Partner Uncertainty and the Dynamic Boundary of the Firm*

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Abstract

We develop a new theory of the dynamic boundary of the firm where asset owners sometimes want to change partners ex-post. The model identifies a fundamental trade-off between (i) a “displacement externality” under non-integration, where a partner leaves a relationship even though the benefit is worth less than the loss to the displaced partner, and (ii), a “retention externality” under integration, where a partner inefficiently retains the other. Renegotiation cannot eliminate these inefficiencies when agents are wealth constrained. When there is more asset specificity, displacement externalities matter more and retention externality less, so that integration becomes more attractive. Our model also predicts that integration always provides stronger incentives for specific investments, and that wealthy owners actually want to commit to ex-post wealth constraints. Our analysis differs from the received theories of the firm because of our emphasis on dynamic partner changes.

Keywords: Asset ownership, control rights, firm boundaries, asset specificity, specific investments, wealth constraints.

JEL classification: D23, D82, D86.

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1 Introduction

The central questions in the theory of the firm are why firms exist, why assets are sometimes held under some form of common ownership, and why economic transactions are not always organized via markets. In this paper we explore the idea that integration (such as through joint asset ownership) may guarantee continued access to critical trading partners. We develop a model of asset ownership that is based on a dynamic trade-off between the commitment to a trading relationship (integration) versus the flexibility of seeking new relationships (non-integration).

We consider an environment in which trading partners cannot take each other for granted, and fear being displaced by alternative partners. In the presence of wealth constraints there can be ex-post inefficiencies, where one partner’s gain from leaving the relationship does not outweigh the loss to the partner left behind. Joint asset ownership protects against such inefficient partner displacements. Then, why does not every trading relationship involve integration? This is because under joint ownership one partner can prevent the other from leaving the relationship, even though leaving would be jointly more efficient.

Our theory focuses on the dynamics of how asset owners match and possibly rematch with each other. We show that integration generates a commitment that makes it more difficult, but not impossible, to change trading partners. We explain how joint asset ownership endogenously raises the costs of leaving a relationship, and how this can be beneficial under some circumstances and detrimental under others. We also introduce the idea that integration is a function of the uncertainty that partners face about how the match with the current partner compares against possible future matches.

Our focus on partner changes is fundamentally different from the property rights perspective, associated with the seminal work of Grossman, Hart and Moore (see Grossman and Hart (1986), Hart and Moore (1990), Hart (1995)). In their models, alternative partners create an outside option that is never exercised in equilibrium. That is, property rights models always assume that the current partner is and will remain the best possible match. Our model introduces uncertainty about future matches, and shows how ex-post inefficiencies, such as displacement and retention externalities, can affect the ex-ante integration decision. Our main insights also do not rely on incentives for specific investments.

Our model is also related to the transaction cost theories associated with Williamson (1975, 1979, 1985), who explicitly worries about ex-post inefficiencies. However, Williamson too focuses mostly on the current match, and worries mainly about opportunism and haggling costs. We focus on different and potentially more important ex-post inefficiencies, related to the inefficient displacement or retention of trading partners. Interestingly, a central prediction of our
theory is that asset specificity favor integration, although the reason is different than in standard transaction cost economics.

In our model there are two symmetric owner-managers with co-specialized assets. Ex-ante they determine the optimal asset allocation. The base model deliberately ignores any specific investments, although a model extension adds them back in. At some ex-post date uncertainty is resolved about whether the original partners have found more attractive outside partners or not. At that stage the original partners can stay together or part their ways.

We allow for the possibility of ex-post inefficiencies in the process of switching partners. One possibility is that a partner gets displaced even though staying together would be jointly efficient – we will call this a displacement externality. Another possibility is that partners stay together even though separating would be jointly efficient – we will call this a retention externality. We endogenously derive these two new ex-post inefficiencies on the basis of two important assumptions. First, we assume that all parties are wealth constrained – we discuss this below. Second, we allow for a team moral hazard problem, where joint production requires the partners to provide non-contractible inputs such as private efforts. The combination of wealth constraints and private efforts generate a concave utility frontier where partners cannot freely transfer utility to each other without affecting incentives and joint efficiency.

We first show that whenever the two partners have symmetric outside options, the ownership structure is irrelevant. Both parties either agree to stay together, or they agree to part ways, but there is no disagreement. Asset ownership only becomes relevant when the two partners have asymmetric outside options.

Consider individual asset ownership and let us call the partners $A$ and $B$. Suppose $A$ found an alternative partner (call him $C$) but $B$ did not, so that $A$ wants to leave $B$ for $C$. There can be what we call a displacement externality, where $A$’s gain from leaving is worth less than $B$’s loss from being displaced. Obviously the two partners would like to renegotiate. Without a wealth constraint this would be straightforward and renegotiation would ensure joint utility maximization. With a wealth constraint, however, the two partners have to renegotiate along a concave utility frontier. The weaker partner ($B$) may offer to give up some share of the profit to the stronger partner ($A$), in order to convince him not to leave. There are two possible outcomes in this renegotiation game. Either $B$’s best offer is not good enough to retain $A$, and despite being jointly inefficient, displacement occurs in equilibrium. Or $A$ accepts a higher profit share and stays. The threat of displacement is then not realized in equilibrium, but the unequal profit shares distort team incentives. The outcome after renegotiation is individually optimal, but fails to maximize the sum of utilities.

Joint asset ownership can give rise to the opposite problem. If $A$ cannot leave without $B$’s consent, there can be a retention externality. This occurs when $A$’s benefit from leaving is
higher than $B$’s benefit from retaining $A$. Without a wealth constraint, renegotiation would again achieve ex-post efficiency. With a wealth constraint, however, the only way that $A$ can compensate $B$ is to give him a stake in his new partnership with $C$. This requires a negotiation among three parties. Again we obtain two possible bargaining outcomes. Either there simply is no offer such that $B$ is willing to let go of $A$, and $A$ still finds it attractive to leave. In equilibrium $B$ retains $A$, even though it is jointly inefficient. Or it is possible to structure some buyout deal, but the outcome does not maximize $A$’s and $B$’s joint utility. This is because $B$ receives a share of profits generated by $A$ and $C$, without actually contributing any productive inputs. The retention externality matters, irrespective of whether inefficient retention occurs, or whether it gets renegotiated into some other outcome that still does not maximize the sum of utilities.

The optimal ex-ante allocation of asset ownership depends on the desirability of changing partners. The main result of our base model is that joint asset ownership is optimal when displacement externalities loom large, whereas individual asset ownership is preferred when retention externalities matter more. The relative importance of displacement and retention externalities depends on how good the original match between partners is. The greater the asset specificity, the greater the displacement externality, and also the smaller the retention externality. A higher asset specificity therefore favors joint asset ownership. Our theory thus delivers one of the key predictions from the transaction cost theory, without referring to the typical ex-post problems emphasized by Williamson.

Our base model deliberately omits specific investments. This allows us to generate a set of predictions about optimal asset ownership that are orthogonal to the standard concerns of the property rights theory. Once we put relation-specific investments back into the model, we find that joint asset ownership always provides stronger incentives for specific investments. The key intuition is that joint asset ownership is efficient when the internal match is good, but can cause retention inefficiencies when the internal match is poor. By contrast, individual asset ownership is efficient when the internal match is poor, but can cause displacement inefficiencies when the internal match is good. Consequently joint asset ownership increases the utility gap between the good and the bad match, whereas individual asset ownership actually narrows the gap.

Our main trade-off arises in the presence of binding wealth constraints, as all ex-post inefficiencies can always be resolved with large amounts of wealth. We also obtain the surprising result that wealthy owners would always want to commit ex-ante to some kind of ex-post wealth-constraint, either partial or complete. Having wealth turns out to be a two-edged sword. On the one hand, wealth helps the partners to mitigate ex-post inefficiencies. On the other hand, it weakens ex-ante incentives for specific investments, precisely because it allows partners to mitigate ex-post inefficiencies when the quality of their match turns out to be poor. We also
show that wealth constraints are only one way of generating ex-post inefficiencies. The main trade-off of our paper also remains valid in a model where there is sufficient wealth, but where there are costs associated with making transfer payments.

For simplicity our base model assumes symmetric partner ex-ante. In reality partners are likely to differ in a variety of ways. We show that asymmetric expectations of finding alternative partners makes joint ownership less likely. This is because the partner with the better outside opportunities is more reluctant to give up the right to unilaterally leave the relationship. We also show that if only one of the two partners is wealth-constrained, all our key results continue to hold. This last finding is interesting, because it suggest that our theory is not only applicable to joint asset ownership for a pair of entrepreneurs, but it also applies to acquisitions of entrepreneurial companies that are wealth constrained, by established corporations that are not wealth constrained. Such acquisitions are empirically very common – see Puri and Zarutskie (2012).

2 Related Literature

Any new theory about the boundaries of firms stands on the shoulder of giants. The economic theory of the firm is dominated by three main schools of thought: transaction cost economics – associated mainly with the work of Williamson; the property rights perspective – associated mainly with the work of Grossman, Hart and Moore; and the incentive theory perspective – associated in particular with the work of Holmström, Milgrom and Roberts (see Holmström and Milgrom (1994), Holmström and Roberts (1998), Milgrom and Roberts (1990)). Our theory does not fit squarely into one of these schools. Instead it borrows a little from each, but then focuses on the dynamics of partner displacement and retention – issues that do not feature prominently in any of the existing schools of thought.

Our theory makes predictions about asset specificity that are consistent with the predictions of transaction cost economics. There is strong empirical support that asset specificity is associated with integration – see Lafontaine and Slade (2007) for a comprehensive survey of the empirical literature. Our theory provides a fresh interpretation as to why asset specificity leads to integration. Transaction cost economics tends to provide verbal arguments about opportunism and ex-post price haggling as the main sources of inefficiencies under non-integration. However, more formal theories have often dismissed these explanations, because rational agents should be able to anticipate any distributional concerns ex-ante, and thus to resolve ex-post inefficiencies. Some recent exceptions are Bajari and Tadelis (2001), Tadelis (2002), Matouschek (2004), and Casas-Arce and Kittsteiner (2011), who develop formal models with costly ex-post adjustments. In our model it is the lack of transferable utility that endogenously creates ex-
post inefficiencies. Asset specificity increases the cost of displacement externalities, and also lowers the cost of retention externalities – both of which favor integration. Our theory therefore combines the very definition of asset specificity, namely that an asset is worth more within than outside a specific trading relationship, with the possibility that in equilibrium partners do go outside the relationship, in order to generate a fresh insight as to why specific assets favor integration.

In the property rights theory it is not the level of asset specificity, but the marginal incentive to increase asset specificity through specific investments, that determines the optimal asset allocation (see Whinston (2003)). Our model differs from the property rights approach in several key aspects: (i) it does not rely on specific investments, (ii) it allows for ex-post inefficiencies, (iii) it allows for partner switching in equilibrium, and (iv), it allows partners to contractually specify prices ex-ante. Moreover, once we add specific investments to our base model we obtain a clear-cut prediction: incentives for specific investments are always stronger under joint than under individual asset ownership.

Our approach clearly differs from the incentive-based theories of the firm, which focus on risk-aversion and multi-tasking. However, we borrow from these incentive-based theories by incorporating the moral-hazard-in-teams problem (Holmström, 1982) into our production function. Together with the assumption of wealth-constrained agents (see also Sappington (1983)), this allows us to endogenously derive a concave utility frontier which generates the ex-post inefficiencies.

Our model does not address differences between integration and relational contracting. In fact, in our model all the outcomes under joint asset ownership can also be achieved using binding long-term contracts. An explanation of relational contracts would require modeling reputations in a repeated game environment. Interestingly, the main theories of relational con-

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4The non-contractibility of prices is crucial for the property rights theory. We assume that prices are contractible at all times. However, our model does have some contractual incompleteness concerning interim information that allows partners to update their profitability forecasts. If these updates were verifiable, then the optimal allocation of assets becomes state-contingent. Even then the underlying trade-off between displacement and retention externalities remains valid.

5Another difference worth mentioning is that in the standard property rights model, joint asset ownership is never optimal, and integration always consists of one agent owning both assets. Cai (2003) derives a model with both specific and general investments, and shows that joint asset ownership becomes optimal when the two types of investments are substitutes. Halonen (2002) provides conditions under which joint asset ownership is optimal in a repeated game framework; see also Blonski and Spagnolo (2003). Our model provides an alternative reason for the optimality of joint asset ownership: to prevent the dissolution of otherwise efficient partnerships.
tracts, such as Baker, Gibbons and Murphy (2002), focus on a given set of partners, and do not consider how in equilibrium agents may change their trading partners.\footnote{One interesting exception is Board (2011), who considers relational contracting between a principal and an agent, where the latter can be replaced in every period depending on changing production requirements. However, Board does not address issues of asset ownership.}

Our model is not the first to consider wealth constraints. Aghion and Bolton (1992) introduce wealth constraints into a financial contracting model. In their model there are fixed non-transferable private benefits that can lead to ex-post inefficient decisions, depending on the allocation of control rights. The main difference to our model is that Aghion and Bolton look at a model with a single asset, and focus on the inherently asymmetric relationship between an investor and entrepreneur. Our model is closer to the Grossman, Hart and Moore set-up with two assets and two productive agents. In our model we also do not rely on fixed private benefits, but instead consider renegotiation of ownership shares in the presence of moral hazard and wealth constraints. In a related vein, Aghion and Tirole (1994) consider the importance of wealth constraints in a model with a single asset. They focus on intellectual property as an asset, and ask whether it should be owned by the developer or user of the innovation. None of these models considers the possibility of switching trading partners at the ex-post stage.

There is a prior literature that looks at ex-post inefficiencies.\footnote{Gibbons (2005) identifies them broadly as adaptation-based theories of the firm; Segal and Whinston classify them as theories with imperfect bargaining.} Of historic interest, in addition to their seminal 1986 paper, Grossman and Hart published a less known book chapter in 1987 with a model where there are ex-post inefficiencies and no specific investments (Grossman and Hart, 1987). More recently, Hart (2009) examines asset ownership in a model of reference points with ex-post irrational and inefficient behaviors. Dessein (2012) outlines a model with inefficient ex-post decisions, leading to a trade-off between adaptation and coordination. Aghion et al. (2012) provide a model with ex-post asymmetric information, showing how the ex-ante asset allocation plays a role over and above any contractual arrangements.

Our paper is loosely related to the large literature on vertical foreclosure. Aghion and Bolton (1987) examine how a seller can lock buyers into long-term contracts to reduce the threat of entry from a competing seller. Bolton and Whinston (1993) use a property-rights approach to study how concerns about supply assurances can motivate vertical integration. Segal and Whinston (2000) further examine how exclusive contracts may (or indeed may not) affect specific investments. Matouschek and Ramezzana (2007) use a search model with sellers and buyers to study how exclusive contracts can reduce ex-post inefficiencies arising from price haggling.\footnote{See also Jing and Winter (2011) for an overview over the recent literature on exclusionary contracts.} One important difference to the literature on exclusive contracts is that our model focuses on
a set-up with two symmetric partners, who are both concerned about retaining their trading partner.

Our paper is also related to parts of the literature on partnerships. Hellmann and Thiele (2014) consider the formation of entrepreneurial teams. De Frutos and Kittsteiner (2008) examine the efficient dissolution of partnerships.

The remainder of this paper is structured as follows. The next section introduces our main model. Section 4 examines how partners make choices about staying versus leaving a relationship, and identifies the optimal asset ownership in the absence of specific investments. In Section 5 we then analyze the role of asset ownership for the partners’ incentives to make relation-specific investments. In Section 6 we identify the optimal allocation of control rights over critical assets, accounting for specific investments and ex-post transfer payments. In Section 7 we discuss robustness properties of our model. Section 8 provides some empirical implications from our model. Section 9 summarizes our main results, and explores avenues for future theoretical and empirical work. All proofs are in the Appendix.

