OPTIMAL FISCAL AND MONETARY POLICY WITH HETEROGENEOUS AGENTS AND NONLINEAR TAXATION

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Motivation: Monetary Policy

- Chari, Christiano and Kehoe (1996) show that the Friedman Rule holds in three different representative agent models of money demand: money-in-utility model, cash-credit model and shopping-time model.
Motivation: Monetary Policy

- Chari, Christiano and Kehoe (1996) show that the Friedman Rule holds in three different representative agent models of money demand: money-in-utility model, cash-credit model and shopping-time model.

- Is the Friedman Rule still optimal in the economy with heterogeneous agents? Eroso and Ventura (2002), based on US data, show that low income households use cash for a greater fraction of their total purchases relative to those households with high income.
Motivation: Fiscal Policy

When consider the heterogeneous, they are two different approaches:

Ramsey Approach: When all activities of agents are observable, choose the optimal linear taxes. (Chari, et al (1994); Chamley (1986); Judd (1985))

Mirrlees Approach: When pose the information friction, find the optimal incentive-insurance tradeoff. (Mirrlees (1971); Saez (2001))
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- What is the optimal fiscal policy when money is essential in the economy?
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  Ramsey Approach: When all activities of agents are observable, choose the optimal linear taxes. (Chari, et al. (1994); Chamley (1986); Judd (1985))
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- What is the optimal fiscal policy when money is essential in the economy?

- Try to partially answer the questions mentioned in Kocherlakota (2005) and Albanesia (2008).
Main Results

- Supply a framework to consider the optimal fiscal and monetary policy with heterogeneous agents in an environment where money is essential.
- Friedman rule is no longer optimal when combined with nonlinear taxation of income.
- The capital income taxation is not zero.
Related Literature: Heterogeneous agents

- Extend the Mirrless approach into dynamic economy: New Public Finance
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- Extend the Mirrless approach into dynamic economy: New Public Finance

- Consider the role of money in reduced-form models
- Costa and Werning (2008) use the MIU model to find that the Friedman rule is Pareto optimal if cash holdings and labor effort are gross complements.
- Wen (2010) illustrates heterogeneity matters the CIA economy.
Related Literature: Search-theoretical Models

- Propose a framework based on a explicit micro foundations
- Lagos and Wright (2005) and Williamson and Wright (2010).
Related Literature: Search-theoretical Models

- Propose a framework based on explicit micro foundations.
- Lagos and Wright (2005) and Williamson and Wright (2010).
- Include capital in the economy.
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Basic Setup: Households

- A continuum unit measure of agents with identical preferences, whose skills differ across households and over time.

- An agent privately knows his history \( \theta^t = (\theta_0, ..., \theta_t) \in \Theta^t \) of current and past skill vectors but not his future skill vectors which are governed by Markov process.

- The probability measure of type \( \theta \) is denoted by \( \mu(\theta) \) is stable over time.

- The skill processes are independent across households and there is no aggregate uncertainty in the environment.
Basic Setup: Two subperiods

- Each period is divided into two subperiods: day (centralized market), night (decentralized market).

- In the CM, households rent their previously accumulated capital and supply labor in the competitive market, also choose their consumption level in the good market, and adjust their asset holdings.

- Entering the DM, each household knows his trading status: A given household is a buyer in the DM with fixed probability $\sigma$, a seller with fixed probability $\sigma$, neither a buyer nor a seller with probability $1 - 2\sigma$.

- In the DM, households interact with anonymous bilateral matching.
Production

- Both in the CM and DM, the inputs of production are capital and effective labor. Effective labor $h_i = \theta_i l_i$, where $l_i$ is the household’s labor input. Effective labor $h_i$ is observable, but actual labor $l_i$ and skill $\theta_i$ are not.

- In the CM, a general good is produced by constant-returns technology $Y = F(K, H)$, here $K = \int k(\theta) d\mu(\theta)$, $H = \int h(\theta) d\mu(\theta)$.

