# Microeconomic Theory
# Qualifier Exam
# July 2007

**Answer all ten questions on the exam.**

***Part I***

1. A household has non-labor income T. The household can earn additional income by supplying labor (L). The total income from both sources is spent on "goods" (X). (We are aggregating all goods into a single composite commodity.) The wage rate is w and the price of "goods" is p. The decisions about L and X are the only decisions that the household makes, and the conditions faced are competitive (w and p are independent of the decisions about L and X).

   Assume that you have enough data to estimate a supply of labor equation and a demand for goods equation that approximate how this household behaves with respect to changes in p, w, and T, in a neighborhood of a point ($p_0$, $w_0$, $T_0$) (i.e. these fitted equations correctly represent the true labor supply and goods demand functions up to the first order):

$$L = \alpha + \alpha_w (w - w_0) + \alpha_p(p - p_0) + \alpha_T(T - T_0)$$
$$X = \beta + \beta_w(w - w_0) + \beta_p(p - p_0) + \beta_T(T - T_0)$$

   a) What restrictions does the theory of rational consumer behavior impose on the 8 parameters in these equations? (8)

   b) Is there a subset of the 8 parameters that can be freely estimated, with the remaining parameters determined from these by theoretical considerations? How many degrees of freedom are there? (4)

   c) Suppose that $\alpha_w = 0$. Someone who has had a little economics tries to convince you that a labor income tax would not result in a deadweight loss because the labor supply curve is completely inelastic. Do you agree? Explain. (5)

   d) What would be your estimate of the deadweight loss of a small tax on labor income that lowers the wage rate from $w_0$ to $w_0 - \delta$? (8)

2. A person is known to be a risk averse expected utility maximizer with a monotonically increasing utility for money. Initial wealth is non-stochastic.

She asserts that she would be indifferent between a 50-50 gamble to lose \$100 or gain \$200, and remaining with the initial wealth for sure.

She also says that she would be indifferent between a sure gain of \$100 and a 50-50 gamble that will either gain \$300 or gain \$0.

Consider a risky prospect which will gain \$300 with probability p, or lose \$100 with probability 1 - p. Based on the above data about the individual's preferences:

For which values of p can you be sure that this prospect will be accepted by this individual?

3. Suppose Y is a strictly convex, closed production set that satisfies the free disposal property. Let $\pi(\cdot)$ be the profit function of the production set Y and that $y(\cdot)$ is the associated supply function. State and prove formally five properties of these functions. Explain what each means and why it is true in intuitive economic terms.

4. Consider a firm with a production function given by

$$f(k,s,u) = \left[u^{\sigma} + (s^{\alpha} + ak^{\alpha})^{\frac{\sigma}{\alpha}}\right]^{\frac{1}{\sigma}}$$

where both $\sigma$ and $\alpha$ are strictly less than 1 and $a > 0$. Here s respectively u denote the amount of skilled respectively unskilled labor employed by the firm, while k denotes physical capital (e.g. machines, computers). The price (wage) of a unit of skilled respectively unskilled labor is $w_s$ respectively $w_u$, and r is the price of a unit of capital. The skill premium is then given by the ratio $w_s/w_u$.

a) Without any calculation, describe the production function. In particular, indicate what do $\sigma$, $\alpha$, and b measure. (Hint: consider the case if $\sigma = \alpha$.]

b) Assume perfect competition, and normalize the price of the final output to 1. Let $s^d$ and $u^d$ denote the firm's optimal demands for skilled and unskilled labor. Show that

$$\Psi\left(\frac{aw_s}{r}\right)^{\frac{\sigma}{\alpha}-1}\left(\frac{u^d}{s^d}\right)^{1-\sigma} = \frac{w_s}{w_u},$$

where $\Psi$ is a function you should find explicitly. Interpret this condition.

