Microeconomic Theory

Qualifier Exam

May 2009

Part I

1. [30 total points] The following question concerns the weak axiom of revealed preferences (WARP).

   (a) [5 points] In your own words, briefly explain what the weak axiom means.

   (b) [2.5 points] State the weak axiom in terms of choice sets (i.e., $C(B)$).

   (c) [10 points] Recall Sen's property α and β:

   Sen's Property α. *For all $B, B' \in \mathfrak{B}$ if $x \in B \subset B'$, and $x \in C(B')$, then $x \in C(B)$.*

   Sen's Property β. *If $x, y \in C(B), B \subset B'$ and $y \in C(B')$, then $x \in C(B')$*

   Prove that if a choice rule structure satisfies WARP it satisfies Sen's α and β.

   (d) [2.5 points] State the weak axiom in terms of demand functions, price and wealth levels (i.e., $x(p, w)$).

   (e) [10 points] Prove that if $x(p, w)$ is homogenous of degree zero and satisfies the property γ (below), it satisfies the weak axiom.

   Property γ. *For some $w > 0$ and all p, p' we have $p' \cdot x(p, w) > w$ whenever $p \cdot x(p', w) \leq w$ and $x(p', w) \neq x(p, w)$.*

2. [35 total points] Consider the following utility function

$$u(x_1, x_2) = x_1 - \frac{\beta}{x_2^2}$$

where $\beta > 0$.

(a) [5 total points] Set up the Lagrangian and write out the Kuhn-Tucker conditions for the utility maximization problem (UMP) with constraint $w \geq p_1x_1 + p_2x_2$ where $w, p_1, p_2 > 0$. Note that you do not need to solve anything at this point.

(b) [5 total points] Under what conditions for β do we have a corner solution? Your answer should not include λ.

For the remaining items assume there is an interior solution (i.e. ignore corner solutions in your answers).

(c) [10 total points] Derive the demand function $x(p, w)$ in the case of an interior solution. You may use any method you choose.

(d) [5 total points] Derive the indirect utility function $v(p, w)$.

(e) [10 total points] Derive the Hicksian demand $h(p, u)$ and expenditure $e(p, u)$ functions using the duality identities.

3. [35 total points] An individual has expected utility preferences over levels of *wealth* given by

$$u(x) = \alpha x^2$$

where $\alpha > 0$

(a) [5 total points] What is his certainty equivalent for a gamble to gain \$100 with probability $\frac{2}{3}$ and lose \$50 with probability $\frac{1}{3}$? Assume his current wealth is w.

(b) [2.5 total points] Suppose instead his utility functions are defined over income. What is his certainty equivalent for a gamble to gain \$50 with probability $\frac{1}{3}$ and gain \$25 with probability $\frac{2}{3}$?

(c) [2.5 total points] What might be different if this individual had prospect theory preferences?

For the remainder of the questions assume the individual has preferences described in part 3a (expected utility preferences over wealth).

(d) [2.5 total points] Is this individual risk averse, risk seeking, or risk neutral? How do you know?

(e) [2.5 total points] When choosing between two gambles, a or b, which gamble would this individual prefer, if a first order stochastically dominated b? a second order stochastically dominated b?

(f) *Insurance.* [20 total points] Suppose there are two possible states of the world. State 1 occurs with probability $\frac{2}{3}$; state 2 occurs with probability $\frac{1}{3}$. In state 1 the individual will have wealth of \$100. In state 2 he will have wealth of \$50. The individual's utility function is the same in either state. He may pay $-z_1$ if state 1 occurs and receive z_2 if state 2 occurs.

   i. [10 points] Prove this individual prefers no insurance to any insurance policy that is actuarially fair. You may assume reasonable bounds for the insurance payment (i.e., $z_1 < 0, z_2 < 50$).

   ii. [5 points] Suppose insurance covers all uncertainty, that is, $100 + z_1 = 50 + z_2$. What is the most the individual will pay in state 1 ($-z_1$) and still prefer to have insurance? What is the expected loss/gain for the individual on such policy?

   iii. [5 points] Suppose this individual was offered the following insurance policy: pay \$5 if state 1 occurs, receive \$15 if state 2 occurs. Yet, given the same preferences and level of wealth as before, the individual did not accept the policy. This result could be explained by subjective utility theory (SEU). What is the highest subjective probability of state 2 occurring that could explain this result?

Part II

II.(1) **(25 points)** Consider a pure exchange economy with L commodities and n consumers, $i = 1, \ldots, n$, each having initial endowment vector $w_i \in \mathbb{R}^L_+$ and preference orderings $\succcurlyeq_i$ which are defined on the consumption set $X_i = \mathbb{R}^L$ and satisfy strict convexity and strict monotonicity.

(a) **(5 pts)** What can be said about the aggregate excess demand $\hat{z}$ of this economy? (be sure to specify the domain and range of the mapping $\hat{z}$.)

(b) **(5 pts)** For each property of $\hat{z}$ you state in part (a), identify which assumption(s) on preferences are needed for the property and then prove that the assumption(s) you identify imply the property.

(c) **(15 pts)** State a theorem that guarantees the existences of competitive equilibrium and outline the proof. Please add additional assumptions if necessary.