- Real wage $w = F_H(K, H)$ and rental rate $r = F_K(K, H)$.

- In the DM, firms do not operate, the sellers’ own effective labor $e$ and capital $k$ are used to produce with the technology $q = f(e, k)$.

- The effort is $e = c(q, k)$ and disutility is $c(q, k)/\theta$, here $c_q, c_{qq}, c_{kk} > 0, c_k, c_{qk} < 0$. 
Government

- Government consumption takes place in the CM.

- The expenditure is financed by taxes, money creation and debt issuance.

- The taxation is a nonlinear function of agent’s effective labor supply and capital stock in that period, denoted by \( T(h_t, k_t) \).

- The budget constraint for government in period \( t \) is

\[
M_t + B_t + P_t \int T(h_t, k_t) d\mu(\theta) = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}
\]

- No-Ponzi constraint

\[
\sum_{t=0}^{\infty} \left[ \frac{P_t}{P_0} \prod_{s=0}^{t-1} \frac{1}{R_{s-1}} \left( G_t - \frac{M_t - M_{t-1}}{P_t} \right) - \int T(h_t, k_t) d\mu(\theta) \right] \leq 0
\]
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In the CM

- Household enters the CM with $m_{t-1}$, $b_{t-1}$ and $k_t$, and knows his own skill type $\theta_t$.

- Instantaneous utility in the period $t$ CM is $U(x_t) - l_t$, where $x_t$ is consumption level and $l_t$ is labor.
The household’s problem in CM

\[ W(\theta_t; m_{t-1}, b_{t-1}, k_t) = \max_{x_t, l_t, m_t, b_t, k_{t+1}} U(x_t) - l_t + V(\theta_t; m_t, b_t, k_{t+1}) \]

subject to

\[ P_t(x_t + (k_{t+1} - (1 - \delta - r_t)k_t)) + m_t + b_t = P_t(w_h - T(h_t, k_t)) + m_{t-1} + R_{t-1}b_{t-1} \]

here, \( W(\theta_t; m_{t-1}, b_{t-1}, k_t) \) be the value function for an agent with type \( \theta_t \) at the beginning of CM, and \( V(\theta_t; m_t, b_t, k_{t+1}) \) be the value function in the DM.
The First-order Conditions

\[ U'(x_t) = V_k(\theta_t; m_t, b_t, k_{t+1}) = P_t\lambda_{\theta_t} \]  \hspace{1cm} (1)
\[ W_m(\theta_t; m_{t-1}, b_{t-1}, k_t) = V_m(\theta_t; m_t, b_t, k_{t+1}) = \lambda_{\theta_t} \]  \hspace{1cm} (2)
\[ W_b(\theta_t; m_{t-1}, b_{t-1}, k_t) = R_{t-1}V_b(\theta_t; m_t, b_t, k_{t+1}) = R_{t-1}\lambda_{\theta_t} \]  \hspace{1cm} (3)
\[ W_k(\theta_t; m_{t-1}, b_{t-1}, k_t) = (1 + r_t - \delta - T_k(h_t, k_t))V_k(\theta_t; m_t, b_t, k_{t+1}) \]  \hspace{1cm} (4)

Here, \( \lambda_{\theta_t} = \frac{1}{P_t\theta_t(w_t-T_h(h_t,k_t))} \) as the agents with skill type \( \theta_t \)' marginal value of entering period \( t \) with one extra unit of money.

