Model Selection Testing for Diffusion Processes with Applications to Interest Rate and Exchange Rate Models

Abstract

A model selection test for nonnested Misspecified diffusion models is developed by using a criterion based on the Kullback-Leibler information criterion in a new asymptotic framework that accounts for the relative significance of diffusion functions to drift functions depending on sampling frequencies. The test examines the hypothesis that two competing models are equivalent in the criterion. Our approach differentiates the roles of diffusion and drift functions; it is of primary importance for a model to have a diffusion function close to a true diffusion function for superiority when the sampling frequency is high, and we use drift functions if the models can not be distinguished by the diffusion functions. The comparative importance of diffusion functions increases with sampling frequencies. We show the informative signal for ranking drift functions is weaker as sampling frequencies become higher compared to a sampling time span. The drift functions become meaningful only when we sample data for long enough. Our new asymptotics deals with the different rates of information by considering both the sampling interval $\Delta$ and the sampling span $T$, and we show the sampling span must increase at a relative speed faster than $1/\Delta^2$ (or $\Delta^2T \rightarrow \infty$) to ensure sufficient information to be collected for distinguishing two models by drift functions. The limiting distribution of the test statistic is normal, and we compare different asymptotic approximations to the sampling distribution of the test statistic using the subsampling, and the nonparametric block bootstrap methods, as well as the standard normal approximation for the test statistics standardized by the heteroskedasticity autocorrelation consistent variance estimators. We apply our test to the model selection problems for spot interest rate models and exchange rate models. We find that many popular models are observationally equivalent.