House Price Dynamics: Fundamentals and Expectations

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Abstract

We investigate whether expectations that are not fully rational have the potential to explain the evolution of house prices and the price-to-rent ratio in the United States. First, a stylized asset-pricing model solved under rational expectations is used to derive a fundamental value for house prices and the price-rent ratio. Although the model can explain the sample average of the price-rent ratio, it does not generate the large and persistent fluctuations observed in the data. Then, we consider a rational bubble solution and two extrapolative expectations solutions: one that features a constant extrapolation parameter and one in which the extrapolation coefficient is an increasing function of the deviation of the growth rate of rents from its mean. In this last solution the degree of extrapolation is stronger in good times than in bad times, generating waves of overoptimism. We show that under this solution the model not only is able to match key moments of the data but can also replicate the run up in the U.S. house prices observed over the 2000-2006 period and the subsequent sharp downturn.

\textit{JEL classification:} E; E3; E6; R21

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1 Introduction

Fluctuations in house prices can have a strong impact on real economic activity. Because housing is typically the most important component of household wealth, changes in house prices affect household wealth and expenditure. Moreover movements in house prices can impact the real side of the economy through their effect on the financial system: the rapid rise and subsequent collapse in U.S. residential housing prices is widely considered as one of the major determinants of the financial crisis of 2007-2009, which has in turn led to a deep recession and a protracted decline in employment (U.S. Financial Crisis Inquiry Commission, 2011). In light of these considerations it is important to identify the determinants of house prices dynamics.

Large movements in house prices, like the recent U.S. boom and bust episode, are hard to generate in standard macroeconomic models with fully-rational expectations. One way to close the gap between the model and the data is to postulate extremely large and persistent shocks to household preferences for housing (Darracq Pariès and Notarpietro 2008, Iacoviello and Neri 2010, Justiniano et al. 2013). Besides being hard to motivate, these preference shocks imply a housing service flow, as measured by the imputed rent, that is too volatile compared to the data. Another way to obtain a substantial rise in prices relative to rents consists in assuming time varying risk aversion, for example via financial liberalization, as in Favilukis et al (2011). Liberalization in housing finance and cyclical shocks endogenously determine the risk premium of the rational agents: episodes of easing lending standards and low mortgage costs decrease risk premia and determine an increase in house prices as well as a decrease in expected returns. This last prediction contradicts survey evidence suggesting that investors typically expect higher future returns after a protracted rise in prices (Case et al. 2012).

Given the shortfalls of these two explanations, in this paper we explore the role of an alternative driver of house price dynamics: expectations. In particular we investigate whether not fully rational expectations can explain the recent evolution in the price to rent ratio and house prices in the United States. We apply, to the housing market, a stylized model in which households own an asset (house) that can be rented out in exchange for an exogenous and stochastic stream of dividends (rents) used for consumption. Houses are treated merely as a financial asset and agents are viewed as real estate investors; from their perspective, rents are analogous in cash flow terms to dividends that stock market investors receive from holding stocks. The choice of such a stylized model is justified by the fact that this framework allows
to clearly isolate the contribution of expectations alone from other mechanisms that could affect the dynamics of house prices. Also, Lucas tree type models or simple present value models have been used extensively in the finance and real estate literature to characterize house price movements.

We explore the ability of four solutions of this model to match the data. All solutions adopt the same stochastic structure of the dividend growth process and the same preferences but they differ in the way agents form their expectations. We view the model solved under rational expectations as the benchmark. As alternatives, we consider a solution that includes a rational bubble component and two solutions that feature extrapolative expectations. These three solutions have been developed for the analysis of the stock market to generate momentum and volatility, characteristics common also to the housing market.

Solving the model under rational expectations and assuming an autoregressive process for the growth rate of rents, we obtain a fundamental solution for the price-rent ratio that matches the average over the sample 1987-2011. Motivated by claims both in the media and in academic circles that the recent housing boom might in fact have been a bubble, we relax the assumption of rational expectations and allow for a rational bubble solution of the model as in Froot and Obstfeld (1991).

Then, we abandon the assumption of rational expectations and we explore the implications of alternative expectation formation mechanisms. In particular we follow the approach developed in Lansing (2006, 2010) for the study of the stock market and assume that agents form expectations in an extrapolative fashion so that their conditional expectations of future values are based on past realizations of the variable to forecast. Although agents are not fully rational in forming their expectations, they solve optimally their wealth allocation and consumption problem.

The assumption of extrapolative behavior is supported by numerous microfunded studies: lab experiments which show that observed beliefs are well described by extrapolative or ‘trend following’ expectations (De Bondt 1993, Hey 1994) and field data analysis that document how extrapolation of the most recent price increase can determine asset allocation choices (Benartzi 2001, Vissing-Jørgensen 2004). More recently, Piazzesi and Schneider (2009), using data on house price expectations from the Michigan Survey of Consumers, study household

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1 Recent macroeconomic studies are assessing the role of non-fully rational expectations in conjunction with other factors for the dynamics of housing prices. For example, see Adam et al. (2011) for an open economy asset pricing model, Burnside et al. (2011) for a matching model.


3 See for example Burnside et al. (2011), Galí (2014), Nneji et al. (2013).
beliefs during the recent US housing boom and provide evidence that expectations of future increases in prices strengthen with the increase in prices, consistent with the extrapolative behavior analyzed in this study.

In the solutions that assume extrapolative behavior of the agents, expectations are related to lagged realizations through an extrapolation coefficient which represents the weight agents put on past observations to form their expectations. This expectation mechanism is able to generate run-ups and sharp declines in house prices. In particular, these solutions suggest a way to understand housing booms which is based on overoptimism and it relates house prices to fundamentals (rents) but in a non linear way. We consider a case in which the extrapolation parameter is fixed and one in which the extrapolation parameter is an increasing function of the actual realization of the rent growth process. The mechanism in the second model is consistent with the Vissing-Jørgensen (2004) finding that the effect of past returns on expectations is stronger if past returns are positive. Under this solution, the model predicts that house prices should be more sensitive to increases in rents when rents are above their mean. Prices rise when rents grow at a fast pace while they plummet when rents decline, hence house prices booms happen together with consumption booms.

We compare the predictions from the different solutions of the model along many dimensions, through a long simulation exercise that explores the ability of the solutions to match some moments of the data and through a simulation of model trajectories to check whether the solutions can replicate the path of house prices over the sample. We find that only the extrapolative expectation solution with time-varying extrapolation coefficient can simultaneously match the volatility and persistence of the price-rent ratio and account for both the surge and drop in house prices and price-rent ratio observed over the sample.