**Remark**

The value functions \( W(.) \) and \( V(.) \) are linear, and a given household’s marginal utility of wealth is independent of its trading status in the previous DM, but depends on the skill type.
In the DM

The household’s problem in DM is

\[ V(\theta_t; m_t, b_t, k_{t+1}) = \sigma[u(q^b_t) + \beta E_t(W(\theta_{t+1}; m_t - d^b_t, b_t, k_{t+1}))] \]
\[ + \sigma[-c(q^s_t, k_{t+1})/\theta_t + \beta E_t(W(\theta_{t+1}; m_t + d^s_t, b_t, k_{t+1}))] \]
\[ + (1 - 2\sigma)\beta E_t(W(\theta_{t+1}; m_t, b_t, k_{t+1})) \]

(5)

Here, \((q^b_t, d^b_t)\) and \((q^s_t, d^s_t)\) denote the quantity of goods and dollars exchanged when the agent is buyer and seller respectively.

Since the value function \(W(.)\) is linearity, rewrite the equation (5) as follow

\[ V(\theta_t; m_t, b_t, k_{t+1}) = \sigma[u(q^b_t) - c(q^s_t, k_{t+1})/\theta_t - \beta E_t(\lambda_{\theta_{t+1}}(d^b_t - d^s_t))] \]
\[ + \beta E_t(W(\theta_{t+1}; m_t, b_t, k_{t+1})) \]

(6)
Bargaining

Let $\phi \in [0, 1]$ denote the bargaining power of the buyer.

The bargaining problem is

$$\max_{q_{it}, d_{it}} \left[ u(q_{it}) - \beta E_t(\lambda_{\theta_{i,t+1}} d_{it}) \right]^\phi \left[ -c(q_{it}, k_{j,t+1})/\theta_{jt} + \beta E_t(\lambda_{\theta_{j,t+1}} d_{it}) \right]^{1-\phi}$$

subject to

$$d_{it} \leq m_{it} \quad (7)$$
The First-order Conditions

The relation between $d_{it}$ and $q_{it}$ is $d_{it} = z(\theta_{it}, \theta_{jt}, q_{it}, k_{j,t+1})$.
where $z(\theta_{it}, \theta_{jt}, q_{it}, k_{j,t+1}) =
\frac{\phi u'(q_{it})c(q_{it}, k_{j,t+1}) + (1-\phi)c_q(q_{it}, k_{j,t+1})u(q_{it})}{\phi \theta_{jt} u'(q_{it}) \beta E_t \lambda_{j,t+1} + (1-\phi)c_q(q_{it}, k_{j,t+1}) \beta E_t \lambda_{i,t+1}}$

Remark

1. The amount of cash demand $d_{it}$ not only depends on the buyer’s money holding and the seller’s capital level, but also related with their skill type. Here $z_q > 0$ and $z_k < 0$.

2. The constraint (7) may not bind here, which depends on the agents’ skill type.
The first-order conditions in DM

\[ V_m(\theta_t; m_t, b_t, k_{t+1}) = \sigma \left[ u_q \frac{\partial q^b_t}{\partial m_t} - \beta E_t(\lambda_{\theta_{t+1}}) \frac{\partial d^b_t}{\partial m_t} \right] + \beta E_t(\lambda_{\theta_{t+1}}) \]  

(8)

\[ V_k(\theta_t; m_t, b_t, k_{t+1}) = \sigma \left[ \frac{c_q}{\theta_t z_q} - \frac{c_k}{\theta_t} + \beta E_t(\lambda_{\theta_{t+1}}) z_k \right] + \beta E_t(W_k(\theta_t; m_t, b_t, k_{t+1})) \]

(9)

**Remark**

Let \( \gamma(\theta_t, q_t, k_{t+1}) = \frac{c_q}{\theta_t z_q} - \frac{c_k}{\theta_t} + \beta E_t(\lambda_{\theta_{t+1}}) z_k \), we have intertemporal optimal condition:

\[ U'(x_t) = \beta E_t[U'(x_{t+1})((1 + F_K(K_{t+1}, H_{t+1})) - \delta - T_K(k_{t+1}, h_{t+1}))] + \sigma \gamma(\theta_t, q_t, k_{t+1}) \]

