Microeconomic Theory
Qualifier Exam
May 2006

Answer all ten questions on the exam. The questions are equally weighted.

Part I.

1. Given the following data: $x_1^0 = 100$, $p_1^0 = 10$, $\frac{\partial x_1}{\partial p_1} = -5$, $\frac{\partial x_1}{\partial w} = 0.01$ suppose the price of good 1 increases to $p_1 = 12.5$. How much should a public assistance program aimed at maintaining a certain standard of living be increased to offset this price increase?

    a) Explain the relevance of the concept of Compensating Variation (CV) to this question.

    b) Use the Slutsky equation to derive an approximation to the CV for this problem.

    c) What is the relationship between your CV approximation measure and a Marshallian consumer surplus measure of the required compensation?

2. Consider an industry with F identical firms indexed f = 1,...,F. The industry is the sole demander of input 1. Let input 1 have elasticity of supply ε, (aggregate) elasticity of demand η, and let all other inputs have constant price. Let P be the output price and $C(W, Q^f)$ be the cost function, where W is the vector of input prices.

    a) In the "standard" case, where W is fixed for each firm and for the industry, use the first-order condition under the profit-maximization hypothesis to derive an expression for the comparative statics effect of a change in P on firm output Q^f.

    b) Now let W_1 change (rise) as industry output increases in response to a rise in P. Use the first-order condition for the representative firm, combined with the market equilibrium condition for input 1, to derive an expression for the industry supply curve, dQ/dP, where $Q = F \cdot Q^f$, as a function of ε, η, F, and cost function derivatives. Hint: Shephard's Lemma will be useful here.

3. Consider the following version of the utility analysis of a representative household, involving four commodities: current goods consumption (C_1) and leisure (L_1) and "future" consumption and leisure (C_2 and L_2). The household is assumed to maximize utility:

$$U(C_1, C_2, L_1, L_2), \qquad U_1, U_2, U_3, U_4 > 0,$$

subject to the constraint that the present value of consumption cannot exceed the present value of income and subject to the time constraint that $L + \ell = T$, where ℓ = labor time and T is the total time endowment. Present values are computed using a nominal interest rate r, at which the household may lend any amount up to its current assets or borrow any amount up to that which may be secured by future income. The initial nonhuman assets, fixed in money terms, are $\bar{A}$, and present and future goods, prices, and money wage rates are P_1, P_2, W_1, and W_2.

a) Write down the relevant budget constraint for this problem.

Assume that for all positive prices a unique maximum is attained at which $C_1, C_2, \ell_1, \ell_2 > 0$.

b) Explain why, for this neoclassical model, you can express the labor supply function as

$$\ell = F\left(\frac{W_1}{P_1}, \frac{W_2}{P_1(1+r)}, \frac{P_2}{P_1(1+r)}, \frac{\bar{A}}{P_1}\right)$$

c) Interpret W_2 as a measure of a "normal" wage rate and deviations of W_1 from W_2 as being "transitory". Suppose you collected data on the labor supply of cab drivers, and found that the drivers work more hours on days when their wage is higher than normal (even though the nominal fare doesn't vary by day, on good weather days the search costs of finding fares is lower, so the effective wage per hour is higher). How would you interpret this empirical finding using the model of labor supply in this problem?

4. Consider a monopoly firm choosing its level of inputs according to the following non-profit-maximizing criterion:

Choose k,l to maximize employment l subject to a lower bound on profits where $p(q)$, $p' < 0$.

a) Show that the firm using this objective will not minimize the resource costs of producing its chosen output.

b) Can we conclude that this firm will have a lower capital-labor ratio than would a profit-maximizing monopolist?

Part II

1. In expected utility theory:

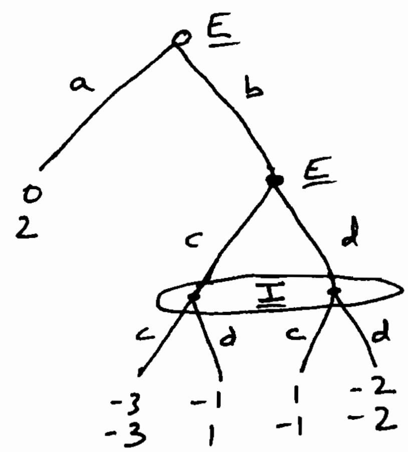
    (a) Formally define when a decision maker said to be risk averse.

