

# The Relative Profitability of the Institution of American Slavery: 1830-1860

Chad Dacus<sup>1</sup>  
*Economics Department*  
*Rice University*

Robin Sickles  
*Economics Department*  
*Rice University*

January 2, 2006

## **Abstract**

The profitability of owning slaves has been a popular area of research in economic history. Existing analyses have focused on estimating the rate of return on slaves and presenting rates of return on other available antebellum investments. Modern stock-portfolio performance measures allow us to rank the owning on slaves with other investments of the period over an extended period of time. In addition, these performance measures provide us with the opportunity to consider both risk and return in conducting this evaluation instead of relying on summary statistics concerning rates of return. Utilizing the distance from the efficient frontier as our metric of performance, we conclude that owning slaves outperformed the other investments of the period. Since the directional distance methodology is deterministic in nature, we perform a probabilistic analysis using bootstrapping to form confidence bounds on our estimates. The dominance of slave owning is confirmed from inspection of the resulting bounds. Our results verify that the institution of slavery was a very financially rewarding enterprise that demanded a military resolution rather than relying on its ultimate economic extinction.

---

<sup>1</sup> The authors would like to thank Don Freeman for numerous helpful comments.

## 1. Introductory Comments

Although many authors have attempted to measure the rate of return of slavery in the antebellum South, a systematic ranking of investments in this period has not been undertaken. Slavery is best viewed as an institution rather than an industry, and this fact lessens the impact of any financial study. That is, some in the South were trying to save slavery more for political (states' rights, for instance) reasons than for financial reasons. Evidence of this can be seen in the vitriolic struggles over tariff policy as well as slavery. In light of this consideration (and others, see Evans, 1962), this ranking may not completely answer the question of whether slavery was going to survive in the absence of the Civil War, but it should help answer the question of whether the plantation economy was underperforming compared to other investments in both the South and North. If slavery was indeed underperforming, even the stoutest of political considerations would not have saved it for perpetuity. In this paper, we intend to investigate whether owning slaves was an underperforming investment through modern portfolio evaluation measures.

The outline of this paper is as follows. In section 2, we discuss the availability of data and the general methodology we will utilize. Conventional methods of measuring risk-adjusted rate of return (RAROR) are presented in section 3. In section 4, the general framework of data envelopment analysis is reviewed. We critique previous estimates of rate of return on slaves and present the modifications we will employ for our own estimates in section 5. In section 6, we present results from a calculation of the Sharpe ratio for each investment. In section 7 we develop the directional distance function, explain our estimation technique, and present the results of statistical tests based on the

estimation. Our bootstrapping methodology is explained in section 8 along with the results, and section 9 concludes.

## 2. Discussion of the Comparison Set and Methods of Analysis

We must first determine how we will rank the investments of the period. The primary consideration in answering this question is the availability of data for the various types of investments. Unfortunately, we are not able to compare the return on slaves to other physical assets due to the unavailability of earnings records for closely-held firms (Martin, 1898). However, it is possible to obtain rates of return on a variety of financial assets such as sixty- and ninety-day bills. In addition, we can obtain records concerning the rates of return of commercial money issued during this period. It seems clear that our comparison universe will, out of necessity, be limited to financial assets.

The question becomes how to compare physical assets and financial assets in risk-return terms. An obvious choice would be to utilize measures of the performance of financial assets, except that relative liquidity of the physical asset versus the financial assets can become a major concern. Stated differently, the physical asset may be much more difficult to resell, so the investor may feel much more 'locked in' with regard to an investment in a physical asset. In that case, any meaningful comparison of risk is elusive. This is not much of a concern in the case of slaves. Since the importation of slaves was banned in 1808, an active secondary market is a given. In fact, Kotlikoff (1979) states that more than 135,000 slaves were sold between 1804 and 1862 in New Orleans. Furthermore, it is a certainty that financial markets were not as liquid in the 1800s as they are today. Gross discrepancies in the liquidities of the assets under discussion are unlikely, but some discrepancy here is unfortunately certain. Markets were simply not as

efficient almost 150 years ago as they are today. However, the inefficiency of a market does not render modern financial ratios uninformative because the purpose of the analysis is comparison of investments rather than arriving at an informative measure in absolute terms. It is far from a settled fact that today's markets are efficient, and that does not stop the prevalent use of modern financial ratios based on benchmarks. Since we have concluded that we should treat all of the investments in the comparison set as financial instruments, we must determine whether to utilize measures based on fixed-income investments or stock-based investments.

Since the majority of the instruments in our comparison universe are fixed-income instruments, this would seem to be an appealing choice. However, slaves and their output definitely were not fixed-income physical assets. Although this mode of analysis would be helpful in analyzing the comparison universe, the entire purpose of this paper is to evaluate investment in slaves versus other representative investments of the day. Clearly, fixed-income analysis is implausible.

The only plausible manner in which to analyze these diverse assets is through the use of modern stock-portfolio measures. Risk-adjusted rate of return (RAROR) is the dominant methodology for analysis of a stock's performance. Although this approach is not without its issues when utilized for such diverse assets, this is the only realistic option for what we would like to accomplish. Applying the RAROR approach, we are able to evaluate both the rate of return to slavery and its attendant risks - placing proper weight on each. In addition, we will be able to construct a ranking of some of the investments available during the period compared to investment in slaves. Therefore, we should now

turn our attention to the various measures of RAROR and which of these measures might be appropriate for the scant data we have at hand.