We emphasize that, although we show that the extrapolative expectation solution with a time varying extrapolation coefficient is consistent with some key characteristics of the housing market, we do not claim that the data is only consistent with this model. Other factors, alone or in conjunction with non fully rational expectations, might play a role in determining the evolution of house prices and price-rent series. In particular many studies identify plausible drivers to the recent house price boom: low real interest rates (Adam et al. 2011), financial liberalization (Duca et al. 2010, Favilukis et al. 2011) and low elasticity of housing supply (Glaeser et al. 2008). The quantitative performance of the stylized model adopted in this paper is even more surprising considering that it abstracts from these factors.

Therefore, the main contribution of our paper is to show that extrapolative expectations embedded in a simple asset-pricing model where rents are the only driving force of house
prices can account for the evolution of the actual price-to-rent ratio and price series. This result suggests that extrapolative expectations with time-varying extrapolation coefficient might be an important ingredient to include into more sophisticated models that aim at replicating the dynamics of the housing market. The analysis in this paper yields another broad message that is not obvious a priori. That is, deviations from rational expectations do not uniformly improve the predictions of the model over the rational expectations solution.

The paper is organized as follows: Section 2 describes the basic model and derives the price-rent ratio under rational expectations as well as under the alternative expectation formation mechanisms. Section 3 reports long simulation results from each solution. Section 4 shows the model-implied trajectories. Section 5 concludes.

## 2 The Model

We treat houses as liquid financial assets that deliver an exogenous stream of consumption (rents) and abstract from the function of houses as stores of value or collaterals and from financing decisions.\(^4\) We use a Lucas tree type model\(^5\) with a risky asset to obtain a fundamental value for the house price and price-dividend ratio \((p_t / d_t)\). We think of the dividend as rent, the stream of consumption and services that is derived from owning\(^6\) (and renting out) a house; we will use the terms dividends and rents interchangeably. Supply of houses is perfectly inelastic, therefore fluctuations in house prices are due exclusively to shifts in demand. In the Lucas model, which is an endowment economy, the representative agent chooses sequences of consumption and equity (shares of the house) to maximize the expected present value of her lifetime utility. In particular the risk-averse representative agent solves the following intertemporal utility maximization problem:

\[
\max_{c_t, s_t} \sum_{t=0}^{\infty} \beta^t U(c_t)
\]

s.t.

\[c_t + p_t s_t = (p_t + d_t) s_{t-1} \quad \text{with } c_t, s_t > 0\]

\(^4\)See Brumm et al. (2015) for implications of collateral requirements in a Lucas’ tree model.


\(^6\)See Davis and Martin (2005) or Piazzesi, et al. (2007) for a consumption-based asset pricing model where housing services enter the utility function.
where \( c_t \) is consumption in period \( t \), \( s_t \) is the equity share purchased at time \( t \), \( d_t \) is the stochastic dividend paid by the share in period \( t \), \( p_t \) is the price of the share in period \( t \) and \( \beta \) is the discount factor. \( \hat{E}_0 \) denotes the agent’s subjective expectations at time zero.

This maximization problem yields the well-known first-order condition:

\[
p_t = \beta \hat{E}_t \left[ \frac{U'(c_{t+1})}{U'(c_t)} (p_{t+1} + d_{t+1}) \right]. \tag{1}
\]

Because there is no technology to store dividends, and houses are available in fixed supply, for simplicity \( s_t = 1 \), consumption will be equal to the dividend\(^7\) in each period (or \( c_t = d_t \) \( \forall \) \( t \)). Substituting this equilibrium condition in (1) and assuming a CRRA utility function, the price-dividend ratio can be rewritten as:

\[
y_t \equiv \frac{p_t}{d_t} = \hat{E}_t \left[ \beta \exp \left( (1 - \alpha) x_{t+1} \right) \left( \frac{p_{t+1}}{d_{t+1}} + 1 \right) \right] \tag{2}
\]

where \( \alpha \) is the coefficient of relative risk aversion and \( x_t \) is defined as the growth rate of dividends: \( x_t \equiv \log(d_t/d_{t-1}) \).

Last, to solve the model it is necessary to specify a stochastic process for the growth rate of dividends which is assumed to be a stationary autoregressive process of order one with mean \( \bar{x} \) and variance \( \sigma^2 = \sigma^2_\varepsilon/(1 - \rho^2) \):

\[
x_t \equiv \bar{x} + \rho (x_{t-1} - \bar{x}) + \varepsilon_t \quad | \rho | < 1, \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \tag{3}
\]

When bringing the model to the data we use the rent series available quarterly from the National Accounts Table and divide it by the housing stock, owned and occupied dwellings, to obtain the rent matured by each owned and occupied house.\(^8\) The lag length was chosen empirically using an AIC selection criterion for the actual series of growth rate of real imputed rents over the sample under analysis.

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\(^7\) For studies where the equivalence between consumption and dividend is broken, see for example Cecchetti et al. (1993).

\(^8\) For robustness, we repeated our analysis using the dataset on rents and house prices constructed by Davis, Lehnert and Martin (2008). Similarly to the results discussed in this paper, we find that the rational expectation solution is not able to generate enough persistence in the price-rent ratio, while the extrapolative expectation and the near rational bubble solution can match the key moments of the series. However, the solutions are not very successful in replicating the evolution of the price-rent ratio over the sample. Results are available from the authors upon request.
2.1 Fundamental Solution

In this section we present the fundamental solution and its implications for the price-rent ratio and real house prices. Solving the model under rational expectations (so that $\mathbb{E}_t$ is the mathematical expectation operator $E_t$) and following the approach in Lansing (2010), an approximate\(^9\) analytical solution for the fundamental price-dividend ratio is obtained as a function of the structural parameters of the economy, i.e. the coefficient of relative risk aversion, $\alpha$, the discount factor, $\beta$, and the parameters governing the stochastic growth rate of the exogenous process for the dividends:

$$y_f^t = \frac{p_t}{d_t} = \exp(a_0 + a_1 \rho (x_t - \bar{x}) + \frac{1}{2} a_1^2 \sigma^2_x)$$  \tag{4}

where

$$a_1 = \frac{1 - \alpha}{1 - \rho \beta \exp\left[(1 - \alpha) \bar{x} + \frac{1}{2} a_1^2 \sigma^2_x\right]}$$

$$a_0 = \log\left[\frac{\beta \exp\left((1 - \alpha) \bar{x}\right)}{1 - \beta \exp\left[(1 - \alpha) \bar{x} + \frac{1}{2} a_1^2 \sigma^2_x\right]}\right]$$

as long as $1 > \beta \exp\left[(1 - \alpha) \bar{x} + (1/2) a_1^2 \sigma^2_x\right]$.

Then the rational expectation solution to this model implies that the price-rent ratio depends on the deviation of the current realization of rent growth from its mean. The fundamental price-rent ratio can be obtained once we assign values to the parameters in (4). Given the frequency of our data we interpret the length of each period as being a quarter. Table 1 summarizes the calibration: the mean, $\bar{x}$, the persistence, $\rho$, and the standard deviation of the errors, $\sigma_x$, of the growth of rents are estimated from the process for the growth rate of the rent series in (3).