3 The Base Model

Consider an initial match of two risk-neutral partners, for ease of exposition called Alice (A) and Bob (B). For example, Bob can be the owner an upstream firm selling an input to Alice as the owner of a downstream firm, which Alice needs to manufacture an end product. The value of their initial outside options is normalized to zero. Each partner initially owns a co-specialized asset, and has wealth \( w \equiv w_A = w_B \geq 0 \).

There are five dates; see Figure 1 for a graphical overview. At date 0, both partners decide on an optimal ownership structure for both assets. While we consider all ownership structures,
the key decision will be whether partners keep individual asset ownership, or they agree on joint asset ownership. As we will discuss in Section 7.3, we can also think of this difference in terms of short-term versus long-term contracts. At date 1, both partners can make relation-specific investments to improve the value of joint production. At date 2, Alice and Bob learn about the prospect of their partnership, and may find alternative partners. They then decide whether to stay with their original partner, or to leave and form a new partnership. Alice and Bob may also renegotiate any division of surplus. At date 3, partners exert private effort to produce output. Finally, at date 4, all returns are realized.

In case of a successful joint production, Alice and Bob generate the profit \( y \) at date 4. We assume that \( y \) is verifiable, and that it has a distribution \(\Omega_{in}(y)\) in \((y)\) over some interval \(y \in [\underline{y}, \bar{y}]\) with \(0 \leq \underline{y} < \bar{y} \leq \infty\). We denote the expected value by \(\pi = \int_y^{\bar{y}} y d\Omega_{in}(y)\), and refer to it as the inside prospect of the match between Alice and Bob. We assume that the inside prospect \(\pi\) is observable by both partners, but non-verifiable by outside parties.\(^9\)

At date 1 Alice and Bob can invest in their relationship to improve the distribution of potential profits \(y\). Specifically we assume that the expected profit \(\pi\) can take on two values: \(\pi \in \{\pi_L, \pi_H\}\), with \(\pi_H > \pi_L > 0\). The inside prospect \(\pi\) will be high with probability \(p = p(r_A, r_B) (\pi = \pi_H)\), and low with probability \(1 - p (\pi = \pi_L)\), where \(p\) is concave increasing in the partners’ relation-specific investments \(r_A\) and \(r_B\). Specific investments are non-contractible, and impose convex private costs \(\psi(r_i), i = A, B,\) with \(\psi(0) = \psi'(0) = 0\). To ensure interior solutions we assume that \(p(0, 0) = 0\) and \(\frac{\partial p(\cdot)}{\partial r_i}|_{r_i=0} = \infty, i = A, B\). We also assume that the cross-partial is not too negative: \(\frac{\partial^2 p(\cdot)}{\partial r_A \partial r_B} > -\kappa\), where \(\kappa > 0\). This ensures that the reaction functions of both partners are well-behaved.\(^10\) Alice and Bob learn the actual inside prospect \(\pi \in \{\pi_L, \pi_H\}\) at date 2.

Depending on the observed inside prospect \(\pi \in \{\pi_L, \pi_H\}\) at date 2, Alice and Bob can decide to break their original partnership and match with alternative partners. Specifically we assume that Alice finds an alternative partner, called Charles (C), with probability \(q > 0\). We assume symmetry so that Bob discovers an alternative partner, called Dora (D), with the same probability \(q\).\(^11\) For simplicity we assume that both alternative partners, Charles and Dora, have zero wealth, and normalize their outside options to zero. The profit \(y\) of a successful alternative

\(^9\)Assuming that \(\pi\) is verifiable does not seem plausible within this context, given that the inside prospect is specific to the collaboration of the two partners. It is not based on objective past performance, but instead, is based on expectations about the benefits of a future joint production.

\(^10\)A sufficient and intuitive assumption is that the specific investments \(r_A\) and \(r_B\) are (weak) strategic complements, so that \(\frac{\partial^2 p(\cdot)}{\partial r_A \partial r_B} \geq 0\).

\(^11\)Recall that Alice and Bob have complementary assets which both are needed for production. This excludes the possibility of Alice partnering with Dora, or Bob partnering with Charles. Moreover, in Section 7.5 we briefly consider the case where Alice and Bob find their alternative partners with different probabilities.
partnership has the distribution $\Omega_{\text{out}}(y)$. We denote the expected value by $\sigma = \int y d\Omega_{\text{out}}(y)$, which we refer to as the outside prospect.\(^{12}\)

At date 3 the partners engage in joint production; this can be either Alice and Bob ($A, B$), or Alice and Charles ($A, C$) and/or Bob and Dora ($B, D$). Joint production requires (i) the use of both of the partners’ complementary assets, and (ii) their private efforts, which we denote $e_i, i = A, B, C, D$. A partner’s disutility of effort is $c(e_i)$, with $c'(e_i) > 0, c''(e_i) > 0$, and $c(0) = c'(0) = 0$. Production either generates a joint profit at date 4 (success), or no profit at all (failure). The success probability is given by $\mu(e_ie_j), i, j \in \{\{A, B\}, \{A, C\}, \{B, D\}\}$, which is increasing and concave in its argument $e_ie_j$, with $\mu(0) = 0$. Thus, the partners’ efforts are complementary, and success requires that both partners apply strictly positive efforts (i.e., $e_i, e_j > 0$).

The realized profit $y$ at date 4 can be divided between the two partners according to any sharing rule, e.g. where Alice obtains $\alpha y$ and Bob receives $\beta y$, with $\alpha + \beta = 1$. Depending on the ownership structure this sharing rule can be implemented in different ways. Under joint asset ownership, we think of $\alpha$ and $\beta$ as a division of ownership shares from the jointly owned venture. Under individual asset ownership there are no ownership shares, so the division of surplus comes from some transfer price – we provide a more formal discussion of this in Section 7.3.\(^{13}\)

Ownership defines control rights over the productive assets. We assume that Alice and Bob initially have full rights of control over their respective assets. Alice and Bob can then choose to retain their control rights at date 0 (individual asset ownership); they can then simply wait until date 2 to see whether in fact they want to partner up.\(^{14}\) If they do, they negotiate a transfer price which determines their profit shares ($\alpha, \beta$) at that time. Alternatively, the partners can agree at date 0 to share control rights over both assets (joint asset ownership). This requires that Alice and Bob negotiate the ownership shares $\alpha$ and $\beta$ at date 0. Section 7.2 shows why we can limit ourselves to individual and joint asset ownership. Within our context ownership matters

\(^{12}\)An alternative interpretation is that Alice and Bob always find alternative partners, but that the outside prospect is then $\sigma_H (= \sigma)$ with probability $q$, and $\sigma_L = 0$ with probability $1 - q$.

\(^{13}\)In principle it is possible to make $\alpha$ and $\beta$ contingent on $y$. In the case of joint asset ownership, it is easy to verify that for any division of surplus with variable $\alpha$ and $\beta$, there exists an equivalent division of surplus with a constant $\alpha$ and $\beta$. W.l.o.g we can therefore focus on constant $\alpha$’s and $\beta$’s. In the case of individual asset ownership, $\alpha$ and $\beta$ depend on how transfer prices are specified, i.e., how they depend on the realization of $y$. To keep our notation as simple as possible we focus on the case of constant $\alpha$’s and $\beta$’s. This is w.l.o.g since all that matters is the expected profit share at date 2.

\(^{14}\)This specific aspect is due to our simplification that no production occurs between the dates 0 and 2. If the two partners were to produce at the earlier stage, then they would write a short-term contract for the early production period, but retain the right to leave the relationship over the long term.
because it affects the ability of a partner to leave: Under individual asset ownership, a partner with a superior outside option can always leave without the consent of the other. Under joint asset ownership, the two partners share control rights over both assets, so that leaving requires consent of the other partner.

The two initial partners Alice and Bob determine asset ownership at date 0. Bargaining can also occur at date 2, where it may involve two or more parties. Because of a potentially binding wealth constraint (in case each partner’s initial wealth $w$ is sufficiently low), we need a bargaining solution for games with non-transferable utilities. We adopt the bargaining protocol of Hart and Mas-Colell (1996), where in each round one member at the bargaining table is selected at random to make a proposal, and where there is a small probability that a partner whose proposal was rejected, is permanently eliminated from the bargaining. We assume that the only members at the bargaining table are those who have the control rights to affect the decision. This means that under individual asset ownership, bargaining takes place between the two partners who want to engage in joint production. Under joint asset ownership, however, leaving requires the consent of the other partner. A new partner (Charles or Dora) therefore has to engage in trilateral bargaining with both of the original partners (Alice and Bob). In the Appendix we show that alternative bargaining protocols may generate different levels of utility, but they do not affect the basic logic of how partners make optimal asset ownership decisions.

4 The Role of Asset Ownership

In this section we analyze how asset ownership affects the renegotiation outcome between Alice and Bob at date 2, and therefore their decision to stay together or to match with alternative partners. We then identify the optimal asset ownership that Alice and Bob agree on at date 0. To show that our key insights do not rely on specific investments, we deliberately shut down this part of the model until Section 5, and assume for now that the probability of a high inside prospect, $p$, is fixed.

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15 Because there is no production between the dates 0 and 2, we could also assume that profit shares are only negotiated at date 2. This alternative interpretation generates identical results.

16 This is a multi-player generalization of the breakdown game by Binmore, Rubinstein, and Wolinsky (1986). This bargaining protocol generates the Maschler-Owen consistent NTU value, which is a generalization of the Shapley value for games with non-transferable utility (Maschler and Owen, 1992). For bilateral bargaining games, the Maschler-Owen consistent NTU value reduces to the Nash bargaining solution. For a more extensive discussion of this, see also Hart (2004).
4.1 Joint Production

We first examine the joint production process for Alice and Bob at date 3, explaining how profit shares affect incentives, success probabilities, and partner utilities. The analysis is analogous for joint production with a new partner (Charles or Dora) except that the inside prospect $\pi \in \{\pi_L, \pi_H\}$ is to be replaced by the outside prospect $\sigma$.

In our model there is team production. Alice and Bob choose their respective efforts $e_A$ and $e_B$ to maximize their expected utilities:

$$U_A(\alpha; \pi) = \alpha \mu(e_A e_B)\pi - c(e_A)$$

$$U_B(\beta; \pi) = \beta \mu(e_A e_B)\pi - c(e_B).$$

The division of joint profit, as reflected by $\alpha$ and $\beta$, affects the effort of each partner, and thus their expected utilities. We denote $\alpha^\text{max}$ and $\beta^\text{max}$ as the individually optimal profit shares for Alice and Bob, respectively.

**Lemma 1** *The joint surplus $V = U_A + U_B$ is maximized when $\alpha = \beta = 1/2$. By contrast, the individually optimal profit shares $\alpha^\text{max}$ and $\beta^\text{max}$ satisfy $1/2 < \alpha^\text{max} = \beta^\text{max} < 1$.*

Figure 2 illustrates the utility-possibility frontier for joint production for different profit shares $\alpha$ and $\beta = 1 - \alpha$. The frontier is backward bending because every partner relies on the productive effort of his co-partner. If one partner exerts no effort (which occurs when

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17Because of the binary nature of outcomes (success or failure), there is no possibility for budget breaking as in Holmström (1982).
$\alpha \in \{0, 1\}$, joint production never succeeds ($\mu(0) = 0$), and Alice and Bob both get a zero utility. We can also see from Figure 2 that the total surplus is maximized when each (symmetric) partner gets exactly half of the expected profit $\pi$ ($\alpha = \beta = 1/2$). However, each partner prefers to get more than half of the expected profit (as $\alpha^{\text{max}} = \beta^{\text{max}} > 1/2$). We also note that any bargaining outcome is always located on the downward sloping part of the utility-possibility frontier. Thus, any equilibrium division of profit, denoted by $\alpha^*$ and $\beta^* = 1 - \alpha^*$, satisfies $\alpha^* \in [1 - \beta^{\text{max}}, \alpha^{\text{max}}]$.

4.2 Symmetric Outside Options

We can now analyze how the allocation of control rights over the two assets, affects Alice’s and Bob’s decision at date 2 to stay together or to dissolve their partnership. For this we first consider the outcome in case they have identical outside options.

Suppose that neither Alice nor Bob found an alternative partner at date 2, which occurs with probability $(1 - q)^2$. Joint production is then the only option, regardless of the asset ownership structure. Because of symmetry, both partners share the profits equally: $\alpha^* = \beta^* = 1/2$; we formally prove this in the Appendix (see Proof of Lemma 1). The expected utility of a partner is then given by

$$U_i(\pi) = \frac{1}{2} \mu(e^*_A e^*_B) \pi - c(e^*_i), \quad i = A, B$$

(1)

where $e^*_A$ and $e^*_B$ denote the equilibrium effort levels. We henceforth suppress the subscript of $U$ whenever the expected utilities of Alice and Bob are identical.

Now suppose that Alice and Bob each found an alternative partner, which occurs with probability $q^2$. They can then either stay together, or split in order to form new partnerships with Charles and Dora. If Alice and Bob decide to stay together, symmetry implies $\alpha^* = \beta^* = 1/2$.

The expected utility for each partners is then $U(\pi)$.

Alternatively, Alice and Bob can decide to match with Charles and Dora respectively. They then bargain over the division of the expected profit $\sigma$ from their new partnerships. Recall that the alternative partners, Charles and Dora, both have zero outside options. The same applies to Alice and Bob during the bargaining. This is because once Alice and Bob approach their new partners, they expect to close a deal with that new partner, and are therefore no longer available as a fall-back option.\(^{18}\) The Nash bargaining solution then suggests that a partner gets exactly

\(^{18}\)Technically, under the Hart and Mas-Colell bargaining protocol, there is an $\varepsilon$ probability that the bargaining fails. Thus, with probability $\varepsilon$, Alice has the fall-back option of going back to Bob, and vice versa. The Hart and Mas-Colell bargaining protocol then assumes that $\varepsilon \to 0$, implying that Alice’s and Bob’s outside options converge to zero.
half of the profits from the new partnership. We denote the profit shares for Alice and Bob in their new partnerships by $\hat{\alpha} = \hat{\beta} = 1/2$.\textsuperscript{19} The expected utility of a partner then becomes

$$U(\sigma) = \frac{1}{2} \mu(e_i e_j^*) \sigma - c(e^*_i), \quad i = A, B, \quad j \in \{C, D\}. \quad (2)$$

We can infer from (1) and (2) that $U(\sigma) = U(\pi)$ when $\sigma = \pi \in \{\pi_{L, H}\}$. Thus, Alice and Bob stay together (joint production) with $\alpha^* = \beta^* = 1/2$ as long as $\pi \geq \sigma$. Otherwise they dissolve their partnership, and match with their alternative partners Charles and Dora. And because leaving is mutually beneficial for $\sigma > \pi$, it is irrelevant whether they agreed on individual or joint asset ownership at date 0.

### 4.3 Asymmetric Outside Options

The most interesting scenario arises when only one partner, say Alice, found an alternative partner, Charles, at date 2. This occurs with probability $q(1 - q)$. The case where only Bob found an alternative partner, Dora, is symmetric and also occurs with probability $q(1 - q)$. The allocation of control rights over the two assets then matters as it defines Alice’s freedom to leave the partnership with Bob. We discuss the implications of individual and joint asset ownership separately.

#### 4.3.1 Individual Asset Ownership

Suppose Alice and Bob agreed on individual asset ownership at date 0, and only Alice found an alternative partner at date 2, Charles. To identify potential inefficiencies that may then arise, we first consider the case where Alice and Bob have no initial wealth at all ($w = 0$). We then relax this assumption and show how Alice and Bob can use their wealth to (partially) offset these inefficiencies.