## Part II

1. Find the subgame perfect equilibria of the following Bargaining games.

   (a) A two-player, two period alternating offer Bargaining game (in which player 1 makes the offer $(x_1, x_2)$ in period 1, where $x_1+x_2 = 1$ and $x_i \geq 0$ for all $i$ and player 2 makes an offer $(y_1, y_2)$ in period 2, with $y_1 + y_2 = 1$ and $y_i \geq 0$ for all $i$). Player $i$ experiences a constant cost of delay of $0 < c_i < 1$. That is, a payoff of $\gamma_i$ in period 2 is worth $\gamma_i - c_i$ for player $i$. If there is no acceptance at the end of the second period than each player gets $-c_i$.

   (b) A two-player three period alternating offer Bargaining game (similar to the above but with three periods) in which the cost to each player $i$ of each period of delay is $c_i$. Treat the cases $c_1 > c_2$ and $c_2 > c_1$ separately.

   (c) A two-player, two period alternating offer Bargaining game (in which player 1 makes the offer $(x_1, x_2)$ in period 1, where $x_1+x_2 = 1$ and $x_i \geq 0$ for all $i$ and player 2 makes an offer $(y_1, y_2)$ in period 2, with $y_1 + y_2 = 1$ and $y_i \geq 0$ for all $i$). Player $i$ experiences a cost of making an offer of $0 < c_i < 1$. If there is no acceptance at the end of the second period than each player gets $-c_i$.

2. Consider a duopoly operating in a market with (inverse) demand function

$$p(q) = a - q,$$

   where $q = q_1 + q_2$ is the aggregate quantity in the market and $q_i$ is the output of firm $i$. Firm $i$ has costs

$$c_i(q_i) = c_i q_i.$$

   Demand is uncertain: It is high $(a = a_h)$ with probability $\mu$ and low $(a = a_l)$ with probability $(1 - \mu)$. Information is asymmetric. Firm 1 knows whether demand is high or low, but firm 2 does not. All this is common knowledge.

   (a) The two firms simultaneously choose quantities. Find the Bayes-Nash equilibrium.

(b) Firm 1 is the Stackelberg leader. Find the Bayes-Nash equilibrium.

(c) Firm 2 is the Stackelberg leader. Find the Bayes-Nash equilibrium.

3. Consider the game between Bill and Tom. Tom can do two things: fight or not. He likes to fight with people who are weak and gets a payoff of 1 if he does and gets nothing if he doesn't fight. Tom, however, doesn't like to fight strong people. He gets a payoff of -1 from fighting with a strong person and a payoff of 2 from not fighting. Tom does not know whether Bill is strong or weak. Bill, of course, knows if he is strong or weak. Tom knows that with probability $p$ Bill is strong and with probability $(1-p)$ that he is weak (and Bill knows this). Tom observes whether Bill is ready or not (for fighting Tom). If Bill is strong, he gets a payoff of 4 by being ready for Tom and Tom chooses not to fight him and he gets a payoff of 2 by being ready for Tom and Tom chooses to fight him. Also if Bill is strong and not ready to fight Tom, then he gets a payoff of 5 if Tom doesn't fight him and a payoff of 3 if he does. If Bill is weak and ready then he gets a payoff of 2 if Tom doesn't fight him and a payoff of 0 if he does. If Bill is weak and unready then he gets a payoff of 5 if Tom doesn't fight him and a payoff of 3 if he does.

   (a) Represent the above game in extensive form.

   (b) Find all the pooling and separating weak perfect Bayesian equilibria for this game (for all values of $p$).

## Part III

(1) Consider an exchange economy with two agents and three commodities. Suppose $X_i = R^3_+$,

$$\begin{aligned} U_1(x_1^1, x_1^2, x_1^3) &= \min\{x_1^1, x_1^2, x_1^3\}; w_1 = (1,1,1) \\ U_2(x_2^1, x_2^2, x_2^3) &= \frac{1}{2}\ln x_2^1 + \frac{1}{3}\ln x_2^2 + \frac{1}{6}\ln x_2^3; w_2 = (1,1,1) \end{aligned}$$

a. Find the demand functions of these two agents.

b. Find the aggregate excess demand functions of this economy,

c. Find a competitive equilibrium allocation and price.

d. Is the competitive equilibrium price unique? Why?

e. Is the competitive equilibrium allocation Pareto efficient? Why?

f. Define a tatonnement price adjustment process; and define global stability for this economy.

g. Is the equilibrium you found in part c globally stable? Why?