The discount factor, $\beta$, and the value of the coefficient of relative risk aversion $\alpha$ are chosen to match the sample average and the standard deviation of the price-rent ratio for the sample 1987Q1-2011Q4.\(^{10}\) Although $\beta$ takes a value greater than one, the mean value of the stochastic discount factor will remain below unity: $E\left[\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha}\right] \approx 0.92$. Kocherlakota (1990) shows that well-defined competitive equilibria exist in infinite horizon growth models even when the discount factor is larger than one. Note that the value of $\alpha$ necessary for the rational expectation solution to match the standard deviation of the price-rent ratio is much

\(^9\)Although Burnside (1998) provides an exact analytical solution, we prefer to work with (4) to facilitate the comparison of the fundamental solutions with the solutions of the not fully rational models.

\(^{10}\)The expected value of the price-rent ratio is decreasing in $\alpha$ and increasing in $\beta$. The variance is increasing in both parameters.
higher than the common estimates of the coefficient of relative risk aversion which lie in the range of one to five.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated to:</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} )</td>
<td>mean of rents growth rate</td>
<td>sample mean growth rate of rents</td>
<td>0.0047</td>
</tr>
<tr>
<td>( \rho )</td>
<td>autocorrelation of rents growth rate</td>
<td>autocorrelation of rents growth rate</td>
<td>0.3623</td>
</tr>
<tr>
<td>( \sigma_\varepsilon )</td>
<td>sd errors of rents growth process</td>
<td>sd residuals of rents growth process</td>
<td>0.0053</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>relative risk aversion</td>
<td>match standard deviation of ( p_t/d_t )</td>
<td>57.5</td>
</tr>
<tr>
<td>( \beta )</td>
<td>discount factor</td>
<td>match mean of ( p_t/d_t )</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Note: Calibration for the asset prices model described in Section 2, equation (4). The parameters \( \bar{x} \), \( \rho \) and \( \sigma_\varepsilon \) are the sample mean, autocorrelation and variance of the residuals for the rent series over the sample 1987Q1-2011Q4, the discount factor \( \beta \) and the value of \( \alpha \) are chosen to match the sample average and the standard deviation of the price-rent ratio.

We conduct a sensitivity analysis exercise for the parameter \( \alpha \). We simulate 100 observations of the price-rent ratio and prices implied by the model from equations (4) and from the conditions for \( a_0 \) and \( a_1 \),\(^{11}\) given the parameterization of the rent growth process in Table 1. The simulated data are obtained by feeding into the model the actual realizations of the growth rate of rents over the sample 1987Q1-2011Q4. The price series is based on the Case and Shiller Composite 10 house prices index while the dividend series is obtained as the average rent of the owned and occupied housing stock. Both series are deflated using the PCE deflator.\(^{12}\) Figure 1 plots the actual price to rent ratio (upper panel) and actual house prices (lower panel) for the United States and the simulated data from the model for several values of the coefficient of risk aversion.

Given the rent growth parameters and any chosen value of \( \alpha \), we solve for the value of \( \beta \) such that expected value of the price-rent series in the model is equal to the mean of the price-rent ratio in the data. Therefore, for any value of the coefficient of relative risk aversion the mean of the simulated data equals the sample average of the price-rent ratio. Then \( \alpha \) can be calibrated so that the variance of the model-implied price-rent ratio matches the variance of the price-rent ratio in the data. The figure shows that higher values of the coefficient of relative risk aversion generate a more volatile price-rent ratio. However, as seen

\(^{11}\) Note that there are three possible values of \( a_1 \) that satisfy the non-linear equation. However, given the parameterization in Table 1 only one of these values satisfies the inequality \( 1 > \beta \exp \left[(1 - \alpha) \bar{x} + \frac{1}{2} a_1^2 \sigma_\varepsilon^2 \right] \). We pick this value to simulate the model.

\(^{12}\) A comprehensive description of the data is provided in the Appendix.
in Table 1, in order to obtain the large variance observed in the data, the coefficient of risk aversion needs to take an implausibly high value.

Moreover, Figure 1 shows that the actual price-rent ratio exhibits strong persistence and it fluctuates substantially throughout the sample, while from equation (4) the model delivers the prediction that, for reasonable values of $\alpha$, the price-dividend ratio should be fairly stable across time around the unconditional mean. To increase the persistence implied by this model it would be necessary to observe a considerably more persistent growth rate process for the rents.\footnote{Simulations show that, for $\alpha$ and $\beta$ calibrated as in Table 1, a coefficient of autocorrelation of 0.8 for the dividend growth process would imply an autocorrelation of 0.79 for the price-rent ratio while in the data the autocorrelation is 0.99.} Similarly, although the fundamental model can capture the upward trend in real prices, for sensible parameterizations of $\alpha$, it cannot generate large deviations from its trend and therefore it cannot replicate the housing boom of the years 2000-2006.

Figure 1. Simulated and Actual Prices and Price-Rent Ratio

Note: this figure shows the actual and simulated price-rent ratio (upper panel) and real prices (lower panel) for the US over the sample 1987Q1-2011Q4. The simulated data are generated for various values of the coefficient of risk aversion and discount factor from equation (4) and (3) by feeding the growth rate series into the solution. The values of the discount factor associated with higher values of the coefficient of risk aversion are: 0.98, 0.99, 1, 1.02 and 1.15.
2.2 A Rational Bubble Solution

We have seen in the previous section that a rational expectation model with a plausible parameterization can match the first sample moment of the price-rent ratio, but it cannot generate the large and persistent fluctuations observed in the data. We are interested in identifying models that can generate these features of the data. High volatility and persistence in the price-dividend ratio and prices are key characteristics of the stock market which have been extensively documented in the literature. Therefore, we apply to the housing market models developed for the analysis of the stock market. In particular, we consider models in which volatility and persistence are amplified by allowing deviations from full-rationality. Because the surge in prices from 2000 to 2006 has been referred to as a housing bubble, both in the media and in the macro literature, in this section we consider a rational bubble solution to the Lucas tree model first developed in Froot and Obstfeld (1991), subsequently generalized by Lansing (2010) to allow for CRRA utility function and autoregressive growth rate of dividends. Then, in section 2.3 and 2.4 we will relax the assumption of rational expectations and consider two extrapolative expectation models developed by Lansing (2006, 2010) for the analysis of the stock market.