II.(2) **(25 points)** Consider a pure exchange economy with L commodities and n consumers, $i = 1, \ldots, n$, each having initial endowment vector $w_i \in \mathbb{R}^L_+$ and preference orderings $\succcurlyeq_i$ defined on $\mathbb{R}^L_+$ can be represented by a differentiable function $u_i : \mathbb{R}^L_+ \to \mathbb{R}$ that satisfies $Du_i(x_i) \in \mathbb{R}^L_{++}$. Let A be the set of feasible allocations. Answer the following questions.

(a) (8 pts) Suppose u_i satisfies an additional assumption (please specify the assumption). Derive a system of equations such that a feasible allocation $x^* \in \mathbb{R}^{nL}_{++}$ is Pareto efficient if and only if x^* solves the following maximization problem

$$\max_{x \in A} \sum_{i=1}^{n} a_i u_i(x_i)$$

for some $a \in \mathbb{R}^L_{++}$.

(b) (8 pts) Define *competitive equilibrium with transfer* in this economy. Also derive a system of equations such that $(x^*, p^*) \in \mathbb{R}^{nL}_{++} \times \mathbb{R}^L_{++}$ is a competitive equilibrium with transfer if and only if (x^*, p^*) solves the system of equations for some multipliers (again state any additional assumption you used).

(c) (9 pts) Using the results from (a) and (b), prove the following version of the second welfare theorem in this economy (with an appropriate assumption on utility functions): "if $x^* \in \mathbb{R}^{nL}_{++}$ is Pareto efficient, then there exists a price vector $p^* \in \mathbb{R}^L_{++}$ such that (x^*, p^*) is a competitive equilibrium with transfer".

II. 3 **(25 points)** Consider a public goods economy with one private good, x, one public good, y, and n agents. Each agents consumption set is the nonnegative quadrant. Each agent i is endowed with w_i units of private good. There are no initial endowments for the public good, but the public good can be produced from the private good, according to a production technology $y = v$. Utility functions of agents are given by

$$u_i(x_i, y) = x_i + c_i \ln y, \quad c_i > 0.$$

(a) **(5 pts)** Define the Lindahl equilibrium and Pareto efficient allocation for this economy.

(b) **(5 pts)** Find the set of interior Lindahl equilibria.

(c) **(5 pts)** For the above economy, find the set of interior competitive equilibria (if necessary, specify values c_i for which competitive equilibria exist).

(d) **(5 pts)** Does the First Theorem of Welfare Economics hold for competitive equilibria in the above economy? Justify your answer.

Part III

1. Consider a Cournot Oligopoly Model with a linear inverse demand function given by $p = M - q$, where p is market price, q is quantity of widgets, and M is a constant. Each widget costs a firm c to produce.

(a) Suppose that each firm has to pay a fixed cost of F regardless of the quantity it produces in order to enter the widget industry. Show that no firm's behavior is modified, provided that the fixed cost F is less than each player's equilibrium profit. Find the threshold for F. Compute equilibrium profits with fixed entry cost.

(b) If the fixed cost exceeds the equilibrium profit with n firms, then at least one firm would have been better off if it had not entered the widget industry. Assuming there are no barriers to entry other than payment of the fixed entry cost of F, determine the number of firms that will end up producing widgets.

(c) What happens to the number of firms, market price, firm output, firm profits, and consumer surplus as F goes to 0?

2. Player 1, the "government," wishes to influence the choice of player 2. Player 2 chooses an action $a_2 \in A_2 = \{0,1\}$ and receives a transfer $t \in T = \{0,1\}$ from the government, which observes a_2. Player 2's objective is to maximize the expected value of his transfer, minus the cost of his action, which is 0 for $a_2 = 0$ and ½ for $a_2 = 1$. Player 1's objective is to minimize the sum $2(a_2 - 1)^2 + t$. Before player 2 chooses his action, the government can announce a transfer rule $t(a_2)$.

   a) Draw the extensive form for the case where the government's announcement is not binding and has no effect on payoffs.

   b) Draw the extensive form for the case where the government is constrained to implement the transfer rule it announced.

   c) Give the strategic forms for both games.

   d) Characterize the subgame-perfect equilibria of the two games.

3. Consider the following stag game:

	V	G
V	(1,1)	(-10,10)
G	(10,-10)	(-100,-100)

a) List all Nash Equilibria of this game.

Now consider the repeated game $\Gamma(\delta)$ that consists of playing the stage game twice, where δ is the probability of a second stage. Let the row player be denoted 1 and the column player be denoted 2. Let σ_{it} denote the probability that player $i \in \{1,2\}$ takes action V in period $t \in \{0,1\}$.

b) Find the lower bound $\underline{\delta}$ such that $(\sigma_{10}, \sigma_{20}) = (1,1)$ can be supported in a subgame perfect Nash equilibrium if and only if $\delta \geq \underline{\delta}$.

c) Specify the second-period strategies that support $(\sigma_{10}, \sigma_{20}) = (1,1)$ in a subgame perfect Nash equilibrium if and only if $\delta \geq \underline{\delta}$.