### 3. Conventional Measures of RAROR and Discussion of Their Relevance for the 1800s

The most commonly utilized measure of risk-adjusted return is the Sharpe ratio (Sharpe, 1966). The Sharpe ratio has the advantage of being exceptionally easy to calculate and interpret:

$$\text{Sharpe ratio} = \frac{(\bar{r}_i - r_f)}{\sigma_i} = \frac{\text{average excess return of asset}}{\text{std. deviation of asset's excess return}}$$

The only two informational requirements here are historical returns of the asset, the  $r_i$ , and the risk-free rate of return,  $r_f$ . As one might imagine, we already run into trouble using the ratio in the 1800s because of the lack of a remotely risk-free security. Modern-day Treasury bills did not exist during this time period. As Officer (2005) reveals, federal funds series are available beginning in 1855. Data is available on ten-year Treasury bonds for antebellum years except for the period of 1835-1842. A good substitute rate can be found for the missing years, but the rate on these bonds cannot be considered completely risk free due to interest-rate risk alone. In order to utilize Sharpe's ratio, we can adopt the rate of return on ten-year Treasury bonds as our risk-free rate with a slight abuse of the term. Although Sharpe's ratio is problematic, we will see that it is by far the most reasonably adapted ratio to the finance of the 1830s through 1860s. We will compare the Sharpe ratio for investment in slaves to other investments of the period in section 6.

As has been mentioned often in the literature (for example, Simons, 1998), the Sharpe ratio lacks an intuitive interpretation for the average investor. Modigliani and Modigliani (1997) propose a measure that is closely related to the Sharpe Index but is

measured as an excess return rather than excess return over standard deviation. In equation format, it is:

$$\text{Modigliani measure} = \frac{(\bar{r}_i - r_f)\sigma_{Ind}}{\sigma_i} = (\text{Sharpe ratio}) * (\text{std. dev. of index excess return}).$$

As pointed out by Simons (1998), "the Modigliani measure is equivalent to the return the fund would have achieved *if it had the same risk as the market index* (emphasis in original). This index should be a broadly-based index such as today's S&P 500. Even then, the S&P 500 is but a *proxy* for the market portfolio. This is a problem with almost all of the popular measures of RAROR: a broad-based index is needed. We do not have enough information to construct such an index for the period under study.

Almost all of the popular measures of RAROR are based on the capital asset pricing model (CAPM) which is given by

$$r_i = r_f + \beta_i(r_M - r_f) + e_i + \alpha_i$$

where  $e_i$  is a firm-specific disturbance with a mean of zero and  $\alpha_i$  is the expected abnormal return (Sharpe, 1964 and Lintner, 1965). The coefficient  $\beta_i$  can be thought of as the non-systematic risk of the security. That is,  $\beta_i$  can be thought of as the risk that is remaining after diversification or the risk specific to the security itself as part of a diversified portfolio. This is not a plausible framework for studying the economy of the 19th century. Table 1 lists some of the more popular remaining RAROR measures that are based, in some way, on the CAPM. Their dependence on the CAPM will make it impossible to utilize them as meaningful measures of the risk-return relationship in the 1800s.

Table 1. Conventional RAROR based on the CAPM

MEASURE	FORMULA
Jensen's alpha	$\alpha_i = (r_i - r_f) - \beta_i(r_M - r_f)$
Treynor's ratio	$T_i = (r_i - r_f)/\beta_i$
Appraisal Ratio	$A_i = \alpha_i/\sigma(e_i)$

As can be seen from its formula, Jensen's alpha (Jensen, 1968) is simply a restatement of the CAPM formula with the zero-mean quantity removed. This measure brings with it all of the restrictive assumptions of the CAPM. The Treynor ratio (Treynor, 1966) is a restatement of the Sharpe ratio with a non-systematic risk term rather than the standard deviation of the firm's excess return. Meanwhile, the appraisal ratio (Treynor and Black, 1973) is simply Jensen's alpha divided by standard deviation of the firm-specific disturbance. In other words, the appraisal ratio can be thought of as Sharpe's Index under the CAPM, and Jensen's alpha is somewhat analogous to Modigliani's measure in that it is more easily interpreted. All of these suffer from the benchmark error (Roll, 1978) and are not plausible for the present study. A reasonable market portfolio cannot be constructed with available data, much less held by an investor. For the purposes of our study, it appears as if the Sharpe ratio is the only plausible measure of investment performance from the set of conventional RAROR measures since an appropriate benchmark is not available.

Another option would be to implement selection procedures based on the entire return distribution. Sengupta (1991) and Sengupta and Park (1993) suggest such a methodology. These approaches are based on stochastic dominance and are very

appealing if one has enough data to perform rigorous tests. Our dataset will consist of, at most, 31 observations for each investment. This is not likely to result in a test of sufficient power to be useful. Moreover, without such a test, these techniques are not as useful because they do not yield results that can be easily explained to those not technically trained in finance. We will utilize directional distance techniques which avoid the need for a benchmark portfolio and any distributional assumptions.