To recover the rational bubble solution, note that the fundamental solution (4) is a particular solution to the stochastic difference equation described by the Euler equation. In the first step towards the fundamental solution the law of iterated expectations is applied to the Euler equation (2). This leads to the present value equation:

\[ y_t = E_t[\beta \exp((1 - \alpha) x_{t+1}) + \beta^2 \exp((1 - \alpha)(x_{t+1} + x_{t+2})) + \ldots + \beta^j \exp((1 - \alpha)(x_{t+1} + \ldots + x_{t+j})) E_{t+j}(y_{t+j} + 1)]. \]

The second step consists in imposing the transversality condition

\[ \lim_{j \to \infty} \beta^j E_t[\exp((1 - \alpha)(x_{t+1} + \ldots + x_{t+j})) y_{t+j}] = 0 \]

to the above present-value equation. Equation (2) admits solutions other than the fundamental solution (4). These solutions satisfy the no arbitrage condition only from period \( t \) to \( t+1 \) but they do not satisfy the transversality condition: the rationale behind this assumption is that agents are still forward looking, but they lack the infinite-horizon foresight required by the transversality condition. Then, the price-dividend ratio can be decomposed into two components: the fundamental solution \( y^f_t \), which is the discounted value of expected future
dividends, and the rational bubble \( y_t^b \):

\[
y_t^{rb} = y_t^f + y_t^b.
\] (5)

The rational bubble component satisfies the period-by-period condition:

\[
y_t^b = E_t \left( \beta \exp \left( (1 - \alpha) x_{t+1} \right) y_{t+1}^b \right).
\]

Therefore, the rational bubble considered in this paper is intrinsic as it does not depend on time or any other variable extraneous to the fundamental solution. For the case of autocorrelated dividends and CRRA utility function, the solutions to the rational bubble component can be expressed as:

\[
y_t^b = y_{t-1}^b \exp \left( \lambda_0 + (\lambda_1 - (1 - \alpha)) (x_t - \bar{x}) + (\lambda_2 + (1 - \alpha)) (x_{t-1} - \bar{x}) \right) \quad y_0^b > 0 \quad (6)
\]

together with the equilibrium conditions

\[
\lambda_2 = - (\rho \lambda_1 + (1 - \alpha)),
\]

and

\[
\frac{1}{2} (\lambda_1)^2 \sigma_e^2 + (1 - \alpha) \bar{x} + \log(\beta) + \lambda_0 = 0
\]

The dynamics of the bubble component depend on the sign of the drift parameter \( \lambda_0 \). The rational bubble solution with negative drift \( \lambda_0 < 0 \) will eventually shrink the bubble component to zero, and imply that no other bubble can occur in the future, while the solution with positive drift \( \lambda_0 > 0 \) will force the price-dividend ratio to grow unboundedly. Both these implication are at odds with the U.S. data, which exhibit a mean reverting price-rent ratio, therefore we select a solution with zero drift by imposing \( \lambda_0 = 0 \).

2.3 Extrapolative Expectations

Extrapolative expectations arise when agents form conditional expectations of future variables based on past observations, therefore extrapolating future behavior from past behavior. Many studies in the behavioral finance literature confirm the presence of extrapolative behavior: Graham and Harvey (2001) use survey data to document that after periods of negative market returns agents reduce their forecasts of future risk premia. A study by Vissing-
Jørgensen (2004) finds evidence of extrapolation in survey data about beliefs of stock market investors; in particular, she documents that investors who experienced high past portfolio returns expect higher future returns. Motivated by these findings, we apply, to the housing market, two models of extrapolative expectations developed for the stock market by Lansing (2006, 2010).\(^\text{14}\)

To derive the solutions of the model under extrapolative expectations it is convenient to rewrite the equilibrium condition (2) in terms of the variable \(z_t \equiv \beta \exp \left((1 - \alpha) x_t \right) \left( y_t + 1 \right)\):

\[
z_t = \beta \exp \left((1 - \alpha)x_t \right) \left( \hat{E}_t z_{t+1} + 1 \right)
\]

that is, the Euler equation is expressed in terms of a composite variable, function of the future price-dividend ratio and the future realization of the growth rate of the dividends. First we consider the simple and parsimonious expectation rule:\(^\text{15}\)

\[
\hat{E}_t [z_{t+1}] = Hz_{t-1} \hspace{1cm} H > 0
\]

where \(H\) is a positive extrapolation coefficient that measures the weight agents put on the last observation in order to form conditional expectations of the forecast variable.\(^\text{16}\) As in the previous solutions, agents are assumed to know the process for the growth rate of dividends.\(^\text{17}\) However they lack the knowledge of the law of motion of the forecast variable \(z_t\) as well as of the mapping between \(x_t\) and \(z_t\). Therefore, agents form their forecasts based on a perceived law of motion (PLM) that does not nest the actual law of motion (ALM).\(^\text{18}\) The forecast rule (8) can be obtained when agents use a geometric random walk as their PLM for the variable \(z_t\). In this case, the parameter \(H\) can be selected to ensure that the representative agent optimizes the forecasting rule for her perceived law of motion: \(H = \exp \left((1 - \alpha)^2 \sigma_x^2 \right)\). We will use this condition to pin down \(H\) in our simulations. Note that, conditional on the forecast for \(z_{t+1}\), the ALM is consistent with the Euler equation indicating that, although

\(^{14}\)Given the similarities in the framework, derivations in Sections 2.3 and 2.4 of this paper follow Lansing (2006, 2010).

\(^{15}\)Note the distinction between subjective expectations, denoted by \(\hat{E}_t\), and the rational expectations, denoted by mathematical expectation operator \(E_t\).

\(^{16}\)The expectation at time \(t+1\) depends on the past realization (at \(t-1\)) rather than the current realization (at \(t\)). This ensures that expectations and actual realization are not simultaneously determined.

\(^{17}\)Fuster, Hebert and Laibson (2011) consider a Lucas’ tree model where agents estimate the dividend growth rate process.

\(^{18}\)Lansing (2006) shows that lock-in of extrapolative expectations can occur if agents are concerned about minimizing forecast errors. This is "because the (atomistic) representative agent fails to internalize the influence of his own forecast on the equilibrium law of motion of the forecast variable", page 318.
agents are not fully rational in forming their expectations, they solve optimally their wealth allocation and consumption problem.

Replacing the expectation in (7) with (8) and applying the definitional relation for $z_t$, the solution\(^\text{19}\) for the extrapolative model with fixed extrapolation coefficient is:

$$y_t^{ee} = \hat{E}_t [z_{t+1}] = (y_{t-1}^{ee} + 1) \beta H \exp ((1 - \alpha)x_{t-1}) \tag{9}$$

From (9) it follows that, as in the case of the rational bubble, the extrapolative expectation solution includes the additional state variable, $y_{t-1}^{ee}$, therefore the rational bubble and the extrapolative expectations solutions can generate a higher persistence than the fundamental solution.

### 2.4 A Near Rational Bubble Solution

A key finding in the Vissing-Jørgensen (2004) study is that positive past returns have a stronger effect on expectations than negative past returns. In this section we outline a model consistent with this finding.