Individual asset ownership allows Alice to unilaterally take her asset and form a new partnership with Charles without Bob’s consent. In the bargaining game between Alice and Charles, Alice’s outside option is to go back to Bob, who does not have an alternative partner to bargain with. According to the Hart and Mas-Colell bargaining protocol, this outside option would only be realized if the bargaining between Alice and Charles breaks down, so that Alice loses Charles as a potential trading partner. In this case, both Alice and Bob would have zero outside options, so that they split the equity in half. The outside option of Alice when bargaining

\textsuperscript{19}Throughout the paper we use an asterisk ($^*$) to indicate equilibrium profit shares under joint production (Alice-Bob match); a hat ($\hat{\cdot}$) indicates the equilibrium profit shares in alternative matches (either Alice-Charles match, or Bob-Dora match).
with her alternative partner Charles is thus given by $U(\pi)$. Let $\hat{\alpha}_I$ denote the equilibrium profit share for Alice when partnering with Charles, where the subscript ‘$I$’ indicates individual asset ownership.\footnote{More formally, $\hat{\alpha}_I$ maximizes the Nash product $[U_A(\hat{\alpha}_I; \sigma) - U(\pi)]^{1/2}[U_C(1 - \hat{\alpha}_I; \sigma)]^{1/2}$. It is easy to see that for any $U(\pi) > 0$ we have $\hat{\alpha}_I \in (1/2, 1)$.} Alice’s expected utility is then $U_A(\hat{\alpha}_I; \sigma)$.

When Alice leaves Bob and matches with Charles, Bob’s expected utility becomes $U_B = 0$. This is clearly smaller than his expected utility $U_B(\pi)$ under joint production with Alice. Thus, Alice imposes a displacement externality on Bob when displacing him with the alternative partner Charles. And leaving Bob is jointly inefficient when $U_A(\hat{\alpha}_I; \sigma) < 2U(\pi)$.

Alice could also stay with Bob but use her better outside option to renegotiate a higher profit share. Alice would then only use her outside option of matching with Charles if the bargaining with Bob breaks down. Again, according to the Hart and Mas-Colell bargaining protocol, Alice would then lose Bob as a potential trading partner. In that case, Alice and Charles would both have zero outside options, and they would split the surplus in half. The outside option of Alice when renegotiating the profit shares with Bob is therefore given by $U(\sigma)$. Let $\alpha^*_I$ denote the equilibrium profit share for Alice when staying with Bob.\footnote{Using Nash bargaining, $\alpha^*_I$ maximizes $[U_A(\alpha^*_I; \pi) - U(\sigma)]^{1/2}[U_B(1 - \alpha^*_I; \pi)]^{1/2}$. Moreover, note that $U(\sigma) > 0$ implies $\alpha^*_I \in (1/2, 1)$.)}

The renegotiation at date 2 under individual asset ownership then leads to the expected utility $U_A(\alpha^*_I; \pi)$ for Alice, and $U_B(\beta^*_I; \pi)$ for Bob, with $\beta^*_I = 1 - \alpha^*_I$. Relative to the equal division of profits with $\alpha = \beta = 1/2$, this outcome is more favorable to Alice, and less favorable to Bob. Most importantly, it is jointly inefficient since joint surplus is maximized at $\alpha = \beta = 1/2$.

We have seen that individual asset ownership can lead to two different types of ex-post inefficiencies: inefficient displacement (leaving) and unequal profit shares (staying). Which inefficiency eventually arises depends on the inside project $\pi \in \{\pi_L, \pi_H\}$ that Alice and Bob observe at date 2. In the Appendix we derive a threshold of the inside prospect, $\hat{\pi}_I(\sigma) = \sigma$, so that asymmetric outside options under individual asset ownership with zero wealth lead to displacement when $\pi < \hat{\pi}_I(\sigma)$, and unequal profit shares when $\pi \geq \hat{\pi}_I(\sigma)$.

If Alice and Bob have some wealth $w > 0$, they could make transfer payments to – at least partially – offset these inefficiencies. It is easy to see that with unlimited wealth the ex-post inefficiencies can be completely eliminated. In the Appendix we characterize the minimum amount of wealth, denoted $\overline{w}_I$, that is required to fully eliminate the inefficiencies. We also characterize $\overline{w}_I$ as the lower bound above which wealth actually changes the renegotiation outcome. For $\pi < \hat{\pi}_I(\sigma)$, renegotiation without wealth leads to inefficient displacement. The partner with the better outside option, say Alice, has a strict preference for working with Charles, rather than accepting Bob’s most generous retention offer ($\alpha = \alpha^{\text{max}}$). Sweetening the retention offer with
a small transfer payment is therefore not enough to win over Alice. In this case $w_I > 0$, so that a small amount of wealth does not affect the renegotiation outcome at all. For $\pi \geq \hat{\pi}_I(\sigma)$, renegotiation without wealth leads to no displacement but unequal profit shares. The partner without outside option, say Bob, can then use his wealth to pay Alice in order to retain a greater profit share. This in turn improves the incentive balance, and therefore joint efficiency. In this case $w_I = 0$, so that any small amount of wealth changes the renegotiation outcome.

We can now identify the optimal choice of the partner with the better outside option under individual asset ownership for any wealth level $w \geq 0$.

**Lemma 2** Consider individual asset ownership and suppose that the two original partners have asymmetric outside options. Then, there exists a threshold $\hat{\pi}_I(\sigma, w)$ such that the partner with the better outside option leaves if $\pi < \hat{\pi}_I(\sigma, w)$. Otherwise, if $\pi \geq \hat{\pi}_I(\sigma, w)$, he stays but renegotiates his share on the expected joint profit $\pi$. The threshold $\hat{\pi}_I(\sigma, w)$ is decreasing in wealth $w$ for $w_I \leq w < \bar{w}_I$.

Joint production between Alice and Bob is the outcome under individual asset ownership with asymmetric outside options whenever the inside prospect $\pi$ is sufficiently high ($\pi \geq \hat{\pi}_I(\sigma, w)$). The partner with the better outside option then renegotiates the division of surplus, which is optimal from a selfish perspective but compromises the efficiency of joint production (as long as the partners do not have sufficient wealth for transfers to settle on an equal split of profits). Displacement, on the other hand, occurs whenever the prospect of the original partnership is sufficiently low ($\pi < \hat{\pi}_I(\sigma, w)$). This imposes a displacement externality on the partner without outside option. Finally, more initial wealth $w$ allows the partner without outside option, say Bob, to offer his partner, Alice, a larger transfer payment. This in turn makes staying (with renegotiation) more attractive for Alice, so that joint production between Alice and Bob is then more often the outcome (as $d\hat{\pi}_I(\sigma, w)/dw < 0$).

### 4.3.2 Joint Asset Ownership

Now consider joint asset ownership, where both partners share control rights over their two assets. This has two implications: First, joint ownership prevents Pareto-inefficient renegotiation between Alice and Bob, so that $\alpha^* = \beta^* = 1/2$ as long as they both prefer to stay together. Second, leaving requires the permission from the other partner, which will necessitate an appropriate compensation. Again we first identify potential inefficiencies for the case without wealth ($w = 0$), and then show how Alice and Bob can use wealth to mitigate these inefficiencies.

Suppose again that only Alice found an alternative partner, Charles, at date 2, and wants to leave Bob. This requires an adequate compensation for Bob; otherwise he would refuse to let
go of Alice. When Alice has no wealth, she can only offer Bob a share on the future return $\sigma$ from her new partnership with Charles. Productive effort is then only applied by Alice and Charles, so that Bob is a shareholder who does not add any value. We define $\hat{\alpha}_J$ and $\hat{\beta}_J$ as the equilibrium shares on the return $\sigma$ for Alice and Bob, respectively. The equilibrium share for Charles is denoted by $\hat{\gamma}_J$, where efficiency requires $\hat{\gamma}_J = 1 - \hat{\alpha}_J - \hat{\beta}_J$. We provide a complete characterization of $\hat{\alpha}_J$, $\hat{\beta}_J$, and $\hat{\gamma}_J$ in the Appendix, using the Maschler-Owen consistent NTU value. For this scenario we denote the expected utility for Alice as $U_A(\hat{\alpha}_J; \sigma)$, and for Bob as $U_B(\hat{\beta}_J; \sigma)$.

Overall we note that while joint asset ownership prevents inefficient displacement, it can lead to a retention externality. This occurs whenever the partner without outside option refuses to let go of the partner with outside option, even if leaving maximizes their joint utility. Renegotiation requires that the productive partner, say Alice, gives up part of the future returns from her joint production with Charles in order to compensate Bob. This buyout arrangement impairs effort incentives, and thus lowers the expected payoff from Alice’s partnership with Charles. Thus, the profit share $\hat{\beta}_J$ offered to Bob may not suffice to buy his consent, so that Alice is forced to stay despite leaving being jointly efficient, which is the case when $U(\sigma) > 2U(\pi)$.

Which of the two inefficiencies – inefficient retention and inefficient buyout – arises in equilibrium, depends again on the inside project $\pi \in \{\pi_L, \pi_H\}$ that Alice and Bob observe at date 2. In the Appendix we characterize the threshold $\hat{\pi}_J(\sigma)$, so that asymmetric outside options under joint asset ownership with zero wealth lead to retention when $\pi \geq \hat{\pi}_J(\sigma)$, and inefficient buyout when $\pi < \hat{\pi}_J(\sigma)$.

When Alice and Bob have some wealth $w > 0$, they can make side-payments to mitigate, or even eliminate, these inefficiencies. In the Appendix we characterize the minimum amount of wealth, denoted $\bar{w}_J$, that is necessary to eliminate all ex-post inefficiencies under joint asset ownership. Likewise we characterize the lower bound of wealth, $\underline{w}_J$, which is required to change the renegotiation outcome. For $\pi \geq \hat{\pi}_J(\sigma)$, renegotiation without wealth leads to retention. For example, if only Alice found an alternative partner, a small amount of wealth is not enough for her to persuade Bob to let her go. In this case $\underline{w}_J > 0$, so that a small amount of wealth does not change the renegotiation outcome. For $\pi < \hat{\pi}_J(\sigma)$, renegotiation without wealth leads to an inefficient buyout. Alice can then use her wealth to make a transfer payment to Bob, thereby preserving her profit share in the new partnership with Charles. This strengthens effort incentives for Alice and Charles in their new partnership, and therefore improves joint efficiency. In this case $\underline{w}_J = 0$, so that any small amount of wealth changes the renegotiation outcome.

The next lemma identifies the choice of the partner with the better outside option under joint asset ownership for any wealth level $w \geq 0$. 

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Lemma 3 Consider joint asset ownership and suppose that the two original partners have asymmetric outside options. Then, there exists a threshold \( \hat{\pi}_J(\sigma, w) \), such that the partner with the better outside option leaves with consent if \( \pi < \hat{\pi}_J(\sigma, w) \). Otherwise, if \( \pi \geq \hat{\pi}_J(\sigma, w) \), he stays with \( \alpha^* = \beta^* = 1/2 \). The threshold \( \hat{\pi}_J(\sigma, w) \) is increasing in wealth \( w \) for \( w_J \leq w < w_J \).

Joint production between Alice and Bob is the equilibrium outcome as long as the inside prospect \( \pi \) is sufficiently high (\( \pi \geq \hat{\pi}_J(\sigma, w) \)). They then split everything in half (\( \alpha^* = \beta^* = 1/2 \)), so that total surplus is maximized. In contrast, Alice and Bob agree to break up whenever the alternative partnership is attractive enough (\( \pi < \hat{\pi}_J(\sigma, w) \)). Unless the partner with the better outside option, say Alice, has sufficient wealth (\( w \geq w_J \)), she needs to offer Bob a stake in the new partnership with Charles in exchange for regaining control rights over her asset. And more wealth \( w \) enables Alice to make larger payments to Bob, thereby allowing her to retain more of the equity of the new partnership with Charles. This makes buying out her asset more attractive for Alice, so that leaving is then more often the outcome (as \( d\hat{\pi}_J(\sigma, w)/dw > 0 \)).

4.4 Optimal Asset Ownership

With symmetric outside options, we know that Alice and Bob stay together at date 2 with an equal split of profits, unless they both strictly benefit from dissolving their partnership (case of two alternative partners and \( \pi < \sigma \)). It is then irrelevant whether both partners agreed on individual or joint asset ownership. For asymmetric outside options, however, the allocation of control rights matters.\(^{22}\) Individual ownership provides flexibility to dissolve an inefficient partnership, but can also lead to unbalanced effort incentives or an inefficient partner displacement. Joint ownership, on the other hand, protects a partner without outside option from opportunism, but hampers the dissolution of inefficient partnerships, thus creating a retention externality.

We have shown how Alice and Bob can eliminate these ex-post inefficiencies, provided they have sufficient wealth for side-payments. If their wealth constraint is not binding under only one ownership structure, then it is easy so see that they choose exactly this ownership structure at date 0 to maximize their expected utilities (individual ownership for \( \bar{w}_I \leq w < \bar{w}_J \), and joint ownership for \( \bar{w}_J \leq w < \bar{w}_I \)). If their wealth constraint is not binding under either ownership structure (\( w \geq \max\{\bar{w}_I, \bar{w}_J\} \)), it is irrelevant what allocation of control rights they choose at date 0: they can then eliminate all ex-post inefficiencies trough transfer payments, so that the equilibrium outcome is always the same.

\(^{22}\)Ex-ante Alice and Bob know that they will have asymmetric outside options at date 2 with probability \( 2q(1 - q) \). Thus, asset ownership matters as long \( 0 < q < 1 \), so that outside options will be asymmetric with a strictly positive probability. However, the specific value of \( q \) is then irrelevant (as long as \( 0 < q < 1 \)), because both Alice and Bob have the same chance of finding an alternative partner.
Figure 3: Asset Ownership and Joint Efficiency

For the remainder of this section we focus on the more interesting case where Alice’s and Bob’s wealth constraint is always binding (i.e., $w < \min\{\overline{w}_I, \overline{w}_J\}$). The next proposition identifies the optimal asset ownership that the two partners then agree on a date 0.

**Proposition 1** Suppose $w < \min\{\overline{w}_I, \overline{w}_J\}$. Then, there exists a threshold $\hat{\pi}_V(\sigma)$, such that the contract choice for the two partners at date 0 is as follows:

(i) For $\pi_L, \pi_H < \hat{\pi}_V(\sigma)$, they choose individual asset ownership.

(ii) For $\pi_L, \pi_H \geq \hat{\pi}_V(\sigma)$, they choose joint asset ownership.

(iii) For $\pi_L < \hat{\pi}_V(\sigma) < \pi_H$, there exists a threshold $\hat{p} \in (0, 1)$, such that both partners choose joint asset ownership at date 0 whenever $p \geq \hat{p}$; otherwise they choose individual asset ownership.

The threshold $\hat{\pi}_V(\sigma)$ satisfies $\hat{\pi}_J(\sigma, w) < \hat{\pi}_V(\sigma) < \hat{\pi}_I(\sigma, w)$, and is increasing in $\sigma$.

For now let us focus on the scenarios (i) and (ii) to explain the rationale behind Proposition 1. For this we refer to Figure 3, which presumes that Alice and Bob have asymmetric outside options at date 2. Dissolving their partnership is then jointly efficient if the inside prospect is sufficiently low ($\pi < \hat{\pi}_V(\sigma)$); otherwise joint production with an equal split of profits maximizes joint surplus.

Consider regions (I) and (II). In these two regions the inside prospect $\pi$ is sufficiently high, so that Alice’s and Bob’s joint utility is maximized when they remain together. Under individual asset ownership there is a displacement externality: In region (II) the outside prospect
σ is sufficiently attractive, so that the partner with outside option simply leaves without renegotiation. In region (I), the partner with outside option merely uses his opportunity to switch as a bargaining chip. Both of these outcomes are ex-post inefficient from a joint perspective. These inefficiencies can be avoided with joint asset ownership, where Alice and Bob always remain together without renegotiation.

In regions (III) and (IV), the inside prospect is weak relative to the outside prospect, so that dissolving the partnership in case of asymmetric outside options would maximize Alice’s and Bob’s joint surplus. In region (IV) the partner with outside option has to buy himself free under joint asset ownership, which compromises effort incentives in his new partnership. However, in region (III) the outside prospect is not high enough to warrant a buyout. As a consequence the partner without outside option inefficiently retains the other. Obviously, these retention inefficiencies can be avoided with individual asset ownership.

