Mircoeconomics Qualifier Examination
August 2008

Answer all questions. Show all your calculations. Each question, and each part of the exam, is with the same number of points. Write Legibly. Good Luck.

Part I

1. *(25 points)* A household consists of two individuals who are both potential workers and who pool their budgets. The preferences are represented by a single utility function $U(x_0, x_1, x_2)$ where x_1 is the amount of leisure enjoyed by person 1, x_2 is the amount of leisure enjoyed by person 2, and x_0 is the amount of the single, composite consumption good enjoyed by the household. The two members of the household have, respectively (T_1, T_2) hours which can either be enjoyed as leisure or spent in paid work. The hourly wage rates for the two individuals are w_1, w_2 respectively, they jointly have non-wage income of $\bar{y}$, and the price of the composite consumption good is unity.

   a) Write down the budget constraint for the household.

   b) If the utility function U takes the form

   $$U(x_0, x_1, x_2) = \sum_{i=0}^{2} \beta_i \log(x_i - a_i).$$

   where a_i and β_i are parameters such that $a_i > 0$ and $\beta_i > 0$, $\beta_0 + \beta_1 + \beta_2 = 1$, interpret these parameters. Solve the household's optimization problem and show that the demand for the consumption good is:

   $$x_0^* = a_0 + \beta_0[[\bar{y} + w_1 T_1 + w_2 T_2] - [a_0 + w_1 a_1 + w_2 a_2]].$$

   c) Write down the labor supply function for the two individuals.

   d) Suppose this model is developed to address the issue of whether labor for worker one in the household is a substitute or complement for labor for worker two.
      i) What is the theoretical prediction?
      ii) Comment on the empirical usefulness of this model for the question at hand.

2. *(25 points)* An individual taxpayer has an income y that he should report to the tax authority. Tax is payable at a constant proportionate rate t. The taxpayer reports x where $0 \le x \le y$ and is aware that the tax authority audits some tax returns. Assume that the probability that the taxpayer's report is audited is π, that when an audit is carried out the true taxable income becomes public knowledge and that, if $x < y$, the taxpayer must pay both the underpaid tax and a surcharge of s times the underpaid tax.

a) If the taxpayer chooses $x < y$, show that disposable income c in the two possible states of the world is given by

$$c_{NOAUDIT} = y - tx,$$
$$c_{AUDIT} = [1 - t - st]\, y + stx.$$

b) Assume that the individual chooses x so as to maximize the utility function

$$[1 - \pi]\, u(c_{NOAUDIT}) + \pi u(c_{AUDIT})$$

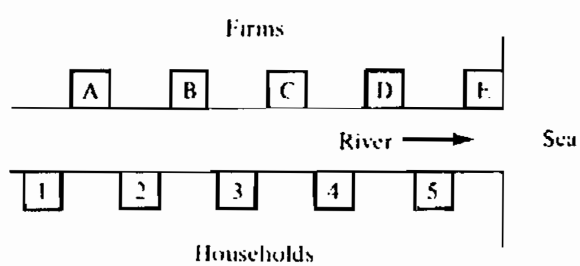
where u is increasing and the individual is risk-averse.

i) Write down the FOC for an interior maximum.

ii) Show that if $1 - \pi - \pi s > 0$ then the individual will definitely under-report income.

c) Assume the optimal income report x^* satisfies $0 < x^* < y$: If true income increases will under-reported income increase or decrease? Explain.

3. *(25 points)* Consider the following map:



i) Each firm exudes 12 gallons of detergent (x) per period, which flows downstream.

ii) Each household likes swimming in clean water and is willing to pay $1 for every gallon of detergent less that there is in the water.

iii) Each firm could avoid producing detergent if it put its effluent through a filter costing $25 per period.

iv) Each household could install its own swimming pool (and so provide a swim in clean water) at a cost of $40 per period.

a) What private arrangements, if any, do you expect to emerge if the firms are not liable?

b) What, if anything, do you think the government should do? What additional information, if any, is relevant?

Part II

1. Consider a game between a (potential) Entrant and an Incumbent. The Entrant can stay out of the market, or enter in a prepared manner or enter in an unprepared manner. The Incumbent does not observe how prepared the Entrant is in case she chooses to enter. All payoffs are measured in utils. If the Entrant stays out of the market, she gets a payoff of 2 and the Incumbent gets a payoff of 4. If the Entrant chooses to enter in a prepared manner and the Incumbent chooses to fight entry then both players get a payoff of 1 and if the Incumbent chooses to accommodate entry then the Entrant gets a payoff of 3 and the Incumbent gets a payoff of 2. If the Entrant chooses to enter in an unprepared manner and the Incumbent chooses to fight entry then the Entrant gets a payoff of 0 and the Incumbent a payoff of 3, and if the Incumbent chooses to accommodate entry then the Entrant gets 4 and the Incumbent gets 2.

   (a) Represent the game between the Entrant and the Incumbent in Extensive form.

   (b) Propose a suitable equilibrium concept for the game and explain why your proposed equilibrium concept is suitable.

   (c) Find all equilibrium of your proposed equilibrium concept.

   (d) Consider a variant of the above game in which the Entrant makes two rounds of decisions (and, hence, has two information sets): First she decides whether to Enter or not and then she decides whether to be prepared or not. The Incumbent, as earlier, does not get to observe how prepared the Entrant, and chooses between fighting or accommodating (entry). Payoffs are as before. Now propose a suitable equilibrium concept (and defend your proposal) and find all the pure strategy equilibria of your proposed equilibrium concept.

