A simple review of statistical test

Li Gan

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A test is a statement. For example, consider a simple t-test of $\beta = 0$. Given H_0: $\beta = 0$, and H_0: $\beta \neq 0$.

Statement 1: accept H_0 if and only if $\left|\frac{\hat{\beta}}{stdc(\hat{\beta})}\right| \leq 1.95$.

However, since the estimated value of $\hat{\beta}$ has random errors, the statement will not right or wrong. Consider the following table:

		Truth: H_0	Truth: H_1
Action	Accept H_0	No Error	Type II Error
Action	Accept H_1	Type I Error	No Error

Or, Type I error is equivalent to: *Accept* $H_1|H_0$ *is true*; and Type II error is equivalent to *Accept* $H_0|H$ *is true*.

In terms of Statement 1, Pr(Type I error) is given by:

$$\Pr\left(Accept\ H_1 \mid H_0\ is\ true(\beta = 0)\right)$$
$$= \Pr\left(\left|\frac{\hat{\beta}}{sd(\hat{\beta})}\right| > 1.95 \middle| H_0\ is\ true(\beta = 0)\right) = 0.05 .$$

since when β=0, then $\frac{\hat{\beta}}{sd(\hat{\beta})}$ has a Student-t distribution.

Two important points may be mentioned here:

(1) It is possible to have either Pr(Type I error) = 0 or Pr(Type II error) = 0, but not both. In fact:

Pr(Type I error) = 0 ➔Pr(Type II error) = 1
Pr(Type II error = 0) ➔Pr(Type I error) = 1

Statement 2: always accept H_0 regardless of the truth.

This statement implies that there will be no Type I error. However, it also implies that Pr(Type II error) = 1.

Statement 3: always accept H_1 regardless of the truth.

This statement implies that there will be no Type II error. However, it also implies that Pr(Type I error) = 1.

In fact, it is generally true that a lower type I error would imply a higher Type II error, and a vice versa. Therefore, a test is actually a compromise between Type I error and Type II error.

It is the default or tradition that we let:

Pr(Type I error) = 0.01, 0.05, or 0.10.

There are often many different tests that can achieve Pr(Type I error) = 0.01, 0.05, or 0.10. The goal is to find a test that may have the minimum Type II error, or equivalently, the maximum power at a given size.

(2) How to determine H_0 and H_1:

It is often NOT arbitrary to assign H_0 and H_1. The most important factor to determine which hypothesis is H_0 is the distribution of the test statistic under H_0. Such distribution of the test statistic should be generic and standard, and not parameter-dependent. The most often used distributions include:

(a) standard normal distribution.
(b) Student-t distribution.
(c) F-distribution
(d) $\chi^2(k)$ distribution.

For example, consider a regression:

$$y = x_1\beta_1 + x_2\beta_2 + u = x\beta + u$$

If we are interested in testing: H_0: $R\beta = 0$ where R is a matrix of constants.

$$Var\left(R\hat{\beta}\right) = R\,Var\left(\hat{\beta}\right)R'$$

Therefore, asymptotically, and under H_0: $R\hat{\beta} \sim N\left(0, R\,Var\left(\hat{\beta}\right)R'\right)$. It is typical to transform a *k*-dimension multivariate normal distribution to a χ^2(k) distribution:

The test statistic is given by: $\left(R\hat{\beta}\right)'\left[R\,Var\left(\hat{\beta}\right)R'\right]^{-1}\left(R\hat{\beta}\right) \sim \chi_k^2$

Based on this test statistic, we can construct a test. An example is:

Statement 4: Reject H_0 if the test statistic $\left(R\hat{\beta}\right)'\left[R\,Var\left(\hat{\beta}\right)R'\right]^{-1}\left(R\hat{\beta}\right) > critical\ value$. The critical value is obtained using the distribution of χ^2(k).