Now consider the most interesting scenario (iii) from Proposition 1. Alice and Bob then choose joint asset ownership if the inside prospect π is likely to be high (p ≥ ˆp), because preserving the partnership is likely to be valuable. Otherwise they choose individual asset ownership in order to retain the flexibility to dissolve a likely inefficient partnership (p < ˆp). Thus, the threshold ˆp balances (i) the risk of preserving inefficient partnerships (joint ownership with π = π_L), and (ii) the risk of compromising otherwise efficient partnerships (individual ownership with π = π_H). Overall we note that for this scenario the ex-ante optimal allocation of control rights can lead to ex-post inefficiencies, namely a displacement inefficiency (individual ownership), associated with regions (I) and (II) in Figure 3, and a retention externality (joint ownership), associated with regions (III) and (IV).

If the internal learning process was based on verifiable signals so that an ex-ante contract can distinguish between π = π_L and π = π_H, then Alice and Bob could write a contingent contract which stipulates individual asset ownership at date 2 whenever π = π_L < ˆπ_V(σ), and joint asset ownership whenever π = π_H ≥ ˆπ_V(σ). However, assuming that π is non-verifiable seems more plausible within the present context, given that learning about the inside prospect is specific to the collaboration of the two partners. It is not based on objective past performance, but instead, is based on expectations about the benefits of a future joint production. Obviously there is also the theoretical possibility of using mechanism design ex-ante, which might allow the two partners to reveal the observable state through a cleverly designed revelation game, in the spirit of Moore and Repullo (1988) or Maskin and Tirole (1999). We do not pursue this approach for several reasons. From an applied perspective, these mechanisms seem somewhat remote from the contracts used in the ‘real world’. From a purely theoretical perspective, the mechanism of Maskin and Tirole relies on risk-aversion and the ability to implement large punishments. In our model we have risk-neutral players. Most importantly, for our main analysis
we focus on binding wealth constraints so that large punishments – which are central to these implementation games – are not feasible. Moreover, the recent work by Aghion et al. (2012) shows that subgame perfect implementation games are not robust to even small deviations from the common knowledge assumption.

For the remainder of this paper we assume that the inside prospect \( \pi \in \{ \pi_L, \pi_H \} \) is observable but not verifiable at date 2, so that the partners cannot rely on contingent contracts. Moreover, we focus on the most interesting scenario where \( \pi_L < \hat{\pi}_V(\sigma) < \pi_H \). This implies that the ex-ante decision over asset ownership involves a trade-off between the flexibility of individual asset ownership versus the commitment value of joint asset ownership.

4.5 Social Efficiency

The analysis so far focuses on maximizing joint efficiency, which is precisely what the two partners try to achieve. However, one may also ask how the outcomes rank in term of social efficiency, when also accounting for the utilities of alternative trading partners. Because of the moral-hazard-in-teams problem at date 3, we note that there will always be deviations from the first-best outcome. Let us therefore accept that any outcome has some inefficient effort incentives, and focus instead on the question of socially efficient partner choices.

For ease of exposition we focus on the case with zero wealth, so that \( (\pi)_I(\sigma, 0) = \sigma \); the case with wealth is similar. In region \((I)\) of Figure 3 the alternative partnership is less efficient, and Alice and Bob prefer joint asset ownership. Partner switching does then not occur in equilibrium, so that the outcome is also socially efficient. In regions \((III)\) and \((IV)\), the alternative partnership is more efficient than the original one, so switching is socially efficient. Recall that Alice and Bob also prefer individual asset ownership in these two regions, which results in partner switching. Thus, total surplus is also maximized in regions \((III)\) and \((IV)\).

The interesting case concerns region \((II)\) where \( \pi \in (\hat{\pi}_V, \sigma) \) for \( w = 0 \). Since \( \pi < \sigma \), the alternative partnership is more efficient than the original one, so that switching is socially efficient. This would occur in equilibrium under individual asset ownership. However, Alice and Bob actually prefer joint asset ownership in this region, and they stay together. In region \((II)\) we therefore find a divergence between the jointly optimal partner choice of staying together, versus the socially efficient choice of switching partners.\(^{23}\)

\(^{23}\)For \( w > 0 \) we know from Lemma 2 that \( \hat{\pi}_I(\sigma, w) < \sigma \). In addition to region \((II)\) in Figure 3, the outcome is then also socially inefficient in part of region \((I)\).
5 Relation-specific Investments

We now augment our model and allow Alice and Bob to make relation-specific investments at date 1. We can then examine how the allocation of control rights over critical assets affects the partners’ incentives to invest in their relationship.

In Sections 4.3.1 and 4.3.2 we have shown how Alice and Bob can use their wealth to obtain a more efficient renegotiation outcome at date 2. The presence of wealth will therefore affect their incentives to make relation-specific investments at date 1. Moreover, both partners are symmetric at date 1, so that in equilibrium they choose the same level of specific investment. We define \( r^*_I(w) \) as the equilibrium relation-specific investment of a partner under individual asset ownership, and \( r^*_J(w) \) as the equilibrium investment under joint asset ownership. We provide a complete characterization of the partners’ expected utilities and the equilibrium investment levels, \( r^*_I(w) \) and \( r^*_J(w) \), in the Appendix; see Proof of Proposition 2.

For now let us assume that Alice and Bob use their entire wealth to mitigate ex-post inefficiencies. For this the next proposition compares the specific investments under individual asset ownership (\( r^*_I(w) \)) and joint asset ownership (\( r^*_J(w) \)) for different wealth levels.

**Proposition 2** For \( w < \max\{w_I, w_J\} \), joint asset ownership provides greater incentives for relation-specific investments, i.e., \( r^*_J(w) > r^*_I(w) \). Moreover,

(i) \( r^*_J(w) \) is decreasing in the partners’ wealth \( w \) for \( w_J \leq w < w_I \), and

(ii) \( r^*_I(w) \) is increasing in the partners’ wealth \( w \) for \( w_I \leq w < w_J \).

For \( w \geq \max\{w_I, w_J\} \), relation-specific investments are identical and constant under individual and joint asset ownership, i.e., \( r^*_I(w) = r^*_J(w) \).

To explain the key intuition for this result, let us first focus on the case with zero wealth \( (w = 0) \), so that Alice and Bob cannot make any ex-post transfers. In general, incentives for specific investments stem from the difference in the utility levels associated with a low versus high inside prospect. With joint asset ownership, the partner combination is always efficient when the inside prospect is high, but leads to inefficient retention when the inside prospect is low. The latter inefficiency of joint asset ownership widens the difference in utilities between a low and a high inside prospect. With individual asset ownership, the partner combination is always efficient when the inside prospect is low, but causes displacement problems when the inside prospect is high. The latter inefficiency of individual asset ownership narrows the difference in utilities between a low and a high inside prospect. We therefore find that joint asset ownership provides stronger incentives for specific investments (\( r^*_J(0) > r^*_I(0) \)), precisely
because the inefficiency then arises when the partners have failed to develop a strong internal relationship.

The next question is what happens to specific investments when Alice and Bob have some initial wealth \( w > 0 \)? Under individual asset ownership, wealth allows them to smooth out ex-post inefficiencies in the good state \( \pi = \pi_H \). This improves the marginal benefit of specific investments, so that \( r^*_I(w) \) is increasing in \( w \). Under joint asset ownership, wealth helps Alice and Bob to correct ex-post inefficiencies in the bad state \( \pi = \pi_L \). This makes the difference between the bad and the good state relatively smaller, and therefore compromises Alice’s and Bob’s incentives to make relation-specific investments. Thus, \( r^*_J(w) \) decreases in \( w \), while \( r^*_I(w) \) increases.

Alice and Bob can eliminate all ex-post inefficiencies for sufficiently high wealth levels \( w \geq \max\{\overline{w}_I, \overline{w}_J\} \). That is, with enough wealth they can always dissolve their partnership in the bad state \( \pi_L \), so that \( V(\pi_L) = U(\sigma) \); and they can always agree on staying together with an equal split of profits in the good state \( \pi_H \), so that \( V(\pi_H) = 2U(\pi) \). The allocation of control rights is then irrelevant, and the marginal incentives for specific investments are the same \( r^*_I(w) = r^*_J(w) \).

Proposition 2 provides another interesting insight: Relation-specific investments are maximized under joint asset ownership with zero wealth. This is an important and surprising result. Under joint asset ownership, wealth allows Alice and Bob to mitigate ex-post inefficiencies in the bad state \( \pi = \pi_L \). However, this compromises the marginal benefit of specific investments at date 1. Thus, Alice and Bob make the highest specific investments under joint asset ownership when they have no wealth at all.

6 Specific Investments and Optimal Asset Ownership

We can now complete our model and identify the optimal ownership structure for Alice and Bob, accounting for their specific investments and potential ex-post transfers. For this we proceed in two steps. First we characterize the expected utility for each partner under individual and joint asset ownership for different wealth levels.\(^{24}\) We then compare these expected utilities, and identify the optimal allocation of control rights for Alice and Bob at date 0. For now we assume again that Alice and Bob use their entire wealth to mitigate ex-post inefficiencies.

The next lemma characterizes Alice’s and Bob’s expected utilities for different levels of wealth, when they choose individual asset ownership at date 0.

\(^{24}\)The expected utility is obviously increasing in wealth itself, so we focus on the expected utility from the productive activities, net of initial wealth. This expected utility still depends on wealth, since wealth affects both incentives and ex-post payoffs (case of asymmetric outside options).
Lemma 4 Under individual asset ownership, the expected utility of a partner at date 0, denoted by $EU_I(w)$, has three distinct segments:

(i) For $w < w_I$, $EU_I(w)$ is constant in $w$.

(ii) For $w_I \leq w < w_I$, $EU_I(w)$ is strictly increasing in $w$.

(iii) For $w \geq w_I$, $EU_I(w)$ is constant in $w$.

The previous sections identified two distinct facets of wealth. On the one hand, having wealth allows Alice and Bob to mitigate potential inefficiencies arising from asymmetric outside options; and doing so is always optimal ex-post. On the other hand, having wealth affects their incentives for relation-specific investments. Under individual asset ownership the ex-post efficiency effect of wealth and the incentive effect of wealth both go in the same direction: More wealth improves the renegotiation outcome at date 2; it also improves ex-post incentives for specific investments at date 1 because the inefficiencies are associated with a high inside prospect. The expected utility $EU_I(w)$ is therefore increasing in wealth $w$ in the range $w \in [w_I, \bar{w}_I)$, and constant everywhere else.

We now turn to joint asset ownership. For the next lemma we define $w_J^*$ as the wealth level which maximizes the expected utility of a partner under joint ownership at date 0.

Lemma 5 Under joint asset ownership, the expected utility of a partner at date 0, denoted by $EU_J(w)$, has the following properties:

(i) For $w < w_J$, $EU_J(w)$ is constant in $w$.

(ii) For $w_J \leq w < w_J$, there exists a threshold $\hat{\pi}_H$ such that $w_J^* = w_J$ for all $\pi_H \geq \hat{\pi}_H$, and $w_J^* \in (w_J, w_J)$ for all $\pi_H < \hat{\pi}_H$.

If $w_J^* = w_J$, then $EU_J(w)$ is strictly decreasing in $w$ for $w \in [w_J, \bar{w}_J)$.

If $w_J^* > w_J$, then $EU_J(w)$ is strictly increasing in $w$ for $w \in [w_J, w_J^*)$, and strictly decreasing in $w$ for $w \in (w_J^*, \bar{w}_J)$.

(iii) For $w \geq \bar{w}_J$, $EU_J(w)$ is constant in $w$.

Lemma 5 shows that a partner’s expected utility under joint asset ownership is not necessarily monotone in wealth. This is because wealth has two opposite effects: It allows Alice and Bob in case of asymmetric outside options to improve their ex-post payoffs in the bad state $\pi = \pi_L$. However, this concurrently compromises Alice’s and Bob’s incentives to invest in their relationship (see Proposition 2). Which effect dominates then depends on the importance
of relation-specific investments, as reflected by the parameter $\pi_H$. For sufficiently high values of $\pi_H$ ($\pi_H \geq \hat{\pi}_H$), the incentive effect always dominates. In this case the expected utility $EU_J(w)$ is decreasing in $w$, and has its maximum at zero wealth.\footnote{If $\overline{w}_J > 0$, there is a range $[0, \overline{w}_J]$ where $EU_J(w)$ is maximized.} For lower values of $\pi_H$ ($\pi_H < \hat{\pi}_H$), the incentive effect does not always dominate. In the Appendix we show that the expected utility $EU_J(w)$ then first increases in wealth, and then decreases.

We can now derive a condition so that Alice and Bob prefer joint to individual asset ownership at date 0. For parsimony we define $\overline{w} \equiv \max\{\overline{w}_I, \overline{w}_J\}$.

**Proposition 3** There always exists a critical wealth level $w_0$, with $w_0 \in [0, \overline{w}_J^*]$, such that the partners strictly prefer joint asset ownership for all $w \in (w_0, \overline{w})$.}

Figure 4 compares the expected utility levels under individual versus joint asset ownership, using the insights from Lemmas 4 and 5, and Proposition 3. If Alice and Bob have sufficient wealth ($w \geq \overline{w}$), they can eliminate all ex-post inefficiencies in case of asymmetric outside options. The specific ownership structure is then irrelevant (i.e., $EU_I(w) = EU_J(w)$ for $w \geq \overline{w}$). For $w < \overline{w}$, however, there is a divergence between individual and joint asset ownership. In fact, there always exists a region where joint asset ownership is preferred to individual asset ownership. This region extends all the way down to $w_0$. In some cases we have $w_0 = 0$, so that joint asset ownership is optimal for all levels of wealth; see the left graph of Figure 4 where $\pi_H \leq \hat{\pi}_H$. In other cases we have $w_0 > 0$, so that joint asset ownership is only optimal for intermediate levels of wealth; see the right graph of Figure 4 where $\pi_H > \hat{\pi}_H$. All this implies that the two partners, Alice and Bob, only choose individual asset ownership at date 0 when (i) relation-specific investments are not very important for their partnership ($\pi_H \leq \hat{\pi}_H$), and (ii)
they have little or no initial wealth \((w \leq w_0)\). Otherwise they always have a (weak) preference for joint asset ownership.

We can see from Figure 4 that the partners’ expected utilities, and therefore the optimal ex-ante allocation of control rights over their assets, depend on how much wealth they have available for ex-post transfers. An interesting question is what wealthy partners would do if they can commit to limiting the amounts that can be used for ex-post transfer payments? We get the following corollary, which immediately follows from the above, and can be seen directly off Figure 4.

**Corollary 1** If wealthy partners can commit to limiting the wealth available for ex-post transfer payments, then they always choose joint asset ownership and commit to being wealth constrained at \(w = w^*_J\).

The maximum of the expected utilities, \(EU^{\max} = \max\{EU_I(w), EU_J(w)\}\), is always reached at \(EU_J(w^*_J)\). This implies that the combination of joint asset ownership and a wealth constraint at \(w^*_J\) achieves the best trade-off between ex-ante incentives for specific investments and ex-post efficiency. Interestingly, in the case of \(\pi_H \geq \hat{\pi}_H\), we even have \(w^*_J = 0\). The optimal arrangement for wealthy partners is then joint asset ownership with the commitment to a zero wealth constraint ex-post.

### 7 Robustness

#### 7.1 Other Ex-Post Constraints

For our main model we derive ex-post inefficiencies on the basis of binding wealth constraints. In this section we briefly sketch a model where partners do not face wealth constraints and cannot commit to limiting their wealth, but where any ex-post transfer payments are costly. We show that our central trade-off between displacement and extension externalities continue to hold.

Suppose that for any transfer of wealth, \(T\), there is a cost \(\tau T\). That is, if one partner pays \(T\), the other only gets \((1 - \tau)T\). The simplest interpretation is that transfer payments are taxed at a rate \(\tau\). One can also think of financial intermediation costs, or transaction costs more broadly. For simplicity we focus on costs that are linear in \(T\), but it is easy to extend our analysis to the case of non-linear costs (including fixed costs).