We follow the approach in Lansing (2010) and assume agents form expectations for the composite variable $z_t$ in the following fashion:

$$\hat{E}_t [z_{t+1}] = \exp(b(1 + \rho)(x_t - \bar{x}) + \frac{1}{2}b^2\sigma^2_x)z_{t-1}. \tag{10}$$

This can be seen as a model of extrapolative expectations with a time-varying weight assigned to past observations:

$$\hat{E}_t [z_{t+1}] = H_t z_{t-1} \tag{11}$$

where $H_t \equiv \exp(b(1 + \rho)(x_t - \bar{x}) + (1/2)b^2\sigma^2_x)$. The conditional expectation in (10) can be derived by iterating forward the following perceived law of motion:

$$z_t = z_{t-1} \exp (b(x_t - \bar{x})) \quad z_0 > 0, \quad \tag{12}$$

and then taking the conditional expectation at time $t$. However, like in the extrapolative expectation solution presented in the previous section, when constructing their conditional

\(^{19}\)The solution of this model in the case of i.i.d. dividend growth rate is provided in Lansing (2006); the model considered in this section is a straightforward modification to accommodate for time dependence in the rent growth rate.
forecasts, agents use the past, rather than the contemporaneous realization, because the realization $z_t$ depends in turn on the agents’ own forecast. This assumption avoids simultaneity in the determination of the forecasts and of the actuals. The perceived law of motion in (12) implies that the weight agents assign to the lagged observations depends non-linearly on the parameter $b$ and on the deviation of the rent growth rate from its mean. Plugging the conditional expectation (10) into the equilibrium condition (7) and applying the definitional relation $y_t \equiv \beta^{-1} \exp ((1 - \alpha)x_t)^{-1} z_t - 1$ obtains:

$$y_{t+1}^{nr} = \hat{E}_{t+1}[z_{t+1}] = (y_{t-1}^{nr} + 1) \beta \exp \left( b(1 + \rho)(x_t - \bar{x}) + (1 - \alpha)x_{t-1} + \frac{1}{2}b^2\sigma_x^2 \right)$$  \hspace{1cm} (13)

so that the price-dividend ratio is a function of its past values and of the current and past realizations of the dividend growth process.

We consider a near rational solution of the model, which relies on the ‘restricted perception equilibrium’ defined by Evans and Honkapohja (2001). In this solution, the subjective forecast parameter $b$ is selected from moments of the actual data. In particular, because the perceived law of motion (12) implies $\Delta \log z_t = b(x_t - \bar{x})$ we calibrate $b$ to match the covariance between $\Delta (\log z_t)$ and $x_t$:

$$b = \frac{\text{cov}[\Delta \log z_t, x_t]}{\text{Var}(x_t)}$$

where $\Delta \log z_t \equiv \log (z_t / z_{t-1})$. Subsequently, the near rational restricted perceptions equilibrium value for the parameter $b$ is derived as the fixed point from the non-linear map:

$$b = \frac{(1 - \rho)m}{1 - \rho k}$$

where $k$ and $m$ are defined as:

$$k = \beta \exp \left( (1 - \alpha)\bar{x} + \frac{1}{2}b^2\sigma_x^2 \right)$$

$$m = (1 - \alpha) + b(1 + \rho)\beta \exp \left( (1 - \alpha)\bar{x} + \frac{1}{2}b^2\sigma_x^2 \right)$$.

This calibration strategy gives the following parameter values: $b = 3.93$, $m = 3.97$ and

\footnote{Note that there are three values of $b$ that satisfy the non-linear mapping. As in Lansing (2010) we select the value of $b$ that guarantees $0 \leq k(b) < 1$ so that $\log(z_t)$ is stationary.}
As the subjective forecast parameter $b$ is positive, the perceived law of motion (12) implies that the weight put on lagged observations is (non-linearly) increasing in the growth rate of the dividend process. The model predicts that house prices should be more sensitive to increases in rents when rents are above their mean. It also suggests a way to understand housing booms which is based on overoptimism. Finally, because the model implies that prices rise when rents grow at a fast pace and drop when rents decline, it predicts that booms in house prices happen together with consumption booms.

3 Model Simulations

To compare the quantitative performance of the solutions described in section 2.1 through 2.4 we simulate data from each solution for the following variables: the price-rent ratio ($y_t$), the growth rate in the price-rent ratio ($\log(y_t/y_{t-1})$), real net returns ($r_t$) and growth rate of real house prices ($\log(p_t/p_{t-1})$). Table 2 presents several statistics computed from the actual and simulated data: mean, standard deviation, skewness, kurtosis and autocorrelation. Figure 2 plots 2000 simulated observations from each solution for the price-rent ratio and the net returns series.

Given the calibration in Table 1, the fundamental solution matches the mean and the standard deviation of the price-rent ratio by construction, but the simulated price-rent ratio series inherits the persistence of the growth rate of rents, which is only about one third of the persistence of the actual price-rent ratio. Relaxing the assumption of fully rational expectations is enough for the model to generate the same persistence as in the data. However, the not fully rational solutions differ in their ability to match other moments of the price-rent ratio.

The simulation exercise reports two calibration experiments of the parameter $\alpha$ for both the rational bubble and the extrapolative expectations solutions. For all calibrations the discount factor is chosen to match the mean of the price-rent ratio, but in column (A) the coefficient of relative risk aversion is picked to match the trajectory of the data, while in column (B), $\alpha$ is chosen to match the standard deviation of the price-rent ratio. In particular, to give a chance to the solutions to replicated the housing boom-bust episode, in column (A) we select the value of $\alpha$ that minimizes the distance of the model implied series from the

---

21 In the data the price-rent ratio series is stationary, as the ADF test rejects the null hypothesis of unit-root for the price-dividend ratio at the 10 percent significance level.

22 Net returns are defined as $(p_t + d_t)/p_{t-1} - 1$. 

actual price-rent ratio over the subsample 2004Q3-2008Q2.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>U.S. data</th>
<th>Fundamental (A)</th>
<th>Rational Bubble (A)</th>
<th>Extrapolative (A)</th>
<th>Extrapolative (B)</th>
<th>Near Rational</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t = p_t/d_t$</td>
<td>61</td>
<td>61</td>
<td>67</td>
<td>60</td>
<td>61</td>
<td>62</td>
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<tr>
<td>mean</td>
<td>11.08</td>
<td>11.02</td>
<td>67.81</td>
<td>10.37</td>
<td>2.73</td>
<td>11.12</td>
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<tr>
<td>sd</td>
<td>11.08</td>
<td>11.02</td>
<td>67.81</td>
<td>10.37</td>
<td>2.73</td>
<td>11.12</td>
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<tr>
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<td>0.69</td>
<td>0.55</td>
<td>18.61</td>
<td>23.24</td>
<td>0.14</td>
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<tr>
<td>kurtosis</td>
<td>2.54</td>
<td>3.57</td>
<td>457</td>
<td>734</td>
<td>2.88</td>
<td>3.66</td>
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<tr>
<td>autocorrelation</td>
<td>0.99</td>
<td>0.35</td>
<td>0.98</td>
<td>0.97</td>
<td>0.99</td>
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$log(y_t/y_{t-1})$