(10)
Monetary Equilibrium

**Definition**

Given policy processes \( \{T(\cdot), R_t\} \), the government spending process \( \{G_t\} \), and the initial conditions \( \{M_0, B_0, K_0\} \), an monetary equilibrium is a collection of \( \{x_t, q_t, k_t, l_t, P_t, m_t, M_t, b_t, B_t, r_t, w_t\} \) such that

i. households optimize \( \{x_t, l_t, k_t, m_t, q_t\} \) to maximize their utility, subject to the budget constraint, taking the price \( \{P_t, r_t, w_t\} \) and the policy processes as given;

ii. Government budget constraints hold every period;

iii. market clear: \( \int m_t d\mu(\theta) = M_t; \int b_t d\mu(\theta) = B_t; \int k_t d\mu(\theta) = K_t; \int h_t d\mu(\theta) = H_t \) and resource constraint

\[
\int x_t d\mu(\theta) + G_t + K_{t+1} = F(K_t, H_t) + (1 - \delta)K_t
\]
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**Feasible Condition**

**Definition**

Define an allocation \((x, h, K, q, e)\) to be feasible if

\[
\int x_t d\mu(\theta) + G_t + K_{t+1} = F(K_t, \int h_t d\mu(\theta)) + (1 - \delta)K_t \tag{11}
\]

\[
\int q_t d\mu(\theta) = f(\int e_t d\mu(\theta), K_{t+1}) \tag{12}
\]

for all \(t\)
Incentive-compatible

Since the skill history $\theta^t$ is private information, the allocations must respect incentive-compatibility conditions. Let
\[
\tilde{W}(\xi; x, h, q, e) = E_t\left(\sum_{i=0}^{\infty} \beta^i \left[ U(x_{t+i}(\xi)) - \frac{h_{t+i}(\xi)}{\theta_{t+i}} + \sigma(u(q_{t+i}(\xi)) - \frac{e_{t+i}(\xi)}{\theta_{t+i}}) \right]\right)
\]
which is the utility from reporting strategy $\xi$ for agent with skill type $\theta_t$.

**Definition**

An allocation $(x, h, K, q, e)$ is incentive-compatible if
\[
\tilde{W}(\xi^*; x, h, q, e) \geq \tilde{W}(\xi; x, h, q, e)
\]
for any $\xi \in \Sigma^T$, while $\xi^*(\theta^t) = \theta^t$ for all $\theta^t$ is the truth-telling strategy.
Social Planner’s Problem

The social planner’s problem is

$$\max_{x,h,K,q,e} \sum_{i=0}^{\infty} \beta^i \int U(x_{t+i}) - \frac{h_{t+i}}{\theta_{t+i}} + \sigma(u(q_{t+i}) - \frac{e_{t+i}}{\theta_{t+i}})d\mu(\theta)$$

subject to feasible condition (11)(12) and incentive-compatible condition (13).
PROPOSITION

There exists \( \{x^*, h^*, K^*, q^*, e^*\} \) solve the social planner’s problem, which satisfies,

\[
1 - \delta + F_K(K_{t+1}^*, \int h_{t+1}^* d\mu(\theta)) + \sigma f_K(\int e_t^* d\mu(\theta), K_t^*) \leq U'(x_t^*) \leq E_t \frac{1}{\beta U'(x_{t+1}^*)} \tag{14}
\]
Optimal Policy

PROPOSITION

- The optimal nominal interest rate is greater than 1, which means the Friedman rule is not optimal.
- If the probability of being seller is small, the optimal taxation on capital income is positive.

Note: Use the Jansen Inequality, then compare the result with the intertemporal optimal conditions, such as equation (10), we could get the results
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Further Study

- Do the numerical analysis of this model and find out the welfare cost of taxation and inflation.

- Besides the money as a payment instrument, consider the other payment methods, such as credit in the economy. What is the effect of monetary policy?

- Introduce the financial intermediation in the environment. As different agents face different financial constraints, try to explain the phenomenons in the asset market.