In our model without specific investments, the ownership decision at date 0 is solely driven by the ex-post inefficiencies associated with asymmetric outside options. We therefore focus on the asymmetric case, and assume that Alice found an alternative partner, Charles, at date
Figure 5: Utility-possibility Frontier for Joint Ownership with Ex-post Transfers

2. Consider the case of individual asset ownership (the case of joint asset ownership follows a similar logic). If Alice leaves Bob, then she gets the utility $U(\sigma)$ from her new relationship with Charles. If Alice stays and renegotiates with Bob, she gets some shares $\alpha(\tau)$, as well as a transfer payment $T_{BA}(\tau)$ from Bob. We are interested in the cutoff level $\hat{\pi}_I(\tau, \sigma)$, above which it is possible for Alice and Bob to come to an agreement, and Alice therefore stays.

Figure 5 provides a graphical analysis for how $\hat{\pi}_I(\tau, \sigma)$ depends on $\tau$. In the renegotiation, Alice is asking Bob for a more favorable deal than the status quo of $\alpha = \beta = 1/2$. We can see from Figure 5 how costly transfer payments affect the utility frontier between Alice and Bob. Giving Alice a higher utility can initially be done by increasing her profit share up to some $\alpha^*(\tau) > 1/2$, where the marginal rate of substitution satisfies $dU_A/dU_B = -(1 - \tau)$. Beyond this point is would be inefficient to offer Alice an even higher profit share, since the incentive costs exceed the costs of making transfer payments. Bob then prefers to transfer more utility to Alice through costly transfer payments, so that the utility frontier has a slope of $-(1 - \tau)$. The maximum transfer $T_{BA}^{\text{max}}$ that Bob is willing to offer, satisfies $U_B(1 - \alpha^*(\tau); \pi) - T_{BA}^{\text{max}} = 0$. This gives Alice the utility $U_A(\alpha^*(\tau); \pi) + (1 - \tau)T_{BA}^{\text{max}}$, or equivalently, $U_A(\alpha^*(\tau); \pi) + (1 - \tau)U_B(1 - \alpha^*(\tau); \pi)$. The cutoff $\hat{\pi}_I(\tau, \sigma)$, above which it is possible for Alice and Bob to reach an agreement, therefore satisfies

$$U_A(\sigma) = U_A(\alpha^*(\tau); \pi) + (1 - \tau)U_B(1 - \alpha^*(\tau); \pi).$$
For τ = 1 the model reverts back to our main model with perfect wealth constraints. The highest utility that Alice can get from staying with Bob is then given by $U_A(\alpha^{\text{max}}; \pi)$, and the cutoff $\hat{\pi}_I(\tau = 1, \sigma)$ satisfies $U_A(\sigma) = U_A(\alpha^{\text{max}}; \pi)$ as before. This point is represented by $\pi_1$ in Figure 5.

For τ = 0 we have a model with perfectly efficient ex-post wealth transfers. The highest utility that Alice can get from staying with Bob is then given by $U_A(\alpha^*(\tau); \pi) + U_B(1 - \alpha^*(\tau); \pi)$, where $\alpha^*(\tau = 0) = 1/2$. And the cutoff $\hat{\pi}_I(\tau = 0, \sigma)$ satisfies $U_A(\sigma) = V(\alpha = 1/2; \pi)$; the cutoff is thus equivalent to the efficient level, i.e., $\hat{\pi}_I(\tau = 0, \sigma) = \hat{\pi}_V(\sigma)$. This point is represented by $\pi_3$ in Figure 5.

The most important insight is that for any $\tau \in (0, 1)$, the maximum utility that Alice can get from Bob lies somewhere between the two extremes $\pi_1$ and $\pi_3$. In Figure 5 we denote this point by $\pi_2$. Relative to the model with perfect wealth constraints, the range where renegotiation succeeds is increased from $\pi_1$ to $\pi_2$. However, relative to the model with perfect wealth transfers, there is a region between $\pi_2$ and $\pi_3$ where the joint utility would be maximized if Alice stayed with Bob, but transfer costs prevent renegotiation so that Alice leaves. Put differently, as long as transfer payments are costly, there is a region $\pi \in (\pi_2, \pi_3)$ where inefficient displacement occurs. Moreover, even though displacement does not occur for $\pi > \pi_2$, there are still some inefficiencies whenever $\tau > 0$. This is because the profit share for Alice remains above the efficient level $\alpha = 1/2$, and because Bob may need to make some costly transfers in equilibrium.

Overall we note that even a small cost of making ex-post transfers is enough to generate the trade-off between displacement externalities (individual ownership) and retention externalities (joint ownership). Thus, while we consider binding wealth constraints a natural choice for the base model, our key insights do not depend on this assumption. In fact, the trade-off between the benefit of retaining flexibility under individual asset ownership versus the benefit of commitment under joint asset ownership, carries over to a model with unlimited wealth but costly transfer payments.

### 7.2 Alternative Ownership Structures

So far we focused on individual and joint asset ownership as the only possible ownership structures. We now briefly explain why we can safely ignore all other ownership structures.

The main alternative ownership structure is full asset ownership in the hands of one of the two partners. This ownership structure plays a large role in the property rights literature. With ex-ante symmetric partners, it does not matter which partner owns both assets; w.l.o.g. we assume it is Alice. It is easy to verify that whenever Alice finds an alternative partner and
Bob does not, then the model behaves just like under individual asset ownership. And if Bob finds an alternative partner and Alice does not, then the model behaves just like under joint asset ownership. From Proposition 1 we know that either individual or joint asset ownership is optimal; thus, mixing individual and joint ownership is never optimal.

In fact, asymmetric asset ownership creates additional inefficiencies when Alice controls the two assets. If both Alice and Bob find alternative partners, then Alice can hold up Bob before releasing his asset. Thus, Bob would have to share some of the profits from his new partnership with Dora, which is clearly inefficient. At the ex-ante stage, Bob would also not relinquish his asset for free. In fact, Alice would have to give Bob a larger profit share ex-ante, which would lead to further inefficiencies. Asymmetric ownership is therefore never optimal within the present context.

From Proposition 1 it is immediate that randomization among the symmetric ownership structures is suboptimal (except for the special case of \( p = \hat{p} \) where it is a matter of indifference). Giving ownership to outsiders is not optimal in our model either, since owner-managers want to retain maximal effort incentives. Moreover, the type of outside ownership suggested by Gans (2005) is not an equilibrium, because we allow asset owners to coordinate on joint asset ownership. Finally, because there are no sequential investments in our model, there is no role for options on ownership as in Nöldeke and Schmidt (1998).

### 7.3 Joint Ownership versus Long-term Contracts

Can long-term contracts be used instead of joint asset ownership? We now show that our main trade-off between individual versus joint asset ownership can also be recast as a trade-off between short-term versus long-term contracts.

Suppose that Alice and Bob retain individual asset ownership at date 0, but commit to a contract that specifies the price at which they transact at date 4. If the contract is only structured as an option without commitment, then it obviously has no effect at all. However, if the contract is binding, then the two partners face a very similar renegotiation game as under joint asset ownership: If they want to switch partners, they cannot do so without the consent of their original trading partner.

The only difference between a long-term contract and joint asset ownership concerns the division of surplus. Under joint asset ownership, Alice and Bob each get a constant fraction of the profits, as defined by \( \alpha \) and \( \beta \). With a long-term contract the partners agree on a pre-specified price (or pricing formula) that determines the division of surplus. What matters for the model is not the actual distribution at date 4, but the expected distribution at dates 2 and 3. Consider the easiest example where a seller offers a single good to a buyer. Let \( \overline{v} \) be the
value of the good for the buyer at date 4, and $\tilde{c}$ be the cost of the seller. The joint profit is then given by $y = \tilde{v} - \tilde{c}$. We conveniently denote the joint distribution of $\tilde{v}$ and $\tilde{c}$ by $\Omega_{vc}(\tilde{v}, \tilde{c})$, so that

$$\int y d\Omega_{in}(y) = \int (\tilde{v} - \tilde{c}) d\Omega_{vc}(\tilde{v}, \tilde{c}).$$

Let $\lambda$ be the transfer price specified in the long-term contract. This price can be made contingent on verifiable information, in this case on the realizations of $\tilde{v}$ and $\tilde{c}$ at date 4.

With a constant inside prospect $\pi$, it is easy to see that for every $\alpha$ there exists a unique transfer price $\lambda$ that leads to the same expected utilities at date 2 as joint asset ownership. Alternatively, it is possible to define a flexible pricing schedule that actually achieves identical utilities at date 4. The flexible pricing schedule must then satisfy $\alpha y = \tilde{v} - \lambda$, which can always be achieved with $\lambda = (1 - \alpha)\tilde{v} + \alpha\tilde{c}$. By construction this pricing schedule continues to implement the same outcomes as joint asset ownership, even when the inside prospect is uncertain. We thus find that long-term contracts can always be structured such that they generate the same ex-ante utilities as joint asset ownership. Our model is therefore consistent with an interpretation of the integration decision as either joint asset ownership or long-term contracts.

### 7.4 Asymmetrically Wealthy Partners

In our base model we focused on two partners with identical wealth levels. Naturally one may ask whether our key trade-off between displacement and retention externalities remains intact when allowing for asymmetrically wealthy partners. For example, one of the partners may be a wealth-constrained entrepreneur, the other an established corporation with large cash reserves.

Suppose that Alice is fully wealth constrained but Bob faces no such constraints. Consider individual asset ownership with a high inside prospect ($\pi_H$). If only Alice finds an outside partner, she may want to leave. This causes a displacement externality. Now if Bob has wealth, he can make a transfer payment that convinces Alice to stay, making both parties better off. Unfortunately this solution only works in one of the two asymmetric scenarios. If only Bob finds an outside partner, he may want to leave. Alice does not have the wealth to retain Bob, and thus the displacement externality arises again in equilibrium. A similar argument applies to joint asset ownership with a low inside prospect ($\pi_L$). If only Bob finds an outside partner, he normally is affected by the retention externality. If Bob has wealth, he can make a transfer payment to buy out Alice. However, if only Alice finds an outside partner, she cannot buy herself free, and the retention externality arises again in equilibrium. Overall we see that with asymmetrically wealthy partners the same inefficiencies occur, only less frequently. All that

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26 Specifically, we have $\alpha \int (\tilde{v} - \tilde{c}) d\Omega_{vc}(\tilde{v}, \tilde{c}) = \int (\tilde{v} - \lambda) d\Omega_{vc}(\tilde{v}, \tilde{c})$, or equivalently, $\lambda = \int \tilde{v} d\Omega_{vc}(\tilde{v}, \tilde{c}) - \alpha \int (\tilde{v} - \tilde{c}) d\Omega_{vc}(\tilde{v}, \tilde{c})$. 

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Figure 6: Preferences with Ex-ante Asymmetric Outside Options

is needed is that at least one of the partners has insufficient wealth to completely eliminate potential displacement and retention externalities.

### 7.5 Ex-ante Asymmetric Outside Options

In our model both partners, Alice and Bob, are perfectly symmetric. Importantly, they have the same prospects of finding an alternative partner at date 2. In this section we briefly discuss how asymmetric expectations affect the partners’ assets ownership decisions. For this we use a simplified version of our model without specific investments, where Alice and Bob each get a utility $\pi/2$ when staying together, and $\sigma/2$ when matching with alternative partners.\(^{27}\) We also assume that Alice will find an alternative partner, Charles, with probability $q_A$, while Bob will find Dora with probability $q_B(\neq q_B)$.

In the Appendix we show that Alice prefers joint asset ownership if

$$q_A < \tilde{q}_A = \left[ \left( \frac{1 - q_B}{q_B} \right) \left( \frac{\sigma - \pi}{\pi} \right) + 1 \right]^{-1}.$$  

Likewise, Bob favors joint ownership if $q_B < \tilde{q}_B$, where $\tilde{q}_B$ is symmetric to $\tilde{q}_A$. The threshold $\tilde{q}_A (\tilde{q}_B)$ is increasing and convex in $q_B (q_A)$ when $\pi < \sigma/2$. When $\pi > \sigma/2$, it is increasing and concave in $q_B (q_A)$

Figure 6 illustrates Alice’s and Bob’s preferences for joint asset ownership, for different values of $q_A$ (y-axis) and $q_B$ (x-axis). Consider first the left graph where $\pi < \sigma/2$, i.e., where

\(^{27}\)Allowing for ex-ante asymmetric outside options in our main model would significantly complicate the analysis without adding significant additional insights. Asymmetries would lead to an ex-ante unequal split of surplus, where the partner with the better expected outside option obtains more than half of the surplus. This in turn would complicate the renegotiation outcome at date 2. A complete analysis of this is beyond the scope of this paper.
leaving is efficient from a joint perspective. In region $(III)$, Alice is sufficiently unlikely to find an alternative partner, and therefore prefers the protection of joint asset ownership ($q_A < \hat{q}_A$). Likewise, Bob only prefers joint asset ownership in region $(I)$ ($q_B < \hat{q}_B$). We can see that for $\pi < \sigma/2$ the two partners never agree on sharing control rights. Thus, individual asset ownership is the equilibrium outcome for all $q_A, q_B \in (0, 1)$.

The right graph illustrates Alice’s and Bob’s preferences when $\pi > \sigma/2$, i.e., when staying together maximizes their joint surplus. Again, each partner prefers joint asset ownership only when he is sufficiently unlikely to find an alternative match. For Alice this happens in regions $(II)$ and $(III)$, for Bob in regions $(I)$ and $(II)$. In region $(II)$ Alice and Bob both prefer to share control over their assets. Joint asset ownership therefore requires that the two partners are not too dissimilar in terms of their outside prospects.

### 8 Empirical Implications

Our model allows us to derive some new empirical predictions about the boundaries of the firm. Most of the theoretical work has focused on comparative statics, and consequently most of the empirical work has focused on cross-sectional determinants of the integration decision. Our theory suggests an empirical research agenda about the time-series properties of integration.

For a given level of asset specificity, our model predicts that dynamic partner switching is more common under individual than under joint asset ownership. The same applies to short-term versus long-term contracting. It seems highly intuitive that switching to an outside supplier or outside buyer is more rare in a vertically integrated setting, so we would be surprised if this prediction was not found in the data. The interesting point here is actually that this simple prediction cannot be obtained from theories with ex-post efficiency, such as the property rights theories. This is because in those models all efficient partner changes always occur in equilibrium, irrespective of asset ownership.

Our model predicts that non-integration (or short-term contracting) is more common in environments with considerable uncertainty about what the best trading partners are. Previous empirical work typically focused on general measures of uncertainty, often with mixed results (see Lafontaine and Slade, 2007). We contend that these measures fail to distinguish between uncertainty about production and demand, versus uncertainty about partner choices. Moreover, our theory suggests that the dynamics of partner choices also depends on the internal match quality, i.e., the interim learning about how well partners are suited for each other.
9 Conclusion

This paper develops a new theory of the firm, based on how partners dynamically switch in and out of relationships. The model identifies a fundamental trade-off between two ex-post inefficiencies. Under non-integration (i.e., individual asset ownership) there can be a displacement externality, where a partner may leave, even though the benefit is worth less than the loss to the displaced partner. Under integration (i.e., joint asset ownership) there can be a retention externality, where a partner may hold on to the other, even though the benefit to the departing partner would exceed the loss to the remaining partner. These inefficiencies arise endogenously in our model with wealth-constrained agents and team incentive problems, and are robust to renegotiations. Optimal firm boundaries are determined by the relative importance of displacement and retention externalities: The greater the asset specificity in a given relationship, the greater the displacement and the smaller the retention externality, and hence the more attractive is integration.

