Microeconomic Theory
Qualifier Exam
May 2007

Part 1
Answer all questions.

I(1) Consider a utility function of the form: $U = \sum_{i=1}^{N} \beta_i \ln(x_i - \gamma_i)$, with $\sum \beta_i = 1$ and $\gamma_i \geq 0$. Goods x_i are available at per unit prices P_i, and the individual has a fixed income of W.

(a) Derive the demand functions for x_i.

(b) Verify that the general homogeneity and symmetry restrictions of the utility maximization hypothesis hold for these demands.

(c) Identify important special restrictions which the functional form imposes on demand.

I(2) An individual has the following Bernoulli utility function:

$$u(w) = w^{1/2}.$$

Her initial wealth is 10 and she faces the lottery $\widetilde{X}$: $(-6, \frac{1}{2}; +6, \frac{1}{2})$.

(a) Compute the exact value of the certainty equivalent.

(b) Show that with such a utility function absolute risk aversion is decreasing in wealth while relative risk aversion is constant.

(c) If the utility function becomes

$$v(w) = w^{1/4},$$

answer again part (a). Relate this change to the concept of more risk averse'.

I(3) Discuss each of the following. **Be concise and to the point in your answer.**

(a) The impact of nonconvexities in preferences on the continuity of individual and aggregate demand curves.

(b) The impact of nonconvexities in preferences on the quasiconcavity of indirect utility representations of those preferences.

(c) The impact of assuming strict convexity in output for the representative firm cost functions on the existence of long-run perfectly competitive (identical firm) partial equilibrium.

(d) The impact of assuming quasi-linear preferences on the usefulness of Marshallian Consumer Surplus as an individual welfare measure.

I(4) Consider an agent who lives for three periods, $t = 1, 2, 3$. In period one he maximizes

$$U_1(c) = \ln\ c_1 + \beta\delta\ \ln\ c_2 + \beta\delta^2\ \ln\ c_3$$

where $\beta, \delta \in (0, 1]$; in period two he maximizes

$$U_2(c) = \ln\ c_2 + \beta\delta\ \ln\ c_3$$

(a) Suppose that at time 1 the agent makes a complete plan for his future consumption, solving the problem $max_c U_1(c)$ subject to the budget constraint $c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} \leq W$, where W is his initial wealth (the agent has no income: you may want to think of a retiree) and $r \geq 0$ is the market interest rate. Solve for his ideal consumption path. How do c_1 and first-period savings depend on the interest rate? How would you interpret this in terms of income and substitution effects?

(b) At time 2 the agent decides consumption for periods 2 and 3 solving the problem $max_c U_2(\widehat{c}_2, \widehat{c}_3)$ subject to the budget constraint $\widehat{c}_2 + \frac{\widehat{c}_3}{1+r} \leq (1 + r)(W - c_1)$, the last term being of course the amount he had saved at time 1, plus the interest earned. Solve for his ideal consumption path. If $\beta = 1$, will the agent stick to his original plan (i.e. $c_2 = \widehat{c}_2$ and $c_3 = \widehat{c}_3$)? What if $\beta < 1$? If the ideal consumption paths differ, how do they differ? What is the intuition for these results?

Part II
Answer all questions.

II.(1) Two individuals bargain over the distribution of a "pie" of unit size. Each individual $i \in \{1, 2\}$ makes a demand x_i specifying how much she wishes to obtain of the "pie". The "pie" is then allocated as follows: If $x_1 + x_2 \leq 1$ then individual i gets $x_i + (1 - x_1 - x_2)/2$. If $x_1 + x_2 > 1$ then each individuals gets 0.

(a) Identify all pure strategy Nash equilibria.

(b) Is there any equilibria in mixed strategies. If yes, then specify one.

II(2) Consider a Cournot duopoly operating in a market with (inverse) demand function

$$p(q) = a - q,$$

where $q = q_1 + q_2$ is the aggregate quantity in the market and q_i is the output of firm i. Firm i has costs

$$c_i(q_i) = c_0 + c_i q_i.$$

Demand is uncertain: It is high ($a = a_h$) with probability μ and low ($a = a_l$) with probability $(1 - \mu)$. Information is asymmetric. Firm 1 knows whether demand is high or low, but firm 2 does not. All this is common knowledge. The two firm simultaneously choose quantities.

(a) What are the strategy spaces of the two firms?

(b) What is the Bayes-Nash equilibrium of this game? What assumptions do you need to make concerning a_h, a_l, μ, c_0 and c_i to ensure that equilibrium quantities are positive?

(c) Suppose $\mu = 0$. Suppose the game is repeated twice with each firm observing what both produced after the first period. Propose an equilibrium for the game and argue that it is in fact the equilibrium of the twice repeated game.