#### 4. Data Envelopment Analysis Portfolio Efficiency Index (DPEI)

The most problematic feature of the conventional RAROR models is the need for a benchmark for the market portfolio. As previously discussed, this is not realistic for the period under study. Murthi et. al. (1997) dispense with the need for a benchmark portfolio by developing a comparison procedure based on data envelopment analysis (DEA). In addition, transaction costs are included in this analysis. DPEI can be specified as

$$\max DPEI = \frac{r_0}{\sum_{i=1}^m w_i x_{ij} + v\sigma_0}$$

$$\text{s.t. } \frac{r_j}{\sum_{i=1}^m w_i x_{ij} + v\sigma_j} \leq 1 \quad j = 1, \dots, n.$$

In the numerator, we have the excess return of the security. In the denominator, measures of both risk and transaction costs are included.

DPEI is non-parametric (the other measures discussed so far require normality for statistical testing) and incorporates transaction costs. This is very advantageous for an actively-managed portfolio. The costs of managing the funds can easily be weighed against the benefits of this management.

Despite DPEI's important advantages over the conventional measures of RAROR, its data requirements are cumbersome for this project. Transaction costs are not measurable on the slave transactions, which have been taken from a secondary source. In addition, the comparison investments' transaction costs have not been explicitly measured. Our methodology in calculating directional distance will be similar to the algorithm utilized in determining DPEI.

### 5. Examination and Discussion of Available Data

Some of the more influential early works were Phillips (1929), Gray (1958), and Ramsdell (1929). These works were completed before the advent of modern econometric methods and rely primarily on anecdotal evidence. These studies are useful primarily due to their summarization of data from primary sources. In addition, their theories serve as a starting point for the works to be described in this section.

The work of Conrad and Meyer (1962) utilized basic econometric and financial tools to arrive at their estimates of the rate of return on the investment in slaves. Their methodology is intuitively appealing because they start from the first principle of a production function. We begin with the presumption that their methodology should be employed, and we will abandon that assumption only if it can be shown that the problems inherent in the analysis cannot be overcome with the available data.

The first problem with Conrad and Meyer's work is that they assume that all slaves realized the median life expectancy. As revealed by Evans (1962), this causes their return estimates to be too high due to the high number of slave children who did not live to maturity. An obvious solution here is to more carefully consider the demographic data for death rates before maturity. While exact mortality results are not available, this

shortcoming can be largely alleviated. Conrad and Meyer also assume that slaves are sold at precisely the age of eighteen. The authors implicitly assume the existence of 'breeding states' by relying solely on demographic data to reach this conclusion. As Fogel and Engerman (1974a) point out, differing demographics for the Old South in comparison to the New South do not make for a convincing argument of systematic intervention in the reproduction of slaves; it would be much better to produce actual bills of sale indicating movement of slaves from plantations in the Old South to those in the New South. Since such records do not exist in the quantity needed, it remains unclear whether such intervention was a major factor in Old South profits.

A more troublesome aspect of Conrad and Meyer's research is their estimation of land costs. One would expect much different returns from poor land in Alabama compared to prime cleared land in Mississippi. This leads to an estimate of land costs of between \$90 and \$1,400 per hand. Such a wide range of possibilities is unacceptable for econometric purposes. The authors state that the vast majority of costs were bracketed by the costs of \$180 and \$600, but, in the end they state that 'a typical case' involved a land investment of \$450 per hand without justifying their estimate through econometric analysis. The great variation in land costs leads one to suggest the alternative of sampling representative plantations to determine rates of return. This reveals the primary difficulty in Conrad and Meyer's approach: such records do not exist for enough individual plantations to allow for a systematic study. In addition, as detailed in Fogel and Engerman (1974), Conrad and Meyer also had difficulties in estimating the costs of capital such as plows, gins, and wagons.

Productivity estimates for slaves is another major concern. Fogel and Engerman (1974) assert that Conrad and Meyer's productivity estimates are too low. It is difficult to see that this criticism is warranted because Conrad and Meyer estimate the rate of return throughout a wide range of productivities.<sup>2</sup> In any case, the rates of return could be easily adjusted with an accurate estimation of productivity. It is not clear which estimate of productivity should be chosen; this is another concern that would be alleviated with an investigation of individual plantations instead of attempting to generalize to a level that is not justified by the available data.

Although Conrad and Meyer's methodology is intuitively appealing, we find too many difficulties in estimation due to the dearth of quality data. The analysis should be undertaken with a focus on individual plantations, but this is not possible with existing information. A different approach will be necessary.

Evans (1962) approaches the issue of the profitability of slavery through an entirely different mode of analysis. He utilizes the rates of hire for slaves as the revenue-producing activity of slaves and thus avoids many of the issues Conrad and Meyer faced. However, Evans' analysis is not without its own limitations. For example, Butlin (1971) pointed out a couple of important concerns. First, it is unclear whether the hires Evans found were representative of the time period. Second, Butlin indicates that it is entirely possible that the jobs slaves were required to do as hires were more dangerous than their chores on the plantation. Butlin does not clarify his objection with regard to the first objection other than to suggest that the sample is too small. While this objection should be noted, a source of systematic bias in Evans' data has not been identified. As to the

---

<sup>2</sup> Productivities ranged from 3.75 bales per hand to 7 bales per hand. However, Conrad and Meyer adjusted the cost of capital according to productivity. This is likely the source of Fogel and Engerman's discontent.

second critique, it has not been proven that hired slaves died in any greater frequency than plantation slaves other than an observation that slaves residing in cities experienced a higher mortality rate than those living in rural locations. More importantly, liability for the death would have to be established and financial responsibility assessed. It is mere speculation to state that risk (for the slaveowner) was increased by the hiring of slaves; no data has been found to support this proposition.