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<th>U.S. data</th>
<th>Fundamental (A)</th>
<th>Rational Bubble (A)</th>
<th>Extrapolative (A)</th>
<th>Extrapolative (B)</th>
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<td>-0.000</td>
<td>0.000</td>
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<td>0.017</td>
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<td>-0.235</td>
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<td>kurtosis</td>
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<td>-0.131</td>
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$r_t$

<table>
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<th>Fundamental (A)</th>
<th>Rational Bubble (A)</th>
<th>Extrapolative (A)</th>
<th>Extrapolative (B)</th>
<th>Near Rational</th>
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<tr>
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<td>4.22%</td>
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<td>sd</td>
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<td>3.000</td>
<td>2.988</td>
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<td>0.018</td>
<td>0.025</td>
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<td>0.113</td>
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</table>

$log(p_t/p_{t-1})$

<table>
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<tr>
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<th>Fundamental (A)</th>
<th>Rational Bubble (A)</th>
<th>Extrapolative (A)</th>
<th>Extrapolative (B)</th>
<th>Near Rational</th>
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<td>0.47%</td>
<td>0.47%</td>
<td>0.47%</td>
<td>0.47%</td>
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<tr>
<td>sd</td>
<td>0.023</td>
<td>0.201</td>
<td>0.024</td>
<td>0.015</td>
<td>0.009</td>
<td>0.021</td>
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<td>-0.254</td>
<td>-0.010</td>
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<td>kurtosis</td>
<td>3.37</td>
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<td>autocorrelation</td>
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<td>-0.319</td>
<td>0.017</td>
<td>0.026</td>
<td>0.670</td>
<td>0.093</td>
</tr>
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Note: descriptive statistics for the actual observations over the sample 1987Q1-2011Q4 and data simulated from the models described in Section 2.1 through 2.4. The column ‘Fundamental’ refers to the variables generated under the fundamental solution in Section 2.1, ‘Rational Bubble’ to the rational bubble solution with $\lambda_0 = 0$ from Section 2.2, ‘Extrapolative’ to the extrapolative expectation solution from Section 2.3 and ‘Near Rational’ to the near rational solution from Section 2.4. Models are simulated under the parameterization in Table 1. Additional parameter values are as follows: for the fundamental solution $\alpha_0 = 3.98$, $\alpha_1 = -87.78$, for the rational bubble solution column (A) $\alpha = 0.15$, $\beta = 0.98$, $\lambda_1 = 33.67$, for column (B) $\alpha = 4.85$, $\beta = 1.002$, $\lambda_1 = 34.24$ for the ‘Extrapolative’ model column (A) $\alpha = 0$, $\beta = 0.98$ and $H = 1$, for column (B) $\alpha = 4.9$, $\beta = 1.0015$ and $H = 1.0004$; for the near rational model $\alpha = 2.3$, $\beta = 0.99$ and $b = 3.93$. Net returns $r_t$ are defined as $(p_t + d_t)/p_{t-1} - 1$. Statistics are computed over 19,500 simulations after discarding the first 500 draws.

The price-rent ratio series simulated from the rational bubble solution calibrated as in column (B) has the same standard deviation as the actual series, which is a consequence of our calibration strategy, while for calibration as in column (A) the standard deviation of the simulated series is six times larger than in the data. Figure 2 highlights that, regardless of the calibration method, the rational bubble solution creates housing cycles with characteristics
which are very different from the ones observed in the U.S. data. In the simulated series, housing boom and bust episodes occur rarely and are of large magnitude. Picks of the price-rent ratio reach the value of 2000 for the calibration in column (A) and almost 600 for the calibration in column (B), while the highest value of the price-rent ratio during the last U.S. housing boom was about 90. The asymmetry and fat tails of the distribution of the simulated price-rent ratio series are reflected in extremely high values of skewness and kurtosis.

Figure 2. Model Simulations: Price-Rent Ratio, Net Returns and Growth Rate of Prices

Note: simulated price-rent ratio and net returns obtained from the fundamental (re), rational bubble (rb), extrapolative expectation (ee) and near rational bubble (nr) solutions described in Section 2.1 through 2.4. See Table 2 note for a complete description of calibrated parameter values.

As in the case of the rational bubble, the extrapolative solution is better able to replicate the trajectory of the price-rent ratio when $\alpha < 1$, given that in this case the extrapolative
expectation solution generates a positive correlation between the price-rent ratio and the growth rate of rents. However, for $\alpha < 1$ the simulated price-rent ratio has much lower standard deviation than the data. Conversely, the extrapolative solution with $\alpha \geq 1$ can generate the same volatility as in the data but it predicts a decline, rather than an increase, of the price-rent ratio over the years 2000-2006, as we will see in the next Section. Parameterization in column (A) and (B) differ in their ability to match also the remaining moments: the extrapolative expectation solution with $\alpha = 0$ presents with too little skewness, while the solution with $\alpha = 4.9$ generates excess kurtosis.

The near rational solution, where $\alpha$ and $\beta$ are calibrated to match the mean and standard deviation observed in the data, can account for the high persistence observed in the price-rent ratio. Moreover, it can generate positive skewness, although, as most of the other solutions, it delivers excess kurtosis not present in the data. The good performance of this solution is confirmed by the behaviour of the price-rent ratio series in Figure 2: the simulated housing cycles are frequent and, like in the data, the range of values taken by the simulated price-rent ratio goes from about 40 to 100.

Results regarding the performance of the solutions across the other variables are mixed. For all variables the rational expectation solution generates higher standard deviation than the data, as well as negative correlation, while the actual series are positively autocorrelated. All other solutions, except for the extrapolative solution with $\alpha = 0$, exhibit standard deviations similar to the ones in the data for all variables. However, none of the solutions can replicate for any series the high persistence observed in the data. For both calibrations, the rational bubble solutions imply an autocorrelation close to zero, even negative in the case of the growth rate of the price-rent ratio. The other solutions produce positive, although small autocorrelation for the growth rate of the price-rent ratio, returns and growth rate of prices, ranging from 0.1 to 0.35 while in the data the autocorrelation of these series is about 0.9. The extrapolative solution with $\alpha = 0$ is the one that generates the highest autocorrelation in the net returns and in the growth rate of prices, with values close to 0.65. Last, all models can match the first moment of net returns, growth rate of the price-rent ratio and growth rate of prices. However, none of the solutions is able to match both the negative skewness and kurtosis of the growth rate of the price-rent ratio, prices and returns.

