Microeconomic Theory

Qualifier Exam

July 2009

Part I

1. [30 total points] Consider an economy where only three goods are sold. There are three possible states of the world with three different price levels (p_1, p_2, p_3), defined as vectors, $p_1 = (2, 1, 2)$, $p_2 = (2, 2, 1)$, and $p_3 = (1, 2, 2)$. That is, in state 1, good 1 has price 2, good 2 has price 1 and good 3 has price 2. There is an individual who has wealth of 8 in all three states $(w_1 = w_2 = w_3 = 8)$

   (a) [10 points] Draw this individual's budget sets for each of the three states. What shape are the budget sets? Label the points where they intersect each axis.

   (b) [10 total points] Suppose in state 1 the consumer chooses $x_1 = (1, 2, 2)$.

      i. [5 points] What possible values could he choose in state 2 that would violate the weak axiom? In addition, provide a specific example of such a point that would violate the weak axiom.

      ii. [5 points] What possible values could he choose in state 3 that would violate the weak axiom? In addition, provide a specific example of such a point that would violate the weak axiom.

   (c) [10 points] Suppose $x_1 = (1, 2, 2)$, $x_2 = (2, 1, 2)$, $x_3 = (2, 2, 1)$. Show that for any two pairs of choices the weak axiom is satisfied, but the preference relation revealed from these choices violates transitivity.

2. [40 total points] Suppose a firm has production function

$$f(z_1, z_2) = k + \alpha \log z_1 - \frac{\beta}{z_2^2}$$

where $k > 0$, $\beta > 0$, $\alpha > 0$.

(a) [2.5 points] Is it possible that only input 1 or 2 will be used in production? Explain.

(b) [2.5 points] Suppose for some α, β, and z_1', z_2' we have $f(2z_1', 2z_2') = 3f(z_1', z_2')$, is this an example of constant, decreasing or increasing returns to scale?

(c) [5 points] Write out the profit maximization problem (PMP) for this firm.

(d) [10 points] Solve the PMP to derive the factor demand function.

(e) [5 points] Derive the profit and optimal output level functions.

For the remainder of the problem, suppose the firm had the following factor demand and optimal output level functions (i.e., ignore your answers to part c and d):

$$\begin{aligned} z_1(p, w_1, w_2) &= \frac{\alpha p}{w_1} \\ z_2(p, w_1, w_2) &= \frac{\beta p}{w_2} \\ q(w_1, w_2, p) &= \alpha \log \frac{\alpha p}{w_1} + \beta \log \frac{\beta p}{w_2} \end{aligned}$$

(f) [5 points] Derive the conditional factor demand and cost functions from the factor demand and optimal output functions (given above).

(g) [10 points] Sketch the following four graphs if $\alpha + \beta < 1$. Do not calculate anything, just draw the shape of the curves.

i. A graph of the production set in terms of z_1 (holding z_2 constant).

ii. A graph of the production set in terms of z_2 (holding z_1 constant).

iii. A graph of the cost function.

iv. A graph of the marginal cost and average cost functions.

3. [30 total points] Recall the following types of utility functions, (1) constant absolute risk aversion (CARA) and (2) constant relative risk aversion (CRRA). These utility functions will be defined over *levels of wealth.*

$$u(x) = -\alpha e^{-ax} + \beta \quad (1)$$

$$u(x) = \frac{\alpha x^{1-\rho}}{1-\rho} \quad (2)$$

for $\alpha > 0$.

(a) [2.5 points] For what values of a is the CARA function risk averse?

(b) [2.5 points] For what values of α, ρ is the CRRA function risk averse?

(c) [5 total points] Write an expression for the certainty equivalent for a 50/50 gamble between \$0 and \$100.

i. [2.5 points] for CARA utility?

ii. [2.5 points] for CRRA utility?

(d) [10 points] Show, for an individual with CARA utility, that the certainty equivalent for a lottery between \$100 and \$0 with 50/50 probability is the same for any level of wealth, $w > 0$.

(e) [10 points] Show, for an individual with CRRA utility, that the ratio of the certainty equivalent to wealth (ceq/w) for a lottery between a 10% gain in wealth and \$0 with 50/50 probability is the same for any level of wealth, $w > 0$.