The remaining concerns with Evans' methodology can be easily taken into account. Fogel and Engerman (1974a) found that brokers' fees had not been subtracted from the hiring fee. Since brokers' fees were standard at 7.5%, this is not a catastrophic issue. In addition, Butlin (1971) suggested that it is unreasonable to assume that slaves would be immediately hired by a willing lessor. This is a valid concern, and a waiting time should serve to reduce the average rate of return.

A final concern with Evans' work is the exclusion of the demographic information of slaves during the period. That is, Evans considers the rate of return on a prime field hand which does not represent, in any meaningful sense, the rate of return the owner of a plantation of slaves might realize. One must include slaves of extreme young and old age in determining the average rate of return.<sup>3</sup> Since such demographic information is readily available, this is not a substantive criticism for our purposes and will necessitate a simple recalculation of Evans' return estimates.

In summary, the work of Evans, with appropriate modification for expenses and demographic information, provides the most compelling estimates of rates of return

---

<sup>3</sup> We include this demographic information in our analyses and employ the cost estimates of slaves of both young and old age from Conrad and Meyer (1962).

available. We will correct these deficiencies and utilize the data Evans employed in reaching his conclusions.

#### 6. Evaluation of Investments Using the Sharpe Index and Discussion of Results

As mentioned in section 3, Sharpe's ratio is the only well-known measure of RAROR that is applicable when an appropriate benchmark portfolio cannot be constructed. Therefore, we will calculate Sharpe ratios for the investments under study as a preliminary measure of performance. The Sharpe ratio, in this case, will be

$$\text{Sharpe ratio} = \frac{(\bar{r}_i - r_f)}{\sigma_i} = \frac{\text{average excess return of asset}}{\text{std. deviation of asset's excess return}}$$

with the risk-free rate,  $r_f$ , representing the constant maturity yield on 10-year treasury bonds available during the period.

Table 2. Sharpe ratio for Various Investments during the Antebellum Period

TIME PERIOD	SLAVES	N.Y. BILLS	BOSTON PAPER
1831-1835	2.041	0.993	0.834
1836-1840	-0.025	1.756	1.554
1841-1845	0.428	0.641	0.622
1846-1850	2.176	1.595	1.598
1851-1855	5.363	2.538	2.120
1856-1860	10.113	0.830	0.926

As mentioned in section 3, using the treasury bond rate is not ideal for at least two reasons. First, the ten-year period exposed an investor to significant interest-rate risk. Second, data is not available for when the government issued no debt (1835-1842), so a

proxy rate, the return on high-grade Boston municipal bonds, was used for this time period. A period of five years will be utilized to determine the average excess return, and the standard deviation will be calculated based on the same period. The results of these calculations are presented in Table 2. The results for the years between 1846 and 1860 are particularly striking because of the large differences between the Sharpe ratios for investment in slaves compared to other securities. However, we get mixed results because investment in slaves underperformed during the period 1836-1845. It is clear that investment in slaves was a substantially riskier enterprise than putting money in New York state bonds or in Boston commercial paper. We cannot choose the best investment based on these results, and the Sharpe ratio is not a flexible measure of risk because the possibility of diversification is not taken into account. It is for this reason that we will develop and apply the directional distance function to the same data.

#### 7. Presentation of DDF-RAROR and Comparison of Investments

As discussed in section 3, none of the standard RAROR metrics (other than the Sharpe ratio) are applicable to ranking investments of the antebellum period. The Directional Distance Function (DDF) developed by Chung, Fare, and Grosskopf (1997) will provide a useful metric for our purposes. Based on rate of return data of the various investments, we will first construct the investment opportunity set with its associated efficient frontier. Since estimation of an efficient frontier is done nonparametrically, we avoid the usual assumption of normality. In addition, the mean-variance theory of Markowitz (1959) assures us that the investment opportunity set will be convex as long as we assume a weighted average of portfolios along the efficient frontier is also a feasible portfolio.

In this section, we will present the DDF-RAROR as an alternative ranking for investments during the antebellum period. To arrive at these rankings, we follow the methodology of Jin (2004). First, we group the investments for which we have data in a comparison universe. We then calculate the directional distance associated with each investment. These distances are ordered, and then we perform a statistical test using these sorted rankings.

Each investment can be characterized by a risk-return pair  $(\sigma, r)$  such that  $r \in R_+^1$  and  $\sigma \in R_+^1$ . Now the investment opportunity set can be characterized as the set of financially feasible combinations of  $(\sigma, r)$  defined as

$$F = \{(\sigma, r) \mid (\sigma, r) \in P\}$$

where  $P = \{(\sigma, r) \in R_+^2\}$ . It is necessary to impose some restrictions on the investment opportunity set. First, free disposability of risk and return is assumed as represented by

$$(\sigma, r) \in F \text{ and } \sigma' \geq \sigma, r' \leq r \Rightarrow (\sigma', r') \in F.$$

Simply stated, when an investment is feasible, other investments with higher risk and lower return are also feasible. Another assumption is that reduction of risk is costly in terms of reduced return or

$$(\sigma, r) \in F \text{ and } 0 \leq k \leq 1 \Rightarrow (k\sigma, kr) \in F$$

As discussed earlier, convexity of the investment opportunity set is also assumed.