Overall, the extrapolative expectations calibrated as in column (B) and the near rational bubble are the solutions that can better replicate the moments of the price-rent ratio and match the mean and variance of the other variables. Note that, while matching the first and second moment of the price-rent ratio is a consequence of the calibration strategy, the results
regarding the skewness and autorocorrelation of the series are not obtained by construction. Figure 2 highlights one major difference between the two solutions: for a short sequence of realizations of the dividend growth rate above the mean, the price-rent ratio generated by the extrapolative solution with constant extrapolation coefficient decreases considerably while the one generated from the near rational bubble solution increases substantially. This is because in the simulations the correlation between the growth rate of dividends and the price-rent ratio is positive for the near rational bubble solution while it is negative for the other solution. This will have important implications for the results of the experiment conducted in the next section.

4 Model Predicted Paths

In the previous section we showed that both the extrapolative expectations with constant extrapolation parameter and the near rational bubble solution can match the moments of the price-rent ratio equally well. But is this enough to determine the success of the solutions? And how can we distinguish between the two solutions if they provide with the same predictions in terms of the moments of house prices and price-to-rent ratio? While the finance literature has focused on evaluating the solutions on their ability to match some moments of actual series, we investigate the ability of the models to replicate a sequence of actual realization of the data in the sample under consideration. In particular, we generate model predicted trajectories for the price-rent ratio and the house price series by feeding the observed exogenous rent process, $x_t$, into the models.

Figure 3 shows qualitatively the performance of the solutions by plotting the actual and model-implied price-rent ratio (upper panel) and real house prices (lower panel) over the sample 1987Q1-2011Q4. The parameters are calibrated as in Table 1 and Table 2.

Note that while the fundamental solution is fully characterized by the growth rate of rents and the calibrated parameters, the rational bubble, the extrapolative expectations and the near rational solutions require the additional choice of the initialization value for the price-rent ratio series. To pin down the initial value of the bubble component in the rational bubble solution, $y^b_0$, we make use of equation (5), which states that the actuals are the sum of the fundamental and bubble solution, so that $y^b_0 = y_0 - y^f_0$, where $y_0$ is the realized value of the price-rent ratio observed in 1987Q2 and $y^f_0$ is the fundamental solution obtained from equation (4). The extrapolative expectations and the near rational solutions are initialized to the average of the price-rent ratio over the sample.
As seen in Figure 1, when calibrated as in Table 1, the model solved under rational expectations generates sharp swings of the price-rent ratio around the mean, however these fluctuations are much less persistent than in the data. Moreover, the rational expectation solution predicts a price-rent ratio below the mean, when it is actually at its peak in 2005, and an increase in the price-rent ratio, when it stagnates in 2009-2011.

Figure 3. Actual and Model Predicted Price-Rent Ratio and Price Series

![Figure 3](image_url)

Note: actual price-rent ratio (upper panel), actual real prices (lower panel) and model predicted data over the sample 1987Q1-2011Q4. Simulated data are generated from the models described in Section 2.1 through 2.4 by plugging in the realized growth rate of rents series and the structural parameters as calibrated in Table 1 and Table 2. See Table 2 note for a complete description of the calibrated parameter values.

The rational bubble solution with the coefficient of relative risk aversion calibrated to replicate the data trajectory tracks the behaviour of the price-rent ratio fairly well, as it generates a boom and bust episode in the years 2000-2006. However, the simulated price-rent ratio and price series are higher than the realized series in the 1990s as well as in 2008-2011, after the initial drop in prices. In contrast, the solution with $\alpha$ calibrated to match the standard deviation of the price-rent ratio predicts a flat price-rent ratio series over the sample. This is consistent with the simulation results in Figure 2 which shows that
the latter calibration generates smaller picks in the price-rent ratio for a given realization of rents.

The predicted price-rent ratio series from the extrapolative expectation solution with constant extrapolation parameter and $\alpha < 1$ stays close to the mean and therefore misses the rise and drop in the actual series. The extrapolative solution with $\alpha \geq 1$ implies a price-rent ratio series which evolves in opposite direction with respect to the data: when the price-rent ratio increases sharply in the 2000-2006 period the model generated series plummets. Similarly, when the actual series plunges at the end of the sample the model implied data rise. The behavior of the simulated series for the calibration with $\alpha \geq 1$ is due to the negative correlation between the growth rate of rents and prices that the extrapolative solution generates. Values of the coefficient of relative risk aversion lower than one imply a positive correlation between rents and prices, however they do not produce the substantial fluctuations observed in the price-rent ratio and prices.

Like the rational bubble solution with $\alpha < 1$, the near rational bubble solution of the model overpredicts the rise in price-rent ratio in the first part of the sample, however it can correctly replicate not only the early 2000s surge in the price-rent ratio and in the prices but also the subsequent sharp downturn. Also, the solution predicts an asymmetric boom-bust episode, with a protracted run-up and a sudden, large drop in prices and in the price-rent ratio. Finally, the solution is successful in replicating both the magnitude and the timing of the rise and drop in prices. Differently from the other solutions, the near rational bubble can achieve these results and match key moments of the data, as seen in the previous section, with plausible calibrated values for $\alpha$ and $\beta$. It should be noted that the simulated price to rent series for the near rational bubble solution is generally higher than the data, implying that the mean of the predicted price-rent ratio is above the sample average of the actuals. This is a consequence of the initialization choice.\footnote{To match the sample average the first value for the simulated price-rent ratio should be fixed at 46 and the corresponding price at 112453 dollars. The evolution of both the price-rent ratio and prices series would be essentially unaltered, apart from a downward level shift.}

Next we report some quantitative measures of performance of the solutions: the in-sample Root Mean Squared Error (RMSE), the Mean Correct Forecast Direction (MCFD) and the correlation between the observed and the model generated series from the different solutions, all shown in Table 3. Looking at the RMSE, which is a measure of how much on average the model is accurate in predicting the actual series, the extrapolative solution (A) displays the lowest RMSE for all variables, followed by the rational bubble solution, the near rational,
the fundamental and the extrapolative expectation solution (B). The fundamental solution is by far the worst for net returns and the growth rate of prices. In interpreting these results we should keep in mind that the RMSE is sensitive to the initialization choice: for this reason, for each solution of the model that deviates from rational expectations, we consider an alternative initialization strategy, in which the first value of the model-implied price-rent ratio series is selected to minimize the RMSE over the sample. We report these alternative RMSE values in square brackets in Table 3. Although the RMSE decreases for almost each solution using this new strategy, the gains are modest and the ranking across solutions is unchanged, except for the near rational bubble solution, which now performs as well as the rational bubble solution (A).