Part II

II.(1) **(25 points)** Consider a pure exchange economy with L commodities and n consumers, $i = 1, \ldots, n$, each having initial endowment vector $w_i \in \mathbb{R}^L_+$. Define the equilibrium correspondence $CE : \mathbb{R}^{nL}_{++} \twoheadrightarrow \mathbb{R}^{nL}_{++} \times \mathbb{R}^{L-1}_+$ by

$$CE(w) = \{(x, p) \in \mathbb{R}^{nL}_{++} \times \mathbb{R}^{L-1}_+ : (x, p) \text{ is competitve equilibrium } \},$$

where p^1 is normalized to be 1, i.e. $CE(w)$ is the set of competitive equilibria when the initial endowment vector is $w = (w_1, \ldots, w_n)$ (the preference is fixed). Consumer i's preference can be represented by a continuously differentiable (i.e. $Du_i(x_i)$ is continuous), strictly increasing, and concave function $u_i : \mathbb{R}^L \to \mathbb{R}$ that satisfies the following two conditions:

(1) $Du_i(x_i) \in \mathbb{R}^L_{++}$

(2) If $u_i(x) \geq u_i(x_i')$ for some $x_i' \gg 0$, then $x_i \gg 0$ for $i = 1, \ldots, n$.

Answer the following questions.

(a) (5 pts) Write down a system of equations to characterize the set of competitive equilibria $CE(w)$ for $w \in \mathbb{R}^L_{++}$.

(b) (5 pts) "$CE(\cdot)$ is upper hemi-continuous at $w \in \mathbb{R}^L_{++}$." Translate this into a precise mathematical statement regarding the equations from (a).

(c) (15 pts) Prove that $CE(\cdot)$ is upper hemi-continuous at any $w \in \mathbb{R}^L_{++}$.

II. 2 **(25 points)** Consider a pure exchange economy $e = \{(X_i, \succcurlyeq_i, w_i) : i = 1, ..., n\}$, where $w_i \in X_i = \mathbb{R}^L_+$.

(a) **(5 pts)** Define *Pareto efficiency* and *competitive equilibrium with transfers* for this economy.

(b) **(5 pts)** State the Second Fundamental Theorem of Welfare Economics (you should specify the assumptions under the theorem holds).

(c) **(15 pts)** Prove the theorem under the assumptions given in (a).

II. 3 **(25 points)** In a regulation model, the principal is a regulator who maximizes a weighted average of the agents' surplus $S(q) - t$ and of a regulated monopoly's profit $U = t - C(q, \theta)$, with a weight $\alpha < 1$ for the firms profit. Here, q is the output which is observed perfectly by the principal, t is the transfer payment from the agents to the monopoly, $S(q)$ is the agents' value function with $S' > 0$, $S'' < 0$ and $S(0) = 0$, $C(q, \theta)$ is the monopoly's cost function with $C_q > 0$, $C_\theta > 0$, $C_{qq} > 0$ and $C_{q\theta} > 0$, where which is not observable and are distinguished by $\theta \in \{\underline{\theta}, \bar{\theta}\}$ with respective probabilities ν and $1 - \nu$, and $\Delta\theta \equiv \bar{\theta} - \underline{\theta} > 0$. Because $\alpha < 1$, it is socially costly to give up a rent to the firm. The contract is signed at the interim stage.

(a) **(5 pts)** Write down the principal's incentive-compatible optimization problem.

(b) **(5 pts)** What are the optimal payments $\bar{t}$ and $\underline{t}$ and informational rent to the firm?

(c) **(5 pts)** Find the first-order conditions that characterize the principal's incentive-compatible problem. Discuss the effect of changes in α on the informational rent and the output distortion.

(d) **(5 pts)** Suppose $S(q) = q$ and $\alpha = 1/2$, $\underline{\theta} = 1$ and $\bar{\theta} = 2$, and $\nu = 1/2$. Find the optimal incentive scheme for $\{(\underline{q}, \underline{t}), (\bar{q}, \bar{t})\}$.

(e) **(5 pts)** Does this scheme yield the first best outcome? Justify your answer.

Part III

1. Consider a price competition (Bertrand) model with profit maximizing firms, *i*, that set prices, p_i and can meet all resulting demand. Let $N = \{1,2, \ldots, n\}$ denote the set of firms. The firms that set the lowest price split the resulting demand equally and the remaining firms sell zero units. Given $(p_i)_{i \in N}$, the sales of firm *i* are $q_i = \frac{D(p_i)}{k}$ if $p_i \leq p_j$ for all $j \in N$, where $k = \#\{j \in N \mid p_j = p_i\}$, and equal to 0 otherwise.

    a. Suppose fixed costs are zero and marginal costs are constant and equal *c*, characterize the Nash Equilibria.

    b. Compare the properties of these equilibria with the case of Monopoly and Perfect Competition.

    c. Suppose there is an avoidable fixed production cost *F* and that a Monopoly would be strictly viable. Let the cost function be $C(q_i) = F + cq_i$ if $q_i > 0$ and $C(0) = 0$. Characterize when an equilibrium does or doesn't exist.

    d. Does your answer to c change if you allow mixed strategies? If so, how?

2. First define essential concepts and then prove that every finite game has a Nash Equilibrium possibly in mixed strategies.

3. First define essential concepts and then prove that if there is a unique strategy combination that survives iterated removal of strictly dominated strategies, the strategy combination is a Nash Equilibrium.