Spring 2009 ECON 410 Macroeconomic Theory
Homework 3 Solutions:

(A) Multiple Choice Questions:

1. A
2. C
3. C
4. D
5. A
6. B
7. B
8. B
9. D
10. B
11. A
12. C
13. B
14. D
15. A
16. B
17. C
18. C
19. A
20. B
21. B
22. C
23. B
24. C
25. D

(B) Essay Questions

1. a) Since at the steady state:

$$sy = (n+\delta)k \rightarrow$$
$$sk^{1/2} = (n+\delta)k \rightarrow$$
$$k^{1/2} = s/(n+\delta) \rightarrow$$
$$y = s/(n+\delta) y = s/(\delta+n)$$

Therefore, the ratio of the per capita output:

$$\frac{y_{US}}{y_{Mexico}} = \frac{\frac{s_{US}}{n_{US}+\delta}}{\frac{s_{Mexico}}{n_{Mexico}+\delta}} = \frac{\frac{0.22}{0.01+0.04}}{\frac{0.16}{0.025+0.04}} = 1.79$$

b) For Mexico, $y=k^{\beta}$.

$sk^{\beta} = (\delta+n)k$. So $k^{1-\beta} = s/(\delta+n)$.

$y = [s/(\delta+n)]^{\beta/(1-\beta)}$.

$y_{us}/y_{mexico} = 4.4/2.46^{\beta/(1-\beta)} = 4$. Solve for β, we get $\beta = 0.096$.

c) Now consider the Solow model with the technology growth. From question a), we have:

$y = s/(\delta+n+g)$.

Here $y = Y/(E*L)$, and $k = K/(E*L)$. The ratio between per effective worker output is given by:

$$\frac{\frac{Y_{US}}{E_{US}L_{US}}}{\frac{Y_{Mexico}}{E_{Mexico}L_{Mexico}}} = \frac{\frac{s_{US}}{n_{US}+\delta+g_{US}}}{\frac{s_{Mexico}}{n_{Mexico}+\delta+g_{Mexico}}} = \frac{\frac{0.22}{0.01+0.04+0.04}}{\frac{0.16}{0.025+0.04+0.01}} = 1.146$$

Rewrite the previous equation,

$$\frac{\frac{Y_{US}}{E_{US}L_{US}}}{\frac{Y_{Mexico}}{E_{Mexico}L_{Mexico}}} = 1.146 = \frac{\frac{Y_{US}}{L_{US}}}{\frac{Y_{Mexico}}{L_{Mexico}}} \cdot \frac{E_{Mexico}}{E_{US}} = 4 \cdot \frac{E_{Mexico}}{E_{US}}$$

$$\rightarrow \frac{E_{Mexico}}{E_{US}} = \frac{1.146}{4} = 0.287$$

Therefore, Mexico's efficiency is only 28.7% of the US.

2. Mankiw textbook, 242, #2
   i. Production function is $y = k^{0.3}$.
   ii. In the steady state, we know that the growth rate of output is 3 percent, so we know $(n+g) = 0.03$
   iii. Depreciation rate $\delta = .04$.
   iv. The capital-output ratio $K/Y = 2.5$. Because $[K/(EL)]/[Y/(EL)] = k/y = K/Y = 2.5$.

a) Begin with the steady-state condition,

$s^*y = (\delta+n+g)k$ ➔ $s = (0.04+0.03)/2.5 = 0.175$.

b) Because $\alpha = MPK^*(K/Y)$, rewrite the equation then we have:

$MPK = \alpha /(K/Y) = 0.3/ 2.5 = 0.12$.

c) The Golden Rule steady state $MPK = (\delta+n+g) = 0.04+0.03 = 0.07$.

At the Golden rule steady state, *MPK* is 7%, where it is 12% in the initial steady-state. Hence, from the initial steady-state we need to increase k to achieve the Golden Rule steady-state.

d) At the Golden Rule, $MPK = \alpha/ (K/Y)$. Therefore,

$$K/Y = \alpha / MPK = .3/.07 = 4.39.$$

This is higher than the current output-labor ratio of 2.5.

e. From part (a), $s = (\delta+n+g)^*k/y = (0.04+0.03) * 4.29 = 0.30$

To reach the Golden Rule steady-state, the saving rate must rise from 17.5% to 30%.