2. Suppose there are $N \geq 2$ players. Each player must simultaneously choose an integer between 1 and 100. The person who chooses the number closest to one-third of the average of all the numbers chosen wins a prize of 100 utils whereas others get nothing. The prize is evenly split if there are ties.

(a) Does any pure strategy strictly dominate 100?

(b) Find a mixed strategy that strictly dominates 100.

(c) Is 99 strictly dominated?

(d) Argue that the iterated removal of strictly dominated strategies yields a unique choice for each player. What is it?

3. Nature informs Alice about her type. With probability ρ she is strong and with probability $(1-\rho)$ she is weak. Bob doesn't know Alice's type but does know the probability that she is weak or strong (in fact, this is common knowledge). Alice can choose between Q or B. Bob observes her choices. Bob can choose between N and F. If Alice is strong and the (pure) strategy profile (Q, N) is realized then Alice and Bob's payoffs are $(2, 1)$ and if (Q, F) is realized then their payoffs are $(0, 0)$. If Alice is strong and the strategy profile (B, N) is realized then payoffs are $(3, 1)$ and if (B, F) is realized then payoffs are $(1, 0)$. If Alice is weak and (Q, N) is realized then payoffs are $(3, 0)$ and if (Q, F) is realized then payoffs are $(1, 1)$. If Alice is weak and (B, N) is realized then payoffs are $(2, 0)$ and if (B, F) is realized then payoffs are $(0, 1)$.

   (a) Represent the above game in extensive form.

   (b) Find all the pooling and separating (weak) perfect Bayesian equilibria of the game.

Part III

III.(1) **(25 points)** Consider a pure exchange economy with L commodities and n consumers, $i = 1, \ldots, n$, each having initial endowment vector $w_i \in \mathbb{R}^L_+$ and preference orderings $\succcurlyeq_i$ which are defined on the consumption set $X_i = \mathbb{R}^L$ and satisfy strict convexity and strict monotonicity.

(a) **(5 pts)** What can be said about the aggregate excess demand $\hat{z}$ of this economy? (be sure to specify the domain and range of the mapping $\hat{z}$.)

(b) **(5 pts)** For each property of $\hat{z}$ you state in part (a), identify which assumption(s) on preferences are needed for the property and then prove that the assumption(s) you identify imply the property.

(c) **(5 pts)** Briefly discuss the economic significance of the characterization of aggregate excess demand.

(d) **(10 pts)** States a theorem that guarantees the existences of competitive equilibrium and outline the proof. Please add additional assumption if necessary.

III. 2 **(25 pts)** Consider a pure exchange economy $e = \{(X_i, u_i, w_i) : i = 1, ..., n\}$, where $w_i \in X_i = \mathbb{R}^L_+$ and $u_i : X_i \to \mathbb{R}$ is continuous for every i.

(a) **(10 pts)** Show that $x^* \in \prod_{i=1}^n X_i$ is a Pareto optimal allocation iff for every i, $x_i^* \in X_i$ is a solution to the following problem:

$$\max_{x_i} \left\{ (u_i(x_i) : u_j(x_j) \geqq u_j(x_j^*), \forall j \neq i, \& \sum_{i=1}^n x_i \leqq \sum_{i=1}^n w_i \right\}.$$

(b) **(10 pts)** Assume that u_i is differentiable for every i. Use the Kuhn-Tucker theorem to provide a Lagrange characterization of a Pareto optimal allocation x^*. Assume that $x^* > 0$. Interpret the Lagrange multipliers.

(c) **(5 pts)** Now suppose $n = L = 2$, $u_1(x_1, y_1) = 2x_1 + 3y_1$, $u_2(x_2, y_2) = 3x_2 + 2y_2$, $w_1 = (4, 2)$, and $w_2 = (2, 4)$. Find the set of Pareto efficient allocation and show them clearly on the diagram.

III. 3 **(25 points)** In a regulation model, the principal is a regulator who maximizes a weighted average of the agents' surplus $S(q) - t$ and of a regulated monopoly's profit $U = t - C(q, \theta)$, with a weight $\alpha < 1$ for the firms profit. Here, q is the output which is observed perfectly by the principal, t is the transfer payment from the agents to the monopoly, $S(q)$ is the agents' value function with $S' > 0$, $S'' < 0$ and $S(0) = 0$, $C(q, \theta)$ is the monopoly's cost function with $C_q > 0$, $C_\theta > 0$, $C_{qq} > 0$ and $C_{q\theta} > 0$, where which is not observable and are distinguished by $\theta \in \{\underline{\theta}\ , \bar{\theta}\}$ with respective probabilities ν and $1 - \nu$, and $\Delta\theta \equiv \bar{\theta} - \underline{\theta} > 0$. Because $\alpha < 1$, it is socially costly to give up a rent to the firm. The contract is signed at the interim stage.

(a) **(5 pts)** Write down the principal's incentive-compatible optimization problem.

(b) **(5 pts)** What are the optimal payments $\bar{t}$ and $\underline{t}$ and informational rent to the firm?

(c) **(5 pts)** Find the first-order conditions that characterize the principal's incentive-compatible problem. Discuss the effect of changes in α on the informational rent and the output distortion.

(d) **(5 pts)** Suppose $S(q) = q$ and $\alpha = 1/2$, $\underline{\theta} = 1$ and $\bar{\theta} = 2$, and $\nu = 1/2$. Find the optimal incentive scheme for $\{(\underline{q}, \underline{t}), (\bar{q}, \bar{t})\}$.

(e) **(5 pts)** Does this scheme yield the first best outcome? Justify your answer.