Our base model does not rely on relation-specific investments; but when added to the model, we find that joint asset ownership always provides stronger incentives for specific investments. Moreover, we find that wealth has two distinct effects. Ex-post, wealth mitigates the displacement and retention externalities. However, ex-ante wealth reduces incentives for specific investments. A surprising result is that wealthy owners would always want to commit ex-ante to limiting ex-post transfer payments.

Our model draws on insights from several of the leading theories of the firm, including transaction costs, property rights, and incentive-based theories. However, it differs in terms of focusing on the dynamics of how partners change relationships. This allows us to provide a fresh perspective on some of the existing explanations for integration. In particular, our model generates the traditional prediction from transaction cost economics that asset specificity favors integration. However, while the verbal arguments of Williamson typically require different explanations for the advantages of integration and non-integration, we provide a unified framework where one underlying force generates the trade-off. Moreover, while transaction cost economics typically relies on problems of ex-post opportunism and price haggling, we focus instead on the problems of displacement and retention as the main sources of ex-post inefficiencies. This allows us to go directly to the core of asset specificity, namely that the primary concern of the owner of a specific asset is to be not displaced by another.

Our analysis suggests avenues for further theoretical work. We focus on team incentives and wealth constraints as source of ex-post inefficiencies; but there may be other sources of inefficiencies, such as asymmetric information (Aghion et al., 2012). Future research may examine how alternative ex-post inefficiencies affect the dynamic properties of firm boundaries.
Moreover, in this paper we chose the simplest possible dynamic specification where partners have at most one opportunity to switch partners. A worthwhile future research agenda is to extend the model to an infinite horizon. This would allow for a more comprehensive analysis of how asset ownership affects the timing and frequency of partner changes. Such a model might also look at the process of wealth accumulation, and at how partners dynamically trade-off ex-post benefits versus ex-ante costs of having wealth. Overall we believe that looking at the dynamics of asset ownership constitutes a new and promising direction in the theory of the firm.
Appendix

Proof of Lemma 1.

The optimal effort levels, denoted \( e_A^*(\alpha) \) and \( e_B^*(\beta) \), are characterized by the first-order conditions

\[
\alpha \mu'(e_A e_B) e_B \pi = c'(e_A) \tag{3}
\]

\[
\beta \mu'(e_A e_B) e_A \pi = c'(e_B). \tag{4}
\]

Using \( \beta = 1 - \alpha \), the joint surplus \( V \equiv U_A + U_B \) is given by

\[
V(\alpha; \pi) = \mu(e_A^*(\alpha) e_B^*(\alpha)) \pi - c(e_A^*(\alpha)) - c(e_B^*(\alpha)).
\]

The jointly optimal profit share \( \alpha^J \) satisfies the first-order condition

\[
\pi \mu'(e_A^* e_B^*) \left[ \frac{d e_A^*}{d \alpha} e_B^* + \frac{d e_B^*}{d \alpha} e_A^* \right] = c'(e_A^*) \frac{d e_A^*}{d \alpha} + c'(e_B^*) \frac{d e_B^*}{d \alpha}. \tag{5}
\]

Symmetry implies \( \frac{d e_A^*}{d \alpha} = -\frac{d e_B^*}{d \alpha} \) and \( e_A^* = e_B^* \) at \( \alpha = 1/2 \). Thus, (5) is satisfied for \( \alpha = \beta = 1/2 \). The solution \( \alpha^J = \beta^J = 1/2 \) is also unique due to the convexity of \( c(e_i), \ i = A, B \). Thus,

\[
\left. \frac{dU_A}{d\alpha} \right|_{\alpha=1/2} = - \left. \frac{dU_B}{d\alpha} \right|_{\alpha=1/2} > 0.
\]

Moreover, \( e_A^*(0) = e_B^*(1) = 0 \). This implies \( 1/2 < \alpha^{max} = \beta^{max} < 1 \).

Now consider the bargaining at date 0, and suppose that Alice gets the profit share \( \alpha > 1/2 \) with probability \( 1/2 \), and \( 1 - \alpha < 1/2 \) otherwise. Alice’s expected utility at date 0 is then given by \( U_A(\alpha; \pi)/2 + U_A(1 - \alpha; \pi)/2 \). However, from the above we note that her expected utility is maximized when \( \alpha = 1/2 \) (symmetric for Bob). Thus, at date 0 both partners agree on splitting the expected joint surplus in half: \( \alpha = \beta = 1/2 \). \( \square \)

Renegotiation under Individual Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. Consider first the case without wealth \( (w = 0) \). Alice is indifferent between staying (and renegotiating her profit share \( \alpha \)) and leaving, if

\[
U_A(\alpha^*_I; \pi) = U_A(\hat{\alpha}_I; \sigma). \tag{6}
\]

Recall that Bob and Charles both have zero outside options. The bargaining protocol à la Hart and Mas-Colell (1996) then implies that (6) is satisfied for \( \pi = \sigma \). Thus, \( \sigma \leq \pi \) implies \( U_A(\alpha^*_I; \pi) \geq U_A(\hat{\alpha}_I; \sigma) \). For \( \sigma > \pi \) we have \( U_A(\alpha^*_I; \pi) < U_A(\hat{\alpha}_I; \sigma) \). We define \( \tilde{\pi}_I(\sigma) = \sigma \) as the threshold below which Alice is better off leaving Bob \( (\pi < \tilde{\pi}_I(\sigma)) \).
Now suppose that Alice and Bob have each initial wealth \( w > 0 \). Let \( V_I(\pi, w) \equiv U_A(\alpha_I^*; \pi, w) + U_B(\beta_I^*; \pi, w) \) denote Alice’s and Bob’s joint surplus under individual asset ownership when staying together. Recall from Lemma 1 that their joint surplus is maximized in case of joint production when \( \alpha_I^* = \beta_I^* = 1/2 \). The joint surplus is then given by \( 2U(\pi) \). Thus, the minimum value of wealth \( w \) required to eliminate displacement externalities under individual asset ownership, denoted \( w_I \), satisfies \( V_I(\pi, w) = 2U(\pi) \). Next we characterize the minimum amount of wealth \( w_I \), which changes the renegotiation outcome. For \( \pi \geq \hat{\pi}_I(\sigma) \) we know that Alice stays with Bob, but profit shares are unbalanced. Bob can then use even small amounts of wealth to buy back some profit shares from Alice, which improves their joint surplus. Thus, \( w_I = 0 \) for \( \pi \geq \hat{\pi}_I(\sigma) \). For \( \pi < \hat{\pi}_I(\sigma) \), Alice leaves the partnership with Bob. For \( w \to 0 \) Bob cannot retain Alice, and therefore cannot change the renegotiation outcome. Thus, \( w_I > 0 \), where \( w_I \) satisfies \( U_A(\alpha_I^*; \pi, w) = U(\hat{\alpha}_I, \sigma) \).

Proof of Lemma 2.

It follows directly from our previous derivations (see Section "Renegotiation under Individual Asset Ownership" in the Appendix) that the threshold \( \hat{\pi}_I(\sigma, w) \) is defined by

\[
U_A(\alpha_I^*; \pi, w) = U_A(\hat{\alpha}_I; \sigma).
\]

Using (7) we can implicitly differentiate \( \hat{\pi}_I(\sigma, w) \) w.r.t. \( w \):

\[
\frac{d\hat{\pi}_I(\sigma, w)}{dw} = -\frac{dU_A(\alpha_I^*; \pi, w)}{dw} \cdot \frac{dU_A(\alpha_I^*; \pi, w)}{d\pi}.
\]

Recall that \( dU_A(\alpha_I^*; \pi, w)/dw > 0 \) for \( w_I \leq w < w_I \). Moreover, applying the Envelope Theorem we find that \( dU_A(\alpha_I^*; \pi, w)/d\pi > 0 \). Thus, \( d\hat{\pi}_I(\sigma, w)/dw < 0 \) for \( w_I \leq w < w_I \). \( \square \)

Profit Shares under Joint Ownership with Asymmetric Outside Options.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. We denote a coalition by \( S \), with \( S \subset 3 \). Let \( \kappa = (\kappa_A, \kappa_B, \kappa_C) \) be a vector which measures the rate at which utility can be transferred. Moreover, \( \eta_T \in V(T) \) denotes the payoff vector for the subcoalition \( T \).

According to Hart (2004), the Maschler-Owen consistent NTU value can be derived by the following procedure: First, for all \( i \in S \), let the payoff vector \( z \in \mathbb{R}^S \) satisfy

\[
\kappa_i z_i = \frac{1}{|S|} \left[ v_\kappa(S) - \sum_{j \in S \setminus i} \kappa_j \eta_{S \setminus i}(j) + \sum_{j \in S \setminus i} \kappa_i \eta_{S \setminus i}(j) \right],
\]

where the maximum possible value \( v_\kappa(S) \) is defined by

\[
v_\kappa(S) = \sup \left\{ \sum_{i \in S} \kappa_i U_i : (U_i)_{i \in S} \in V(S) \right\}.
\]
Second, if $z$ is feasible, then the payoff vector is given by $\eta_S = z$.

The coalition functions for our setting are as follows:

$V_{\{A\}} = V_{\{B\}} = V_{\{C\}} = 0$

$V_{\{A,B\}} = \{(U_A(\alpha;\pi), U_B(\beta;\pi)) \in \mathbb{R}^{\{A,B\}} : \alpha + \beta \leq 1; \alpha, \beta \geq 0\}$

$V_{\{A,C\}} = \{0, 0\}$

$V_{\{B,C\}} = \{0, 0\}$

$V_{\{A,B,C\}} = \{(U_A(\alpha;\sigma), U_B(\beta;\sigma), U_C(\gamma;\sigma)) \in \mathbb{R}^{\{A,B,C\}} : \alpha + \beta + \gamma \leq 1; \alpha, \beta, \gamma \geq 0\}$

where $V_{\{A,C\}} = \{0, 0\}$ follows from the fact that Alice cannot leave without Bob’s consent under joint ownership. Note that the bargaining outcome must satisfy $\alpha^* \in (0, \alpha_{\text{max}})$ and $\beta^* \in (0, \beta_{\text{max}})$ for the Alice-Bob coalition, and $\alpha^* \in (0, \alpha_{\text{max}})$, $\beta^* \in (0, \beta_{\text{max}})$, and $\gamma^* \in (0, \gamma_{\text{max}})$ for the grand coalition (Alice, Bob, and Charles). Thus, $dU_A/d\alpha > 0$, $dU_B/d\beta > 0$, and $dU_C/d\gamma > 0$ for the relevant values of $\alpha$, $\beta$, and $\gamma$. This implies that the inverse of each utility function exists. We define $\alpha(U_A) \equiv U_A^{-1}(\alpha)$, $\beta(U_B) \equiv U_B^{-1}(\beta)$, and $\gamma(U_C) \equiv U_C^{-1}(\gamma)$. Pareto efficiency then requires

$$\alpha(U_A) + \beta(U_B) = 1 \quad \text{for } V_{\{A,B\}}$$

$$\alpha(U_A) + \beta(U_B) + \gamma(U_C) = 1 \quad \text{for } V_{\{A,B,C\}}.$$ 

The payoffs for the single-player coalitions are given by $\eta_1(A) = \eta_1(B) = \eta_1(C) = 0$. For the two-player coalitions, the equilibrium payoffs satisfy the Nash bargaining solution. Due to symmetry, the payoffs are given by

$$\eta_2(A, B) = (U(\pi), U(\pi))$$

$$\eta_2(A, C) = (0, 0)$$

$$\eta_2(B, C) = (0, 0).$$

It remains to derive the payoff vector $\eta_3(A, B, C)$ for the hyperplane game. For a vector $z = (z_A, z_B, z_C)$ the equation of the hyperplane is

$$\alpha'(U_A)z_A + \beta'(U_B)z_B + \gamma'(U_C)z_C = r,$$ (8)

where

$$r = \alpha'(U_A)U_A + \beta'(U_B)U_B + \gamma'(U_C)U_C.$$ (9)
Using the payoffs for the two-player coalitions, we can now define the equilibrium payoffs for the grand coalition:

\[ \eta_3(A) = U_A(\alpha; \sigma) = \frac{1}{3} [U(\pi) + z_A] \]

\[ \eta_3(B) = U_B(\beta; \sigma) = \frac{1}{3} [U(\pi) + z_B] \]

\[ \eta_3(C) = U_C(\gamma; \sigma) = \frac{1}{3} z_C \]

where, using (9),

\[ z_A = \frac{1}{\alpha'(U_A)} \left[ r - \beta'(U_B) \cdot 0 - \gamma'(U_C) \cdot 0 \right] = \frac{r}{\alpha'(U_A)} \]

\[ z_B = \frac{1}{\beta'(U_B)} \left[ r - \alpha'(U_A) \cdot 0 - \gamma'(U_C) \cdot 0 \right] = \frac{r}{\beta'(U_B)} \]

\[ z_C = \frac{1}{\gamma'(U_C)} \left[ r - \alpha'(U_A)U(\pi) - \beta'(U_B)U(\pi) \right]. \]

Using the Inverse Function Theorem we get \( \alpha'(U_A) = (dU_A/d\alpha)^{-1} \), \( \beta'(U_B) = (dU_B/d\beta)^{-1} \), and \( \gamma'(U_C) = (dU_C/d\gamma)^{-1} \). The equations for the fixed point for the grand coalition are thus given by

\[ U_A(\alpha; \sigma) = \frac{1}{3} \left[ U(\pi) + r \frac{dU_A(\alpha; \sigma)}{d\alpha} \right] \] \hspace{1cm} (10)

\[ U_B(\beta; \sigma) = \frac{1}{3} \left[ U(\pi) + r \frac{dU_B(\beta; \sigma)}{d\beta} \right] \] \hspace{1cm} (11)

\[ U_C(\gamma; \sigma) = \frac{1}{3} \frac{dU_C(\gamma; \sigma)}{d\gamma} \left[ r - U(\pi) \left[ \left( \frac{dU_A(\alpha; \sigma)}{d\alpha} \right)^{-1} + \left( \frac{dU_B(\beta; \sigma)}{d\beta} \right)^{-1} \right] \right], \] \hspace{1cm} (12)

where, using (9),

\[ r = U_A(\alpha; \sigma) \left( \frac{dU_A(\alpha; \sigma)}{d\alpha} \right)^{-1} + U_B(\beta; \sigma) \left( \frac{dU_B(\beta; \sigma)}{d\beta} \right)^{-1} + U_C(\gamma; \sigma) \left( \frac{dU_C(\gamma; \sigma)}{d\gamma} \right)^{-1}. \]

The equilibrium payoff vector \( \eta_3(A, B, C) = (\widehat{U}_A(\alpha; \sigma), \widehat{U}_B(\beta; \sigma), \widehat{U}_C(\gamma; \sigma)) \) thus satisfies the system of three equations, (10), (11), and (12), which also defines the equilibrium profit shares \( \widehat{\alpha}_J, \widehat{\beta}_J, \) and \( \widehat{\gamma}_J. \)
Renegotiation under Joint Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner (Charles). We first consider the case without wealth \((w = 0)\). Alice will then stay with Bob under joint asset ownership with an equal split of profits if
\[
U_A(\pi) \geq U_A(\hat{\alpha}_J; \sigma). \tag{13}
\]

Note that (13) is never satisfied when \(\pi = 0\) and \(\sigma > 0\). Using the Envelope Theorem one can show that \(dU_A(\pi)/d\pi > 0\). Moreover, \(\lim_{\pi \to \infty} U_A(\pi) = \infty > U_A(\hat{\alpha}_J; \sigma)\) for any finite \(\sigma\). Thus, there exists a threshold \(\hat{\pi}_J(\sigma)\) such that (13) is satisfied for \(\pi \geq \hat{\pi}_J(\sigma)\). Now consider briefly the case where both Alice and Bob found alternative partners (symmetric outside options). They then stay together if \(U(\pi) \geq U(\sigma)\), which is equivalent to \(\pi \geq \sigma\). Recall that \(U(\sigma) > U_A(\hat{\alpha}_J; \sigma)\) for all \(\sigma > 0\) because \(\hat{\beta}_J > 0\) and \(e^*_B = 0\) in case of asymmetric outside options. Thus, \(\hat{\pi}_J(\sigma) < \sigma\).

