model consists of many identical firms (a continuum) indexed by j. Each produces a differentiated good facing a constant price elasticity demand function: $Y_t(j) = Y_t(P_t(j)/P_t)^{-\theta}$. They all operate a linear production technology $Y_t(j) = A_t L_t(j)$, that uses $L_t(j)$ units of labor to produce $Y_t(j)$ units of output subject to exogenous stochastic labor productivity A_t. They hire labor in the market paying a real wage $W_t(j)/P_t$. The inverse labor supply function is $\omega(\log(L_t(j)), \log(Y_t))$. It increases with the amount of labor supplied, with an elasticity ψ, and increases with aggregate income, with an elasticity of σ, through a standard income effect that makes agents prefer more leisure in good times. The cost of planning is a fraction κ_j of the profit $\pi(0, y^n(E[a]), E[a])$. The cost of planning is stochastic, and $\sqrt{\kappa}$ is the expectation of $\sqrt{\kappa_j}$. The approximate solution (1) in this economy becomes

$$\sqrt{\frac{4\kappa}{\theta(\theta-1)(\psi+1)(\alpha^2 Var[y_t - y_t^n] + 2\alpha Cov[p_t, y_t - y_t^n] + Var[p_t])}}, \quad (12)$$

and the approximate solution (10) in this economy becomes

$$\sqrt{\frac{4\kappa}{\theta(\theta-1)(\psi+1)(\alpha^2 Var[y_t - y_t^n])}} \quad (13)$$

where $\alpha = (\sigma+\psi)/(1+\theta\psi)$. Several parameter sets of (θ, σ, ψ) are considered: (10, 1, 6.7) as a baseline, (10, 1, 1.25) used by Chari et al. (2000), (7.88, 0.16, 0.47) used by Rotemberg and Woodford (1997), and (7.8, 0, 6.7) used by Ball and Romer (1990). y_t^n is measured by log output per hour, and y_t is measured by quarterly real GNP. Several possibilities of the costs of planning are also considered. I use the quarterly GDP deflator as a measure of the price level. Following Reis (2006), I use an Hodrick-Prescott filter to isolate the cycle in the output gap and the price level.

Table 1 reports the optimal expected length of inattentiveness predicted by (12), the approximate solution proposed in this paper. Numbers in the parenthesis are those predicted by (13), the approximate solution proposed by Reis (2006). (12) consistently predicts shorter inattentive length than (13). This is because Reis considers less uncertain circumstances than I in the sense that firms are certain of the future path of the aggregate price level. When the costs of planning are 4.6% of the profit, which is the estimate of Zbaracki et al. (2004), (12) predicts 2.1 to 6.3 quarters of inattentiveness while (13) predicts 10 to 26 quarters. Carroll (2003) and Mankiw, Reis, and Wolfers (2004) use data on inflation expectations to infer the speed at which information disseminates in the economy, and both estimate an average inattentiveness of about one year. Hence, (12) predicts empirically plausible lengths of inattentiveness while (13) predicts much longer lengths of inattentiveness. For the four different parameter combinations of (θ, σ, ψ), costs of planning of 16.2%, 4.6%, 1.8%, and 9.5% of profits respectively, would generate 4 quarters of inattentiveness by (12), while costs of planning of 0.7%, 0.4%, 0.1%, and 0.5% of profits would generate the same length using (13). Hence, the costs required by (12) to generate 4 quarters of inattentiveness are consistent with the costs estimated by Zbaracki et al. (2004), while those required by (13) are consistently lower than their estimates.[1]

5 Conclusion

Approximation and perturbation methods are powerful analytical tools that are increasingly used in economics. However, calculations are sensitive to the point around which the approximation is taken. This paper presents a striking example in which a seemingly small difference in approximation points causes a large difference in obtained results. Reis (2006) approximated optimal inattentive length around a point where all the firms are attentive. I approximated around the point where firms have asymmetric information. It is not surprising that the two solutions are qualitatively different; it is interesting that they are different quantitatively. The solution proposed in this paper conceptually fits the sticky-information model better, in which firms update information at different points in time, and gives empirically more plausible predictions of inattentive length than the one proposed by Reis (2006).

[1] It is noteworthy that 4.6% of profits is still a conservative use of estimates of Zbaracki et al. (2004). They estimate that the total costs of price adjustment are 20% of profits, and managerial costs, which are a part of the total costs, are 4.6% of the profits.

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Optimal expected length of inattentiveness in quarters

		Parameter Combinations (θ, σ, ψ)			
		Baseline	Chari.et al	Rotemberg-Woodford	Ball-Romer
		$(10, 1, 6.7)$	$(10, 1,\ 1.25)$	$(7.88,\ 0.16,\ 0.47)$	$(7.8,\ 0,\ 6.7)$
Costs	0.046	2.1 (10)	4.0 (13)	6.3 (26)	2.8 (12)
of	0.028	1.7 (8)	3.1 (10)	4.9 (20)	2.2 (9)
planning	0.010	1.0 (5)	1.9 (6)	3.0 (12)	1.3 (6)
(κ)	0.001	0.3 (2)	0.6 (2)	0.9 (4)	0.4 (2)

Table 1: This table presents the optimal expected length of inattentiveness in quarters predicted by the approximation proposed in this paper. Numbers in the parenthesis are predicted inattentiveness by the approximation proposed by Reis (2006).