Finally, we assume that rates of return beyond the risk-free rate are not possible without undertaking risk, that is,

$$(\sigma, r) \in F \text{ and } \sigma = 0 \Rightarrow r = r_f.$$

The distance we seek to measure is the distance between the security's risk-return pair and the efficient frontier, which is the boundary of the investment opportunity set,  $F$ .

The directional distance function developed by Chung, Grosskopf, and Fare (1997) will be calculated as:

$$\vec{d} = \sup\{d \mid (\sigma + dg_\sigma, r + dg_r) \in F\}$$

where  $g_\sigma$  and  $g_r$  are, respectively, the elements for risk and return of the direction vector  $g$ . When  $(\sigma, r)$  is not on the efficient frontier,  $\vec{d} \leq 1$  indicates the proportionate reduction of risk and enhancement of return necessary for the investment to reach the efficient frontier.

It is possible that the directional distance will vary according to the direction along which performance is evaluated. In the risk-minimizing direction,  $g = (-\sigma, 0)$ , the directional distance will be

$$\hat{d} = \inf\{d \mid (d\sigma, r) \in F\}$$

where  $\hat{d} \leq 1$  represents the proportionate reduction of risk for a security at  $(\sigma, r)$  to reach the efficient frontier. Alternatively, in the return-maximizing direction,  $g = (0, r)$ , the directional distance will be

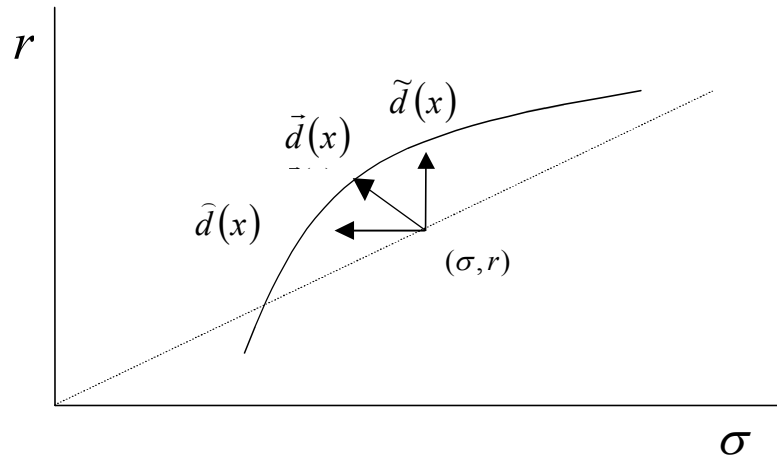
$$\tilde{d} = \sup\{d \mid (\sigma, dr) \in F\}$$

where  $\tilde{d} \geq 1$  represents the proportionate enhancement of return that a security would need to reach the efficient frontier. The directional distance function can be visualized by Figure 1.

For the purpose of defining a RAROR, we invert the directional distance to obtain the DDF-RAROR defined as:

$$\tau = 1/(1 + \vec{d})$$

Figure 1. Directional Distance



The intuition of DDF-RAROR is especially simple: there exists a tradeoff between risk and return, and the closer an investment is to the efficient frontier, the more desirable the investment. In order for a security to reach the efficient frontier, risk should be reduced with a simultaneous enrichment of return.

The investment opportunity set,  $F$ , and efficient frontier are unknown, so the DDF-RAROR will be estimated from a random sample of risk-return pairs represented by  $Y_n = \{(\sigma_i, r_i) \mid i = 1, \dots, n\}$ . From the work of Deprins, Simar, and Tulkens (1984), the Free Disposal Hull (FDH) of the set of observations

$$\hat{F}_{FDH} = \{(\sigma, r) \in R_+^2 \mid \sigma \geq \sigma_i, r \leq r_i, \quad i = 1, \dots, n\}$$

is used to estimate  $F$ . Using the convex hull of  $\hat{F}_{FDH}$ , we arrive at the DEA estimator of  $F$  proposed by Charnes, Cooper, and Rhodes (1978)

$$\hat{F}_{DEA} = \left\{ (\sigma, r) \in R_+^2 \mid \sigma = \sum_{i=1}^n w_i \sigma_i, r \leq \sum_{i=1}^n w_i r_i, \text{ s.t. } \sum_{i=1}^n w_i = 1; w_i \geq 0, i = 1, \dots, n \right\}.$$

Now that the investment opportunity set has been estimated, all that remains is to

calculate the directional distance  $\vec{d}_i$  and its associated DDF-RAROR,  $\hat{\tau}_i$ . We

accomplish this through standard linear programming:

$$\begin{aligned} \vec{d}_i &= \max \beta \\ \text{s.t.} \quad (1 + \beta)r_i &\leq w'r \\ (1 - \beta)\sigma_i &= w'\Sigma w \\ w'c &= 1 \end{aligned}$$

The objective function provides the directional distance. The first constraint ensures that the return on the security augmented by the directional distance lies on the efficient frontier. The second constraint accomplishes the same goal with the covariance matrix. The final constraint verifies that the portfolio weights sum to one and prohibits the taking of short positions.<sup>4</sup> If diversification is considered implausible, we can restate the linear programming problem as

$$\begin{aligned} \vec{d}_i &= \max \beta \\ \text{s.t.} \quad (1 + \beta)r_i &\leq w'r \\ (1 - \beta)\sigma_i &= w'\sigma \\ w'c &= 1 \end{aligned}$$

In this case, the optimization problem becomes the usual linear programming model utilized in conventional productivity analysis. Now the DDF-RAROR is determined from the distance between the Capital Allocation Line (CAL) and the security rather than

---

<sup>4</sup> The taking of short positions was clearly impossible for these investments during the antebellum period.

based on the distance between the investment and the concave efficient frontier. To allow for diversification, which undoubtedly occurred during the antebellum period, we will focus our efforts on the first of the two programs. We can now turn our attention to ranking the investments.