2 [The Second Fundamental Theorem of Welfare Economics] Suppose $(x^*, y^*)$ with $x^* > 0$ is Pareto optimal, suppose $\succcurlyeq_i$ are continuous, convex and strictly monotonic, and suppose that $Y_j$ are closed and convex. Prove that there is a price vector $p \geq 0$ such that $(x^*, y^*, p)$ is a competitive equilibrium with transfer payments, i.e.,

(1) if $x_i' \succ_i x_i^*$, then $px_i' > px_i^*$ for $i = 1, \ldots, n$.

(2) $py_j^* \geqq py_j'$ for all $y_j' \in Y_j$ and $j = 1, \ldots, J$.

3 Consider an economy with two goods, a private (rivalrous) good $x$, say leisure, and a public (non-rivalrous) good $y$, say radio broadcast music. Both goods are measured in hours per day. There are two consumers, 1 and 2, and one firm. The firm produces $y$, using labor $v$ as input. (Labor is leisure given by a consumer to the firm; hence if consumer $i$ supplies $v_i$ units of labor to the firm, then the amount of leisure left to $i$ is $x_i = w_i - v_i$, where $w_i$ is $i$'s initial endowment of leisure.) Let the production function of the firm be linear (constant returns to scale), with $k$ units of $v$ needed to produce one unit of $y$ at any scale of output ($k > 0$). There is no free disposal. The initial endowments $w_i$ of $x$ are positive, but the initial endowment of $y$ is zero.

Assume that each consumption set consists of all points $(x_i, y)$ such that $x_i \geqq 0$ and $y \geqq 0$,for $i = 1, 2$. Furthermore, for any $x_i \geqq 0$ and $y \geqq 0$, the utility function of consumer $i$ is given by:

$$u_i = z_i + \phi_i(y), \tag{1}$$

where the valuation function $\phi_i(y)$ is twice differentiable, and has a positive derivative $\phi_i'(y) > 0$, and a negative second derivative $\phi_i''(y) < 0$ for all $y \geqq 0$, for $i = 1, 2$. (Remember it is assumed $w_i > 0$ for $i = 1, 2$.)

Suppose that each consumer $i$ voluntarily chooses to contribute an amount of labor $v_i \geqq 0$ toward the production of the public good $y$, with $v_i < w_i$. By definition, at a Nash equilibrium allocation, written $(\bar{x}_1, \bar{x}_2, \bar{v}_1, \bar{v}_2, \bar{y})$, each consumer $i$ maximizes $u_i$, treating $v_j$ as given (for $j \neq i$)), and taking into account the equality

$$ky = v_1 + v_2. \tag{2}$$

(a). Find the conditions that characterize Pareto efficient allocations. (These will be equations in $x_1$, $x_2$, $y$ and the original endowments.)

(b). Suppose that, at a Nash equilibrium, consumer 2 contributes a positive amount of labor, but is still left with positive amounts of leisure, i.e., $w_2 > \bar{v}_2 > 0$, while consumer 1 contributes nothing, i.e., $\bar{v}_1 = 0$. Could such an equilibrium be Pareto optimal?

(c). Suppose that, at a Nash equilibrium, both consumers contribute positive amounts of labor, but are still left with positive amounts of leisure. Could such an equilibrium be Pareto optimal?

(d). Suppose that, a Nash equilibrium, neither consumer contributes any labor. Could such an equilibrium be Pareto optimal?

(e). Define the Lindahl equilibrium and prove that every Lindal equilibrium is Pareto efficient under local non-satiation.

Explain your answers to questions (a)-(e) as fully as possible.