We can now consider the case where Alice and Bob have each some initial wealth \(w > 0\). The minimum value of wealth \(w\) required to eliminate retention externalities under joint asset ownership, denoted \(\overline{w}_J\), ensures that Alice can fully compensate Bob without offering him an equity stake in the new partnership with Charles. Thus, \(\overline{w}_J\) satisfies \(\hat{\pi}_J(w) = 0\). It remains to characterize the minimum amount of wealth \(\overline{w}_J\), which changes the renegotiation outcome. For \(\pi \geq \hat{\pi}_J(\sigma)\) we know that Alice stays with Bob. For \(w \to 0\) Alice cannot buy herself free, so the renegotiation outcome does not change. Thus, \(\overline{w}_J = 0\) for \(\pi \geq \hat{\pi}_J(\sigma)\). For \(\pi < \hat{\pi}_J(\sigma)\), Alice leaves Bob, but needs to offer him an equity stake in the new partnership with Charles. Alice can then use even small amounts of wealth to buy back some equity from Bob, which improves Alice’s expected utility when partnering with Charles. Consequently, \(\overline{w}_J > 0\) for \(\pi < \hat{\pi}_J(\sigma)\).

Proof of Lemma 3.

From our previous derivations (see Section "Renegotiation under Joint Asset Ownership" in the Appendix), we can immediately infer that the threshold \(\hat{\pi}_J(\sigma, w)\) is defined by
\[
U(\pi) = U_A(\hat{\alpha}_J; \sigma, w). \tag{14}
\]

Using (14) we can implicitly differentiate \(\hat{\pi}_J(\sigma, w)\) w.r.t. \(w\):
\[
\frac{d\hat{\pi}_J(\sigma, w)}{dw} = \frac{dU_A(\hat{\alpha}_J; \sigma, w)}{dw} \cdot \frac{dU(\pi)}{d\pi}.
\]

We know that \(dU_A(\hat{\alpha}_J; \sigma, w)/dw > 0\) for \(\overline{w}_J \leq w < \overline{w}_J\). Furthermore, using the Envelope Theorem it is straightforward to show that \(dU(\pi)/d\pi > 0\). Consequently, \(d\hat{\pi}_J(\sigma, w)/dw > 0\) for \(\overline{w}_J \leq w < \overline{w}_J\). \(\square\)
Proof of Proposition 1.

We focus on the case with asymmetric outside options because only then the ownership structure matters. Moreover, maximizing a partner’s expected utility at date 0 is equivalent to maximizing the joint surplus of Alice and Bob.

We first derive the cutoff \( \hat{\pi}_V(\sigma) \), so that staying together with \( \alpha^* = \beta^* = 1/2 \) is jointly efficient for \( \pi \geq \hat{\pi}_V(\sigma) \), and dissolving the partnership is jointly efficient for \( \pi < \hat{\pi}_V(\sigma) \). W.l.o.g. suppose that only Alice found an alternative partner at date 2 (the case where only Bob found an alternative partner is symmetric). The joint surplus in case of joint production with \( \alpha^* = \beta^* = 1/2 \), is given by \( 2U(\pi) \). When Alice leaves, the joint surplus of Alice and Bob is maximized when Bob, as unproductive partner, does not get a stake in the new Alice-Charles partnership. The joint surplus is then given by \( U_A(\hat{\alpha}; \sigma) \). Thus, staying together (with \( \alpha^* = \beta^* = 1/2 \)) and dissolving the partnership are both jointly efficient if

\[
2U(\pi) = U_A(\hat{\alpha}; \sigma). \tag{15}
\]

Recall that \( dU(\pi)/d\pi > 0 \). Moreover, note that \( U(0) = 0 \) and \( \lim_{\pi \to \infty} U(\pi) = \infty > U_A(\hat{\alpha}; \sigma) \) for any finite \( \sigma \). Thus, there exists a threshold \( \hat{\pi}_V(\sigma) \), defined by (15), such that \( 2U(\pi) \geq U_A(\hat{\alpha}; \sigma) \) for \( \pi \geq \hat{\pi}_V(\sigma) \), and \( 2U(\pi) < U_A(\hat{\alpha}; \sigma) \) for \( \pi < \hat{\pi}_V(\sigma) \). Using (15) we can implicitly differentiate \( \hat{\pi}_V(\sigma) \) w.r.t. \( \sigma \):

\[
\frac{d\hat{\pi}_V(\sigma)}{d\sigma} = \frac{dU_A(\hat{\alpha}; \sigma)}{d\sigma} \frac{d\pi}{dU(\pi)}. \tag{16}
\]

Using the Envelope Theorem we can show that \( dU_A(\hat{\alpha}; \sigma)/d\sigma > 0 \) and \( dU(\pi)/d\pi > 0 \). Thus, \( d\hat{\pi}_V(\sigma)/d\sigma > 0 \).

Suppose that \( \pi < \hat{\pi}_V(\sigma) \), so that dissolving the partnership is jointly optimal. Under individual asset ownership, Alice would leave if \( \pi < \hat{\pi}_I(\sigma, w) \), where according to Lemma 2, \( \hat{\pi}_I(\sigma, w) \) is defined by

\[
U_A(\alpha_I^*; \pi, w) = U_A(\hat{\alpha}_I; \sigma). \tag{17}
\]

Note that \( 2U(\pi) > U_A(\alpha_I^*; \pi, w) \) for \( w < w_I \), whereas the right-hand sides of (15) and (16) are identical. Thus, \( \hat{\pi}_V(\sigma) < \hat{\pi}_I(\sigma) \). This implies that individual asset ownership is optimal for \( \pi < \hat{\pi}_V(\sigma) \) as it always ensures the jointly efficient dissolution of the partnership in case of asymmetric outside options.

Now suppose that \( \pi \geq \hat{\pi}_V(\sigma) \), so that staying together with \( \alpha^* = \beta^* = 1/2 \) is jointly optimal in case of asymmetric outside options. Under joint asset ownership, Alice stays (with \( \alpha^* = \beta^* = 1/2 \)) if \( \pi \geq \hat{\pi}_J(\sigma) \). Recall from Lemma 3 that \( \hat{\pi}_J(\sigma, w) \) is defined by

\[
U(\pi) = U_A(\hat{\alpha}_J; \sigma, w). \tag{17}
\]
To show that $\hat{\pi}_J(\sigma, w) < \hat{\pi}_V(\sigma)$ for $w < \overline{w}_J$, we define $\hat{\pi}_V^J(\sigma)$ as the value of $\pi$ under joint asset ownership where staying together (with $\alpha^* = \beta^* = 1/2$) and dissolving the partnership (with $\hat{\beta}_J > 0$) lead to the same joint surplus:

$$2U(\pi) = U_A(\hat{\alpha}_J; \sigma, w) + U_B(\hat{\beta}_J; \sigma, w).$$

Note that $U_A(\hat{\alpha}; \sigma) > U_A(\hat{\alpha}_J; \sigma, w) + U_B(\hat{\beta}_J; \sigma, w)$ for $w < \overline{w}_J$, whereas the left-hand sides of (15) and (18) are identical. Thus, $\hat{\pi}_V^J(\sigma, w) < \hat{\pi}_V(\sigma)$. Moreover, we can write (18) as

$$U(\pi) + U(\pi) - U_B(\hat{\beta}_J; \sigma, w) = U_A(\hat{\alpha}_J; \sigma, w),$$

where, according to the Maschler-Owen consistent NTU value, $\chi < 0$ (otherwise Bob would not release his asset). Thus, the left-hand side of (19) is smaller than the left-hand side of (17), while their right-hand sides are identical. Hence, $\hat{\pi}_J(\sigma, w) < \hat{\pi}_V(\sigma)$. Thus, joint asset ownership is optimal for $\pi \geq \hat{\pi}_V(\sigma)$ as it always preserves the partnership with $\alpha^* = \beta^* = 1/2$.

We can now identify the optimal asset ownership for different values of $\pi \in \{\pi_L, \pi_H\}$ and $w < \max\{\overline{w}_J, \overline{w}_J\}$. From the above we can immediately infer that choosing individual asset ownership at date 0 is always optimal when $\pi_L, \pi_H < \hat{\pi}_V(\sigma)$. Likewise, joint asset ownership is always optimal when $\pi_L, \pi_H \geq \hat{\pi}_V(\sigma)$.

Next we derive the optimal asset ownership for $\pi_L < \hat{\pi}_V(\sigma) < \pi_H$. For this we first derive the expected utilities at date 2 when both partners observe the inside prospect $\pi \in \{\pi_L, \pi_H\}$. Consider individual asset ownership. Let $\alpha^+_I$ denote the profit share of the partner with the only outside option (asymmetric case), and $\alpha^-_I$ the profit share of the partner without outside option, where $\alpha^-_I = 1 - \alpha^+_I$. Moreover, let $\hat{\alpha}_I$ denote the equilibrium profit share of the partner with outside option when he leaves. The expected utility of a partner a date 2 is then given by

$$EU_I(\pi, \sigma, w) = q^2 \max\{U(\pi), U(\sigma)\} + (1 - q)^2U(\pi) + q(1 - q)V_I(\pi, \sigma, w),$$

where

$$V_I(\pi, \sigma, w) = \begin{cases} U(\hat{\alpha}_I; \sigma) & \text{if } \pi < \hat{\pi}_I(\sigma) \\ U(\alpha^+_I; \pi, w) + U(\alpha^-_I; \pi, w) & \text{if } \pi \geq \hat{\pi}_I(\sigma) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

Now consider joint asset ownership. Let $\hat{\alpha}_J$ denote the new profit share of the partner with the only outside option when leaving the partnership, and $\hat{\beta}_J$ the profit share of his former partner as compensation. The expected utility of a partner at date 2 is then given by

$$EU_J(\pi, \sigma) = q^2 \max\{U(\pi), U(\sigma)\} + (1 - q)^2U(\pi) + q(1 - q)V_J(\pi, \sigma, w),$$

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where

\[ V_J(\pi, \sigma, w) = \begin{cases} 
U(\hat{\alpha}_J; \sigma, w) + U(\hat{\beta}_J; \sigma, w) & \text{if } \pi < \hat{\pi}_J(\sigma) \\
2U(\pi) & \text{if } \pi \geq \hat{\pi}_J(\sigma)
\end{cases} \]

is the total expected utility of a partner in case of asymmetric outside options.

We can now write the expected utility of a partner at date 0 under individual asset ownership \((EU_I(p))\) and joint asset ownership \((EU_J(p))\) as

\[ EU_k(p) = pEU_k(\pi_H, \sigma, w) + (1 - p)EU_k(\pi_L, \sigma, w), \quad k = I, J \]

where \(EU_I(\pi, \sigma, w)\) and \(EU_J(\pi, \sigma, w)\) are defined by (20) and (21), respectively. Thus, both partners agree on joint asset ownership at date 0 when \(EU_J(p) \geq EU_I(p)\), which is equivalent to

\[ p \geq \hat{p} \equiv \frac{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w)}{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) + V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w)}. \]

where \(V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w) > 0\) and \(V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) > 0\) for \(\pi_L < \hat{\pi}_V(\sigma) < \pi_H\).

**Alternative Bargaining Protocols.**

If both partners have zero outside options, they are perfectly symmetric. Any reasonable bargaining solution then suggests an equal split of surplus. Similarly, if Alice and Bob both found alternative partners, then we have two pairs of symmetric partners. Again we note that an equal split of surplus is the most reasonable bargaining outcome. Alternative bargaining protocols therefore only matter for the case of asymmetric outside options. We distinguish between the bargaining games under individual versus joint asset ownership.

Consider first the bargaining game under joint asset ownership with binding wealth constraints, where Alice wants to leave Bob to partner with Charles. Because the agreement of all three parties is required, any reasonable bargaining involves trilateral bargaining. While there may be many bargaining protocols that affect the distribution of rents between the three parties, the key insight is that the critical threshold \(\hat{\pi}_J(\sigma, w)\) from Lemma 3 does not depend on the specific distribution of these rents. This threshold only depends on the feasibility of obtaining an agreement between Alice, Bob and Charles that satisfies all three participation constraints. Specifically, at \(\pi = \hat{\pi}_J(\sigma, w)\) both Alice and Bob are indifferent between dissolving their partnership and staying together (each getting \(U(\pi)\)), while Charles receives the minimum equity stake \(\gamma = 1 - \alpha_{\text{max}}\). For any \(\pi > \hat{\pi}_J(\sigma, w)\) it is impossible to get a tripartite agreement, and for any \(\pi \leq \hat{\pi}_J(\sigma, w)\) it is always possible get such an agreement. As a consequence, the specific bargaining protocol actually does not matter for the partners’ decision to stay together or to do a buyout.
Under individual asset ownership with binding wealth constraints we know from Lemma 2 that there exists a critical threshold \( \hat{\pi}_I(\sigma, w) \), such that Alice leaves Bob whenever \( \pi < \hat{\pi}_I(\sigma, w) \), and stays whenever \( \pi \geq \hat{\pi}_I(\sigma, w) \). Again we argue that reasonable alternative bargaining protocols may generate different utilities, but the critical threshold remains unaffected. One important restriction of the bargaining protocol by Hart and Mas-Colell (1996) is that at any point in time only one party can make an offer. Consider relaxing this assumption, and suppose that there can be simultaneous offers. In particular assume that the unique partner (Alice) can hold an auction for offers from the non-unique partners (Bob and Charles). Such an auction game results in a standard Bertrand pricing. It is easy to show that these Bertrand offers are more favorable to Alice than the bargaining outcome under the Hart and Mas-Colell protocol. However, since the auction is always won by the player with the highest valuation, it continues to be true that Alice teams up with Bob whenever \( \pi \geq \hat{\pi}_I(\sigma, w) \), and with Charles whenever \( \pi < \hat{\pi}_I(\sigma, w) \). Again we find that the critical threshold \( \hat{\pi}_I(\sigma, w) \) remains unaffected by the specific bargaining protocol.

**Proof of Proposition 2.**

Consider individual asset ownership. At date 1 partner \( i = A, B \) chooses his specific investment \( r_i \) to maximize his expected utility:\(^{28}\)

\[
EU_I(r_i, r_j) = p(r_i, r_j) \left[ q^2 \max\{U(\pi_H), U(\sigma)\} + (1 - q)^2 U(\pi_H) + q(1 - q)V_I(\pi_H, \sigma, w) \right] \\
+ (1 - p(r_i, r_j)) \left[ q^2 U(\sigma) + (1 - q)^2 U(\pi_L) + q(1 - q)V_I(\pi_L, \sigma, w) \right] - \psi(r_i),
\]

where \( j \in \{A, B\} \) and \( j \neq i \). The equilibrium investment levels \( r^*_A(I)(w) \) and \( r^*_B(I)(w) \) under individual asset ownership are then characterized by the first-order conditions:

\[
\frac{\partial p(r_A, r_B)}{\partial r_i} \Phi_I(w) = \psi'(r_i), \quad i = A, B,
\]

where, using \( V_I(\pi_L, \sigma, w) = U(\sigma) \),

\[
\Phi_I(w) = \left[ q^2 \max\{U(\pi_H), U(\sigma)\} + (1 - q)^2 U(\pi_H) + q(1 - q)V_I(\pi_H, \sigma, w) \right] \\
- \left[ q^2 U(\sigma) + (1 - q)^2 U(\pi_L) + q(1 - q)U(\sigma) \right].
\]

Because Alice and Bob are symmetric at date 1, their investment levels \( r^*_A(I)(w) \) and \( r^*_B(I)(w) \) must be also symmetric in equilibrium. We define \( r^*_I(w) \equiv r^*_A(I)(w) = r^*_B(I)(w) \) as the equilibrium relation-specific investment of a partner under individual asset ownership.