The risk-adjusted rate of return of owning slaves was conducted with two separate comparison universes. In the first comparison, returns from owning slaves were compared to those on New York state bonds and Boston commercial paper offerings. In the second comparison, U.S. Treasury bonds were included as part of the comparison set. Since U.S. Treasury bonds were not offered between 1835 and 1842 due to the retirement of all existing government debt, we again substituted returns on Boston municipal bonds during this period. Although the returns are comparable, this is not ideal so we exclude U.S. bonds in the first comparison group. In addition, the Treasury bonds were redeemable at the discretion of the government (Homer and Sylla, 1996), while the Boston bonds did not share this provision. We include these rates in the second study for comparison purposes.

As mentioned previously, the calculations of Evans (1962) were modified and utilized to arrive at estimates for the rate of return one realized from owning slaves. We based our demographics on the population of slaves as a whole so that we are left with a credible estimate of the return on a representative plantation. Conrad and Meyer (1962) was consulted for rates on New York bonds and Boston commercial paper, while U.S. Treasury bond returns were gleaned from Homer and Sylla (1996).

Our testing methodology was straightforward. For each comparison universe, we calculated directional distances for each investment for each five-year period during the

years 1831-1860. Next, we ordered these distances and assigned each of them a rank. At this point, the Wilcoxon rank-sum test was performed. We can state the null and alternative hypotheses as follows:

$H_0$  : The ranking distribution is the same for the investments in the comparison universe.

$H_a$ : The ranking distribution is smaller for the investment in slaves.

Using the exact Wilcoxon rank-sum test, we found  $p$ -values of between 0.01 and 0.025 for both comparison universes. Therefore, we can conclude that slaves returned a higher risk-adjusted rate of return than Boston commercial paper, New York state bonds, and U.S. Treasury bonds. Slaves were clearly a financially remunerative investment for this period, and returns on slaves were getting even stronger as the period came to a close. Slavery was most certainly not a moribund institution.

## 8. Bootstrapping Methodology and Results

The directional distances in the previous section were calculated utilizing the technique of linear programming. Results from linear programming are deterministic, but in the current analysis such a viewpoint is misleading. In the case of the antebellum period, our rates of return cannot be considered deterministic. Especially in the case of slaves, there is every reason to believe that there was substantial variation in earnings by plantation. Since there is significant variation in the returns themselves, the efficient frontier cannot be considered deterministic either. One way to model this variability is to attempt to replicate the data generating process through bootstrapping. Using the methodology presented in Simar and Wilson (1998), we can form confidence intervals around our point estimates of directional distance and reaffirm the statistical significance of our results.

The assumptions necessary to characterize the data generating process can be stated as follows.

(Assumption 1)  $\{(r_i, \sigma_i), i = 1, \dots, n\}$  are i.i.d. random variables on the convex investment opportunity set,  $F = R_+^2$  and both are freely disposable.

(Assumption 2) The actual returns and standard deviations of the investments in the comparison set are realizations of random variables possessing probability density functions whose bounded support in  $R_+$  is compact.

(Assumption 3) Probability mass exists in a neighborhood of the true frontier.

(Assumption 4) The distance function  $d$  is differentiable in its arguments.

Under these assumptions, Kneip et. al. (2001) established that

$$\hat{d} - d = O_p(n^{-1/2})$$

where  $\hat{d}$  is a consistent estimator of  $d$ . For the purposes of our analysis, the rate of convergence is equal to 1/2.

In order to implement bootstrapping methods, we first assume that a random sample  $Z = \{(r_i, \sigma_i), i = 1, \dots, n\}$  is drawn from a data generating process (DGP) under the stated assumptions. The goal of the bootstrap is to replicate the DGP utilizing a large number of pseudo samples. We will represent the set of pseudo samples as

$Z^b = \{(r_i^b, \sigma_i^b), i = 1, \dots, n\}$  where  $b = 1, \dots, B$ . Once each pseudo sample is generated, estimation of the directional distance is performed exactly as in the previous section:

$$\begin{aligned} d_i^b &= \max \beta \\ \text{s.t.} \quad (1 + \beta)r_i &\leq w'r^b \\ (1 - \beta)w'\sigma_i &= w'\sigma^b \\ w'c &= 1 \end{aligned}$$

We will implement this procedure under both comparison universes, yield bootstrap directional-distance estimates of  $\hat{d}^b$  and  $\hat{d}_{TB}^b$  where the *TB* subscript indicates the sample includes information concerning treasury bond returns.

As presented in Simar and Wilson (2000), we follow the 11-step bootstrapping algorithm.