II(3) Consider the game between Bill and the Bully. This game takes place every day (after Bill has had breakfast). The Bully can do two things: fight or not. He likes to fight with people who are feeling cowardly and gets a payoff of 1 if he does (and gets nothing if he doesn't fight). The bully, however, doesn't like to fight with a person who is feeling brave. He gets a payoff of -1 from fighting with a brave person and a payoff of 0 from not fighting. The problem is the Bully does not know whether Bill is feeling cowardly or if he is feeling brave today. Bill, of course, knows how he's feeling in the morning. The Bully knows that with probability .75 Bill is feeling cowardly and with probability .25 that he is feeling brave (and Bill knows this). The Bully observes what Bill has had for breakfast. And Bill can, hence, use what he has for breakfast as a signal to the Bully (regarding whether Bill is feeling brave or cowardly). Bill can have Quiche for breakfast or he can have Beer. Bill prefers to have Quiche for breakfast. And Bill prefers not to fight. He gets a payoff of 2 by having Quiche and not fighting, and a payoff of 1 from having Beer and not fighting and a payoff of 0 from having Quiche and fighting and a payoff of -1 from having beer and fighting.

(a) Represent the above game in extensive form.

(b) Find all the pooling and separating weak perfect Bayesian equilibria for this game.

Part III
Answer all questions.

III.(1) Consider a two-agent two-good exchange economy with no free disposal.

Agent 1 has a preference relation on $\mathbb{R}^2_+$ given by this utility function: for all $(x_1, y_1) \in \mathbb{R}^2_+$,

$$u_1(x_1, y_1) = x_1 y_1.$$

Agent 2 has a preference relation on $\mathbb{R}^2_+$ given by this utility function:for all $(x_1, y_1) \in \mathbb{R}^2_+$,

$$u_2(x_2, y_2) = 2x_2 y_2 - 4.$$

The agents' endowments are $w_1 = (1, 4)$ and $w_2 = (4, 1)$, respectively.

a. Construct an Edgeworth box diagram showing (and labelling) the endowment allocation and typical preferences or indifference (and directions of increasing preference) curves for each consumer.

b. If this economy has Pareto optimal allocations, find them and show them clearly on the indifference curves for each consumer. If there are none, state that.

c. If this economy has competitive (Walras) equilibria, find them and show them clearly on the diagram, indicating price ratios, budget lines, and consumptions. If there are none, state that.

d. Is the competitive equilibrium allocation Pareto efficient? Why?

e. If this economy has core allocations, show them clearly on the diagram. If there are none, state that.

f. Is the competitive equilibrium allocation strictly fair? Why?

g. Define a tatonnement price adjustment process; and define global stability for this economy.

h. Is the equilibrium you found in part c globally stable? Why?

III. 2 Let $L \geq 2$ and let Δ denote the unit simplex in $\Re^L$, that is, $\Delta = \{p \in \Re^L_+ : \sum_{t=1}^{L} p^t = 1\}$, and let $f: \Delta \to \Re^L$ be a continuous function, which we interpret as an aggregate excess demand function.

a. Does there necessarily exist a price-vector $p^* \in \Delta$ with $f(p^*) \leq 0$? If not state an additional assumption which ensures the existence of such a p^*. Give an economic interpretation of your assumption.

b. State the Bouwer fixed-point theorem.

c. Using the Bouwer fixed-point theorem, prove the existence of $p^* \in \Delta$ with $f(p^*) \leq 0$, under the additional assumption, if any, which you stated in your answer to part (a).

d. Now suppose the aggregate excess demand function is well-defined only on $\Delta_{++} = \{p \in \Re^L_{++}\colon \sum_{t=1}^{L} p^t = 1\}$. State the conditions that guarantee the existence of competitive equilibrium.

III. 3 Suppose that there are two types of goods and two consumers. Good m indexes income which can be used to purchase material goods, and good n indexes non-income goods, such as human right, family life, health, etc., that is, we can *roughly* interpret m as those goods included in GDP and n as those not. Consumer i's consumption of the two goods is denoted by a vector (m_i, n_i), $i = 1, 2$. We assume that income consumption exhibits a negative externality, which means that the utility of consumer i is adversely affected by consumer j's consumption on m_j, $j \neq i$. Consumer i's utility function is further specified as

$$u_i(m_i, n_i; m_j) = m_i^{\alpha} n_i^{1-\alpha} - \beta m_j,$$

where $\alpha \in (0,1), \beta > 0,\ i \in \{1,2\}, j \in \{1,2\}, j \neq i$.

(a) Give the critical level of income beyond which not all income should be completely used up at Pareto efficient status.

(b) Find the set of Pareto optimal allocations when all income should be completely used up.

(c) Find the set of Pareto optimal allocations when not all income should be used up.

(d) Use the results you obtained above explain the Easterlin paradox: average happiness levels do not increase as countries grow wealthier.