<table>
<thead>
<tr>
<th>Table 3: Models Performance</th>
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<tr>
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<th>Fundamental</th>
<th>Rational Bubble</th>
<th>Extrapolative</th>
<th>Near Rational</th>
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<td>0.025</td>
<td>0.022</td>
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<td>prices growth rate</td>
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<td>0.028</td>
<td>0.025</td>
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<td><strong>MCFD</strong></td>
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<tr>
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<td>prices growth rate</td>
<td>-0.038</td>
<td>0.298</td>
<td>0.141</td>
<td>0.357</td>
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Note: Root Mean Squared Error (RMSE), Mean Correct Forecast Directions (MCFD) and Correlation between the actual and simulated variables obtained from the solutions described in Section 2.1 through 2.4. The column ‘Fundamental’ refers to the variables generated under the fundamental solution in Section 2.1, ‘Rational Bubble’ to the rational bubble solution with \( \lambda_0 = 0 \) from Section 2.2, ‘Extrapolative’ to the extrapolative expectation solution from Section 2.3 and ‘Near Rational’ to the near rational solution from Section 2.4. Models are simulated under the parameterization in Table 1. See Table 2 note for a complete description of the remaining calibrated parameter values.

The RMSE is a symmetric quadratic loss function which penalizes equally positive or negative prediction errors of the same magnitude. However, for financial assets where positive profits may arise when the sign forecast is correct, it might be important to consider a loss function like MCFD which penalizes cases in which the model predicts a change in the opposite direction than the data, regardless of the size of the prediction error. The MCFD
loss function is defined as:

\[ MCFD = T^{-1} \sum_{t=1}^{T} 1 \left( \text{sign}(f_t) \cdot \text{sign}(\hat{f}_t) > 0 \right) \]

where \( 1(\cdot) \) is an indicator function that takes the value of one if the actual variable \( f_t \) and its prediction \( \hat{f}_t \) have the same sign. Differently from the RMSE, the MCFD (as well as the correlation) shown in Table 3 is basically unaffected by the initialization values chosen for the models. The near rational bubble solution is the one that more frequently predicts correctly the direction of change in both the price-rent ratio (58.6%) and prices (61.6%) delivering a better trading strategy than most other solutions, however gains with respect to the rational bubble (A) and extrapolative expectations (A) are limited. In contrast, a trader would be wrong about 60% of the time when betting on the direction of change of house prices using the extrapolative solution (B) and 50% using the rational expectation solution. In identifying the correct direction of change in the net returns the near rational bubble solution does slightly worse than all other solutions except for the fundamental.

For the fundamental solution the correlation between any actual and simulated series is about zero. The rational bubble and extrapolative expectation solutions, when calibrated to match the trajectories of the data, generate simulated series which are strongly correlated with the data, particularly for the rational bubble solution in the case of the price-rent ratio. The predictions from the near rational bubble solutions exhibit the second highest correlation with the data for the price-rent ratio and the third highest for the net-returns and growth rate of prices. For all variables, the rational solution with \( \alpha \) calibrated to match the second moment of the data produces about half of the correlation between simulated and actual data than the near rational bubble solution. Finally, the extrapolative expectations solution with constant extrapolation parameter, for the parameterization with \( \alpha > 1 \), implies a negative correlation between the actual and model predicted series, the correlation being particularly strong in the case of the price-rent ratio.

Overall, the experiment conducted in this section suggests that even though the extrapolative expectation and the near rational bubble solutions are equally good in predicting key moments of the variables, the near rational bubble solution is more successful in mimicking the evolution of the actual price-rent ratio and price series.
5 Conclusion

The U.S. housing boom-bust episode of the 2000s is difficult to fully explain in terms of fundamental driving forces. By treating houses as a financial asset, this paper uses a stylized asset pricing model to explore the ability of expectations to affect the evolution of house prices and the price-rent ratio in the United States. We consider four solutions to the model, which can be distinguished according to the mechanism agents use to form their expectations.

To evaluate the performance of these solutions we calibrate the model to U.S. quarterly data on house prices and rents. The rational expectation solution requires an implausibly high coefficient of relative risk aversion to match the volatility of the price-rent ratio and is unable to generate enough persistence. The other methods perform better in this dimension. However, neither the rational bubble nor the extrapolative expectation solution with constant extrapolation parameter can simultaneously match the moments of the data and replicate the observed path of the U.S. price-rent ratio and price series. Only the near rational bubble solution is successful in both fronts: for sensible parameterization of the discount factor and the coefficient of relative risk aversion, moments of the variables from a long-run simulation exercise match the moments in the data and the model implied trajectories for the price-rent ratio and price series are consistent with the evolution of the actual data. In particular, the model can replicate the recent asymmetric boom and bust episode experienced in the U.S. housing market. This solution explains real estate cycles merely through shifts in demand driven by exogenous changes in income: it triggers run-ups in prices through waves of overoptimism when rents grow at a pace faster than their mean growth rate and therefore predicts that house prices booms occur together with consumption booms. The simple model adopted in this paper performs surprisingly well, given that it abstracts from other factors that are identified as plausible drivers of the housing market.

Our analysis delivers the additional result, not obvious a priori, that not all deviations from rational expectations can improve the predictions of the model over the rational expectation solution. Although these deviations generate higher persistence than the fundamental solutions, most of them are unable to correctly replicate the path of house prices and the price-rent ratio observed during the latest boom and bust episode in the U.S. residential real estate market.

Overall, we conclude that extrapolative expectations with time variation in the extrapolation parameter might be an important ingredient to include into more sophisticated models that aim at jointly replicating housing market dynamics and business cycle dynamics.
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References


6 Appendix: Data Sources and Transformations

**House Prices**: Case and Shiller Composite 10-city house prices index seasonally unadjusted. The Case-Shiller is a constant-quality home price index constructed using repeated sales. To convert the index into prices we use the November 2011 mean price level from the National Association of Realtors. The house price series is then seasonally adjusted using the U.S. Census Bureau’s X12 seasonal adjustment method and subsequently converted into quarterly data by averaging the monthly observations.

**Rents**: Imputed rental of owner-occupied nonfarm housing seasonally adjusted, series DOWNRC, from NIPA Table 2.4.5U Personal Consumption Expenditures by Type of Product. The imputed rents series available from the National Accounts is constructed as the paid rent adjusted by a coefficient of quality to take into account the higher quality of owner-occupied dwellings. This series is divided by the housing stock of owned and occupied dwellings from the American Housing Survey from US Census, available bi-annually. Bi-annual housing stock data are converted to quarterly by interpolating in the following way: for every year $t$ available we compute the average occupants per dwelling $w_t^A = \frac{POP_t^A}{HS_t^A}$ where POP is the Total Population and HS is the housing stock, owned and occupied dwellings; here the superscript indicates the frequency of the data (A=annual, Q=quarterly). Then we assume the weights are constant over the previous year and set $w_t^Q = w_{t-1}^Q = ... = w_{t-3}^Q = w_t^A$; finally the housing stock is constructed as $HS_t^{Q-j} = w_t^{Q-j} POP_t^{Q-j}$ for $j = 0, ..., 3$ and $\forall t$. For years when the housing stock is not available the occupants per dwelling are computed as the average of the previous and following year.

**Price Indexes**: Rents and house prices are deflated by the personal consumption expenditures deflator: chain-type price index less food and energy, from Bureau of Economic Analysis, seasonally adjusted.