(Step 1) Estimate  $\{\hat{d}, \hat{d}_{TB}\}$  for all investments using the original datasets

$$Z = \{(r_i, \sigma_i), i = 1, \dots, n\}.$$

(Step 2) As revealed in Simar and Wilson (2000), the naïve bootstrap is inconsistent in frontier estimation. In order to ensure consistency, we implement the reflection method proposed by Silverman (1986). From the  $(n \times 1)$  vectors  $P_1 = [\hat{d}_1, \dots, \hat{d}_n]'$  and

$P_2 = [\hat{d}_{TB,1}, \dots, \hat{d}_{TB,n}]'$  where  $n$  is the number of investments, form the  $(4n \times 2)$  augmented matrix

$$A = \begin{bmatrix} P_1 & P_2 \\ -P_1 & P_2 \\ P_1 & -P_2 \\ -P_1 & -P_2 \end{bmatrix}$$

As can be easily seen, constructing the augmented matrix in this way eliminates the concern that the distances are bounded below by zero.

(Step 3) Compute the estimated covariance matrix  $\hat{\Sigma}_1$  from the original data  $[P_1 \ P_2]$  (or the reflected data  $[-P_1 \ -P_2]$ ), and  $\hat{\Sigma}_2$  from the data  $[P_1 \ -P_2]$  (or  $[-P_1 \ P_2]$ ). That is,

$$\hat{\Sigma}_1 = Cov(P_1, P_2) = Cov(-P_1, -P_2)$$

$$\hat{\Sigma}_2 = Cov(P_1, -P_2) = Cov(-P_1, P_2)$$

Next, obtain the lower triangular matrices  $L_1$  and  $L_2$  such that  $\hat{\Sigma}_1 = L_1 L_1'$  and  $\hat{\Sigma}_2 = L_2 L_2'$  via Cholesky decomposition.

(Step 4) Draw  $n$  rows randomly with replacement from  $A$ , and denote the result by the  $(n \times 2)$  matrix  $A^b$ . Then compute  $\bar{A}^b$ , which is the  $(1 \times 2)$  row vector containing the means of each column of  $A^b$ .

(Step 5) Use a random number generator to generate an  $(n \times 2)$  i.i.d. matrix  $\varepsilon$  and construct  $\varepsilon^b$  so that

$$\varepsilon_i^b = \varepsilon_i L_j' \quad j = 1, 2$$

where  $\varepsilon_i^b \sim N(0, \hat{\Sigma}_1)$  or  $N(0, \hat{\Sigma}_2)$ . If  $i \in \{1, \dots, n, 3n+1, \dots, 4n\}$  then the covariance matrix is  $\hat{\Sigma}_1$ , whereas  $i \in \{n+1, \dots, 2n, 2n+1, \dots, 3n\}$  then the covariance matrix is  $\hat{\Sigma}_2$ .

(Step 6) Compute the  $(n \times 2)$  random deviates needed for the bootstrap by the  $\Delta$  function, as in Silverman (1986),

$$\Delta = (1 + h^2)^{-0.5} (M \cdot A^b + h \varepsilon^b) + c_n \otimes \bar{A}^b$$

where  $M = I_n - c_n c_n' / n$ ,  $I_n$  is a  $(n \times n)$  identity matrix, and  $c_n$  is a  $(n \times 1)$  unit vector.

(Step 7) Set  $h = (4/5n)^{1/6}$ , as an appropriate band width  $h$  of bivariate kernel density estimator for the previous step, as suggested in Silverman (1986).

(Step 8) Define the  $(n \times 2)$  matrix of bootstrap pseudo data  $D^b$

$$d_{ij}^b = |\delta_{ij}| \quad \text{where } \Delta = (\delta_{ij})$$

(Step 9) Construct the pseudo sample  $Y^b = \{(r_i^b, \sigma_i^b), i = 1, \dots, n\}$  by setting:

$$r_i^b = r_i(1 + \hat{d}_i) / (1 + d_i^b), \text{ and } \sigma_i^b = \sigma_i(1 - \hat{d}_i) / (1 - d_i^b)$$

(Step10) Compute  $\{\hat{d}^b, \hat{d}_{TB}^b\}$ . These estimates can be obtained by solving the programming problem under the pseudo investment opportunity set consisting of the pseudo sample  $Z^b$  (and  $Z_{TB}^b$ ).<sup>5</sup>

(Step11) Repeat steps (5)-(10)  $B$  times to get a set of bootstrap estimates  $\{\hat{d}^b, \hat{d}_{TB}^b \mid b = 1, \dots, B\}$ . Confidence intervals for these directional distances can now be derived.

The procedure for calculating confidence intervals is straightforward. The bootstrap estimates  $\{\hat{d}^b, \hat{d}_{TB}^b \mid b = 1, \dots, B\}$  are calculated and ordered, and then the extreme ( $\alpha/2$ ) percentages are deleted from both ends of the distribution. The remaining endpoints form the confidence intervals for each investment. The resulting confidence intervals for our data are printed in Tables 3 and 4. The confidence level is 95%.

Table 3. Bootstrap Confidence Intervals – Directional distance without Treasury bonds

INVESTMENT	LOWER BOUND	UPPER BOUND
Slaves	0.0057	0.16485
New York bonds	0.26535	0.7390
Boston Commercial Paper	0.24026	0.6542

In the first comparison universe, the owning of slaves again dominates the other investments in terms of directional distance. In the second comparison universe, we see minimal overlap of confidence intervals which could easily be eliminated if the experiment were repeated.

<sup>5</sup> For observations where this results in infeasible solutions, repeat steps 5-10.