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(^{28}) Note that \( U(\sigma) > U(\pi_L) \) when Alice and Bob each found an alternative partner; thus, \( \max\{U(\pi_L), U(\sigma)\} = U(\sigma) \).
Likewise, the expected utility of partner $i = A, B$ at date 1 under joint asset ownership is given by

$$EU_J(p_i, p_j) = p(r_i, r_j) \left[ q^2 \max\{U(\pi_H), U(\sigma)\} + (1 - q)^2 U(\pi_H) + q(1 - q) V_J(\pi_H, \sigma, w) \right]$$

$$+ (1 - p(r_i, r_j)) \left[ q^2 U(\sigma) + (1 - q)^2 U(\pi_L) + q(1 - q) V_J(\pi_L, \sigma, w) \right] - \psi(r_i).$$

The following first-order conditions define the equilibrium investment levels $r_{A(J)}^*(w)$ and $r_{B(J)}^*(w)$ under joint asset ownership:

$$\frac{\partial p(r_A, r_B)}{\partial r_i} \Phi_j(w) = \psi'(r_i) \quad i = A, B,$$

where, using $V_J(\pi_H, \sigma, w) = 2U(\pi_H)$,

$$\Phi_j(w) = \left[ q^2 \max\{U(\pi_H), U(\sigma)\} + (1 - q)^2 U(\pi_H) + q(1 - q) V_J(\pi_H, \sigma, w) \right]$$

$$- \left[ q^2 U(\sigma) + (1 - q)^2 U(\pi_L) + q(1 - q) V_J(\pi_L, \sigma, w) \right].$$

Again, the Nash equilibrium is symmetric; we thus define $r_J^*(w) \equiv r_{A(J)}^*(w) = r_{B(J)}^*(w)$ as the equilibrium relation-specific investment of a partner under joint asset ownership.

Next, we define

$$F \equiv \frac{\partial p(r_A, r_B)}{\partial r_A} \Phi_k(w) - \psi'(r_A) = 0$$

$$G \equiv \frac{\partial p(r_A, r_B)}{\partial r_B} \Phi_k(w) - \psi'(r_B) = 0,$$

where $k \in \{I, J\}$. Applying Cramer’s Rule we get

$$\frac{dr_{A(k)}^*(w)}{dw} = \frac{\det(X_1)}{\det(X_2)},$$

where

$$X_1 = \begin{pmatrix} -\frac{\partial F}{\partial w} & \frac{\partial F}{\partial r_B} \\ -\frac{\partial G}{\partial w} & \frac{\partial G}{\partial r_B} \end{pmatrix} \quad X_2 = \begin{pmatrix} \frac{\partial F}{\partial r_A} & \frac{\partial F}{\partial r_B} \\ \frac{\partial G}{\partial r_A} & \frac{\partial G}{\partial r_B} \end{pmatrix}.$$

Because $U_i(\cdot), i = A, B$, is concave, $X_2$ must be negative definite, so that $\det(X_2) > 0$. Thus, $dr_{A(k)}^*(w)/dw > 0$ if

$$\det(X_1) = -\frac{\partial F}{\partial w} \frac{\partial G}{\partial r_B} + \frac{\partial G}{\partial w} \frac{\partial F}{\partial r_B} > 0.$$

The second-order condition for $r_{B(k)}^*(w)$ implies $\partial G/\partial r_B < 0$. Moreover,

$$\frac{\partial^2 F}{\partial r_B} = \frac{\partial^2 p(\cdot)}{\partial r_A \partial r_B} \Phi_k(w),$$

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and
\[
\frac{\partial F}{\partial w} = \frac{\partial p(\cdot)}{\partial r_A} \frac{d\Phi_I(w)}{dw} \quad \frac{\partial G}{\partial w} = \frac{\partial p(\cdot)}{\partial r_B} \frac{d\Phi_I(w)}{dw},
\]
where
\[
\frac{d\Phi_I(w)}{dw} = q(1-q)\frac{dV_I(w, \pi_H, \sigma)}{dw}
\]
\[
\frac{d\Phi_I(w)}{dw} = -q(1-q)\frac{dV_I(w, \pi_L, \sigma)}{dw}.
\]

For individual asset ownership, recall that \(dV_I(w, \pi_H, \sigma)/dw > 0\) for \(w_I \leq w < w_I\), which implies that \(\partial F/\partial w > 0\) and \(\partial G/\partial w > 0\) for \(w_I \leq w < w_I\). Thus, \(dr^*_A(w)/dw > 0\) for \(w_I \leq w < w_I\) and \(\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa\), where \(\kappa\) is the lower bound of the cross-
partial satisfying \(\det(X_1) = 0\). Symmetry implies \(dr^*_A(w)/dw = dr^*_B(w)/dw\). For joint asset ownership, recall that \(dV_J(w, \pi_L, \sigma)/dw > 0\) for \(w_J \leq w < w_J\), so that \(\partial F/\partial w < 0\) and \(\partial G/\partial w < 0\) for \(w_J \leq w < w_J\). Thus, \(dr^*_J(w)/dw < 0\) for \(w_J \leq w < w_J\) and \(\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa\). Due to symmetry, \(dr^*_A(w)/dw = dr^*_B(w)/dw\).

For \(w \geq \max\{w_I, w_J\}\) we know that \(V_I(w, \pi_H, \sigma) = 2U(\pi_H)\) (individual ownership), and \(V_J(w, \pi_L, \sigma) = U(\sigma)\) (joint ownership). Thus, we have \(\Phi_I(w) = \Phi_J(w)\) for \(w \geq \max\{w_I, w_J\}\), so that \(r^*_I(w) = r^*_J(w)\). Furthermore, because \(dr^*_J/dw > 0\) for \(w_J \leq w < w_J\) and \(dr^*_J/dw < 0\) for \(w_J \leq w < w_J\), we can infer that \(r^*_J(w) > r^*_I(w)\) for \(w < \max\{w_I, w_J\}\).

Proof of Lemma 4.

Under individual asset ownership the expected utility of Alice at date 0 is given by
\[
EU_I^A(p^*, w) = p^* \left[ q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_I(\pi_H, \sigma, w) \right]
\]
\[
+ (1-p^*) \left[ q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_I(\pi_L, \sigma, w) \right] - \psi(r^*_A),
\]
with \(p^* \equiv p(r^*_A, r^*_B)\) and \(V_I(\pi_L, \sigma, w) = U(\sigma)\). The expected utility of Bob is symmetric. Applying the Envelope Theorem we get
\[
\frac{dEU_I^A(p^*, w)}{dw} = \frac{\partial EU_I^A(p^*, w)}{\partial r_B} \frac{dr^*_B(w)}{dw} + p^*q(1-q) \frac{\partial V_I(\pi_H, \sigma, w)}{\partial w}.
\]
Note that \(\partial EU_I^A(\cdot)/\partial r_B > 0\). We need to consider three cases: (i) \(w \leq w_I\), (ii) \(w > w_I\); and (iii), \(w_I < w \leq w_I\). For the first two cases we know that \(dr^*_B(w)/dw = 0\) and \(\partial V_I/dw = 0\); thus, \(dEU_I^A(p^*, w)/dw = 0\). For \(w_I < w \leq w_I\) we know that \(dr^*_B(w)/dw > 0\) and \(\partial V_I/dw > 0\); thus, \(dEU_I^A(p^*, w)/dw > 0\). This also implies that \(EU_I^A(p^*, w)\) is maximized for \(w \geq w_I\). \(\square\)
Proof of Lemma 5.

Under joint asset ownership the expected utility of Alice at date 0 is given by

\[
EU^A_J(p^*, w) = p^* \left[ q^2 \max \{ U(\pi_H), U(\sigma) \} + (1 - q)^2 U(\pi_H) + q(1 - q)V_J(\pi_H, \sigma, w) \right] + (1 - p^*) \left[ q^2 U(\sigma) + (1 - q)^2 U(\pi_L) + q(1 - q)V_J(\pi_L, \sigma, w) \right] - \psi(r^*_A(I)),
\]

with \( p^* = p(r^*_A(I), r^*_B(J)) \) and \( V_J(\pi_H, \sigma, w) = 2U(\pi_H) \). The expected utility of Bob is symmetric. Applying the Envelope Theorem yields

\[
\frac{dEU^A_J(p^*, w)}{dw} = \frac{\partial EU^A_J(p^*, w)}{\partial r_{B(J)}} \frac{dr_{B(J)}}{dw} + (1 - p^*)q(1 - q)\frac{\partial V_J(\pi_L, \sigma, w)}{\partial w}
\]

and

\[
\frac{dEU^A_J(p^*, w)}{dw} = \phi_J(w) \frac{\partial p(\cdot)}{\partial r_{B(J)}} \frac{dr_{B(J)}}{dw} + (1 - p^*)q(1 - q)\frac{\partial V_J(\pi_L, \sigma, w)}{\partial w},
\]

where

\[
\phi_J(w) = \left[ q^2 \max \{ U(\pi_H), U(\sigma) \} + (1 - q)^2 U(\pi_H) + q(1 - q)2U(\pi_H) \right] - \left[ q^2 U(\sigma) + (1 - q)^2 U(\pi_L) + q(1 - q)V_J(\pi_L, \sigma, w) \right] > 0.
\]

By definition, \( \partial p(\cdot)/\partial r_{B(J)} > 0 \). Moreover, recall from Proposition 2 that \( dr_{B(J)}^*/dw < 0 \) for \( w_J \leq w < \bar{w}_J \). Thus, \( \psi_1 < 0 \) for \( w_J \leq w < \bar{w}_J \). Furthermore, \( \partial V_J(\pi_L, \sigma, w)/\partial w > 0 \) for \( w_J \leq w < \bar{w}_J \), so that \( \psi_2 > 0 \) for \( w_J \leq w < \bar{w}_J \). We define \( w_J^* \) as the wealth level which satisfies \( dEU^A_J(p^*, w)/dw = 0 \) for \( w_J \leq w < \bar{w}_J \), and thus maximizes Alice’s expected utility at date 0. Note that \( w_J \leq w_J^* < \bar{w}_J \) because \( dr_{B(J)}^*/dw = 0 \) and \( \partial V_J(\pi_L, \sigma, w)/\partial w = 0 \) for \( w < w_J \) and \( w \geq \bar{w}_J \). To summarize, (i) \( dEU^A_J(\cdot)/dw = 0 \) for \( w \leq w_J, w \geq \bar{w}_J \), and \( w = w_J^* \) (as \( \psi_1 + \psi_2 = 0 \)), (ii) \( dEU^A_J(\cdot)/dw > 0 \) for \( w_J^* < w < \bar{w}_J \) (as \( \psi_1 + \psi_2 > 0 \)); and (iii), \( dEU^A_J(\cdot)/dw < 0 \) for \( w_J^* < w < w_J \) (as \( \psi_1 + \psi_2 < 0 \)).

Finally note that \( \lim_{\pi_H \to \infty} \phi_J(w) = \infty \) as \( dU(\pi_H)/d\pi_H > 0 \) with \( \lim_{\pi_H \to \infty} U(\pi_H) = \infty \). This implies that \( \lim_{\pi_H \to \infty} \psi_1 = -\infty \) for \( w_J \leq w < \bar{w}_J \), while \( \sup(\psi_2) < \infty \). Thus, there exists a threshold \( \hat{\pi}_H \) such that \( dEU^A_J(\cdot)/dw < 0 \) for all \( \pi_H \geq \hat{\pi}_H \) and \( w \in (w_J, \bar{w}_J) \), which implies a corner solution with \( w_J^* \leq w_J \).

Proof of Proposition 3.

Suppose \( w \geq \bar{w} = \max \{ \bar{w}_I, \bar{w}_J \} \). Under individual asset ownership, \( V_I(\pi_H, \sigma, w) = 2U(\pi_H) \) for \( w \geq \bar{w}_I \). Under joint asset ownership, \( V_J(\pi_L, \sigma, w) = V_I(\pi_L, \sigma, w) = U(\hat{\alpha}_I; \sigma) \) for \( w \geq \bar{w}_J \). Moreover, recall from Proposition 2 that \( r^*_I(w) = r^*_J(w) \) for all \( w \geq \bar{w} \). Thus, \( EU_I(r^*_I, w) = EU_J(r^*_J, w) \) for \( w \geq \bar{w} \).

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Next, recall from Lemma 4 that \( dEU_I(\cdot)/dw > 0 \) for \( \overline{w}_I < w \leq \overline{w}_I \), where \( EU_I(\cdot) \) is maximized for \( w \geq \overline{w}_I \). Moreover, we know from Lemma 5 that \( dEU_J(\cdot)/dw > 0 \) for \( \overline{w}_J < w < w^*_J \), and \( dEU_J(\cdot)/dw < 0 \) for \( w^*_J < w \leq \overline{w}_J \), where \( EU_J(\cdot) \) is maximized when \( w = w^*_J \). This implies that \( EU_J(\cdot) > EU_I(\cdot) \) for \( w \in [w^*_J, \overline{w}] \).

Finally we examine whether \( EU_I(\cdot) > EU_J(\cdot) \) for some \( w < w^*_J \). Suppose \( \pi_H \to \pi_L \). We can then immediately see that \( r^*_I(w) = r^*_J(w) = 0 \), and hence, \( EU_I(\cdot) > EU_J(\cdot) \). We define \( w_0 \) as the critical wealth level so that \( EU_I(\cdot) > EU_J(\cdot) \) for \( w \in [w_0, \overline{w}] \). Note that \( w_0 < w^*_J \) because \( EU_J(\cdot) > EU_I(\cdot) \) for \( w^*_J \leq w < \overline{w} \). Moreover, \( w_0 \geq 0 \) because, when \( \pi_H \) is sufficiently high, \( EU_J(\cdot) > EU_I(\cdot) \) even for \( w = 0 \). Thus, joint asset ownership is strictly optimal for \( w_0 \leq w < \overline{w} \), with \( w_0 \in [0, w^*_J] \). According to Lemma 5, the optimal wealth level is then \( w^*_J \in [0, \overline{w}] \), with \( w^*_J \leq w^*_J \) for all \( \pi_H \geq \pi_H \).

**Preferences – Ex-ante Asymmetric Outside Options.**

W.l.o.g. we focus on Alice’s preference for the allocation of control rights; Bob’s preference is symmetric. Let \( \overline{q}_i \equiv 1 - q_i, i = A, B \). Alice’s expected utility at date 0 under joint asset ownership is

\[
U^J_A = q_A \overline{q}_B \sigma \overline{q}_B + (\overline{q}_A q_B + q_A \overline{q}_B + \overline{q}_A \overline{q}_B) \frac{\pi}{2}.
\]

Likewise, Alice’s expected utility at date 0 under individual asset ownership is

\[
U^I_A = q_A \overline{q}_B \sigma \overline{q}_B + q_A \overline{q}_B \sigma \overline{q}_B + \overline{q}_A \overline{q}_B \frac{\pi}{2}.
\]

Alice prefers joint asset ownership if \( U^J_A > U^I_A \), which is equivalent to

\[
q_A < \overline{q}_A = \left[ \left( \frac{1 - q_B}{q_B} \right) \left( \frac{\sigma - \pi}{\pi} \right) + 1 \right]^{-1}.
\]

Moreover, after some simplifications we find that

\[
\frac{d\overline{q}_A}{dq_B} = \left[ (1 - q_B) \left( \frac{\sigma - \pi}{\pi} \right) + q_B \right]^{-2} \left( \frac{\sigma - \pi}{\pi} \right) > 0
\]

\[
\frac{d^2 \overline{q}_A}{dq_B^2} = -2 \left[ (1 - q_B) \left( \frac{\sigma - \pi}{\pi} \right) + q_B \right]^{-3} \left( \frac{\sigma - \pi}{\pi^2} \right) (2\pi - \sigma).
\]

Note that \( d^2 \overline{q}_A / dq_B^2 < 0 \) if \( \pi > \sigma / 2 \), and \( d^2 \overline{q}_A / dq_B^2 > 0 \) if \( \pi < \sigma / 2 \).
References