Table 4. Bootstrap Confidence Intervals – Directional distance with Treasury bonds

INVESTMENT	LOWER BOUND	UPPER BOUND
Slaves	0.0071	0.2244
New York bonds	0.31857	0.74685
Boston Commercial Paper	0.21894	0.7127

### 9. Concluding Remarks

We have undertaken the first modern study of the risk, as well as the return, involved in owning slaves during the antebellum period. We have found that the owning of slaves outperforms other investments available during the period using distance from the efficient frontier as the ranking criterion. Bootstrap confidence intervals were formed with unchanged results: the directional distance associated with investment in slaves dominated the other antebellum investments. One can conclude that the owning of slaves was a very profitable enterprise, and that the economic climate was not likely to change so dramatically that this would no longer be the case. Indeed, the rates of return close to the end of the period are the most impressive of the study, with the variation in returns steadily decreasing at the same time. Slavery was a strong institution that was getting even more profitable with less risk in the late 1850s. The Civil War was not a gratuitous exercise to end an already moribund institution; it was an imperative struggle to quash a ruthless and inhuman but quite profitable institution of subjugation.

## References

Butlin, K.G. (1971). *Ante-bellum Slavery – Critique of a Debate*. Canberra: Department of Economic History, Australian National University.

Charnes, A., Cooper, W. W., and Rhodes, E. (1978), "Measuring the efficiency of decision making units," *European Journal of Operational Research*, 2, 429-44.

Chung, Y. H., R. Fare, and S. Grosskopf (1997), "Productivity and Undesirable Outputs: A Directional Distance Function Approach," *Journal of Environmental Management*, 51, 229-240.

Conrad, A. and J. Meyer (1958). "The Economics of Slavery in the Ante Bellum South," *The Journal of Political Economy*, 66, 95-130.

Deprins, D., Simar, L. and H. Tulkens (1984), "Measuring labor inefficiency in post offices," in *The Performance of Public Enterprises: Concepts and measurement*, M. Marchand, P. Pestieau and H. Tulkens (eds.), Amsterdam, North-Holland, 243-267.

Efron, B. (1979), "Bootstrap methods: another look at the jackknife," *Annals of Statistics* 7, 1-16.

Efron, B., and R. J. Tibshirani (1993), *An Introduction to the Bootstrap*, New York: Chapman and Hall.

Evans, R. (1962). "The Economics of American Negro Slavery," In *Aspects of Labor Economics*. Princeton, N.J.: Princeton University Press.

Fogel, R. and S. Engerman (1974). *Time on the Cross*. London, England: University Press of America.

Fogel, R. and S. Engerman (1974a). *Time on the Cross Evidence and Methods: A Supplement*. London, England: University Press of America.

Gray, Lewis (1958). *History of Agriculture in the Southern United States to 1860*, 2 vols., Gloucester, MA.

Homer, S. and R. Sylla (1996). *A History of Interest Rates*. New Brunswick, NJ.

Jensen, M (1968), "The performance of mutual funds in the period 1945-1964," *Journal of Finance*, 23, 389-416.

Jin, I. (2004). *Media Conglomeration, Market Concentration, and Stock Market Response*. Unpublished Dissertation, Rice University.

Kotlikoff, L. (1979). "The Structure of Slave Prices in New Orleans, 1804 to 1862," *Economic Inquiry*, 17, 496-518.

Lintner, J. (1965). "The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Journal of Finance*, 23, 389-416.

Martin, J. (1898). *One Hundred Years' History of the Boston Stock and Money Markets*. Boston, MA.

Modigliani, F and Modigliani, L (1997), "Risk-Adjusted Performance," *Journal of Portfolio Management*, 23(2) (Winter), 45-54.

Murthi, B., Choi, Y. and Desai, P. (1997), "Efficiency of mutual funds and portfolio performance measurement: A non-parametric approach," *European Journal of Operational Research*, 98, 408-418.

Officer, L. (2005). "What was the Interest Rate Then? A Data Study," University of Illinois-Chicago. Working paper.

Phillips, U.B. (1929). *American Negro Slavery*. New York, NY: D. Appleton & Company.

Ramsdell, C. (1929). "The Natural Limits of Slavery Expansion," *Mississippi Valley Historical Review*, 16, 151-171.

Roll, R. (1978), "Ambiguity when Performance is Measured by the Securities Market Lines," *Journal of Finance*, 33, 1051-1069.

Sengupta, J. K. (1991), "Maximum probability dominance and portfolio theory," *Journal of Optimization Theory and Applications* 71, 341-357.

Sengupta, J. K., and H. S. Park (1993), "Portfolio efficiency tests based on stochastic dominance and cointegration," *International Journal of Systems Science* 24, 2135-2158.

Sharpe, W. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance*, 19, 425-442.

Sharpe, W. (1966), "Mutual fund performance," *Journal of Business*, 39, 119-138.

Silverman, B. W. (1986), *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, London.

Simar L. and P. Wilson (1998), "Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models," *Management Science* 44, 49-61.

\_\_\_\_\_ (2000), "Statistical inference in nonparametric frontier models: The state of the art," *Journal of Productivity Analysis* 13, 49-78.

Simons, K. (1998), "Risk-Adjusted Performance of Mutual Funds," *New England Economic Review* September/October, 33-48.

Treynor, J. L. (1966), "How to Rate Management Investment Funds," *Harvard Business Review*, 43.

Treynor, J. L. and Black, F. (1973), "How to Use Security Analysis to Improve Portfolio Selection," *Journal of Business*, 46, 66-86.