    (b) What is the relationship between the curvature of a decision makers Bernoulli utility function and her attitudes towrds risk? Provide some intuition either by sketching a proof or giving an example.

    (c) What is the certainty equivalent of a decision maker with Bernoulli utility function $u(x) = \sqrt{x}$ for the lottery which gives either \$0, \$4 and \$16, each with equal probability. Is the certainty equivalent greater than the expected value of the lottery?

    (d) Suppose $x > 0$. Compare the risk attitudes of Rob who has Bernoulli utility function $u^R(x) = \sqrt{x}$ with Carrie who has Bernoulli utility function $u^C(x) = 1 - (1/e^x)$.

2. For the following extensive form game:



    (a) Does player I have any strictly dominated strategy? Does he have a rationalizable strategy? Is the game dominance solvable?

    (b) Find all Nash equilibrium (in pure and mixed strategies).

    (c) Find all subgame perfect equilibria (in pure and mixed strategies).

    (d) Do any of the subgame perfect equilibria seem unreasonable (predictions for you to make)?

3. Consider a duopoly with (one-sided) incomplete information. The cost function of Firm 2 being uncertain to Firm 1. The market (inverse) demand function is $p = a - bq$, where p denotes price, $q = q_1 + q_2$ denotes total output and a and b are parameters. Firm 1 has zero marginal cost and Firm 2's cost function is $c_2(q_2) = \gamma q_2$ with probability π and $c_2(q_2) = -\gamma q_2$ with probability $(1 - \pi)$, where $\gamma\pi - \gamma(1 - \pi) = 0$.

   (a) Suppose the firms are Cournot duopolists. Find the Bayes-Nash equilibrium of this game (in pure strategies).

   (b) Suppose Firm 1 is the Stackelberg leader. Find the Stackelberg equilibrium of this game (in pure strategies).

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Part III

III.(1) Consider a two-person two-commodity economy with

$$X_i = \{x_i \in R^2_+ : x_i^1 + x_i^2 \geq 1\}, \quad i = 1, 2.$$

$$\begin{aligned} w_1 &= (1,0) \\ w_2 &= (1,2) \\ u_1 &= x_1^1 + 2x_1^2 \\ u_2 &= \ell n x_2^1 + \ell n x_2^2 \end{aligned}$$

1. Draw the Edgeworth Box with the consumption set, indifference curves, and the initial endowment point clearly marked.

2. Calculate demand functions for agents 1 and 2.

3. Find the set of Pareto optimal allocation and draw them in the Edgeworth Box.

4. Does a competitive equilibrium exist? If so, is it a Pareto optimal? If not, which of the standard assumptions for the existence theorem is violated?

5. Change the consumption sets to

$$x_i = R^2_+, \quad i = 1, 2.$$

Does the change make a difference in your answer to part (4) above?

III. 2 [The Second Fundamental Theorem of Welfare Economics] Suppose (x^*, y^*) with $x^* > 0$ is Pareto optimal, suppose $\succcurlyeq_i$ are continuous, convex and strictly monotonic, and suppose that Y_j are closed and convex. Prove that there is a price vector $p \geq 0$ such that (x^*, y^*, p) is a competitive equilibrium with transfer payments, i.e.,

(1) if $x_i' \succ_i x_i^*$, then $px_i' > px_i^*$ for $i = 1, \ldots, n$.

(2) $py_j^* \geqq py_j'$ for all $y_j' \in Y_j$ and $j = 1, \ldots, J$.

III. 3 There are a single consumption good, two states, and two consumers. Note that this allows the use of Edgeworth boxes. Utility functions are of the expected utility from. Bernoulli utility functions are identical across states. That is,

$$U_1(x_{11}, x_{21}) = \pi_{11}u_1(x_{11}) + \pi_{21}u_1(x_{21})$$

and

$$U_2(x_{12}, x_{22}) = \pi_{12}u_2(x_{12}) + \pi_{22}u_1(x_{21})$$

where x_{si} is the amount of s-contingent good consumed by consumer i and π_{si} is the subjective probability of consumer i for state s. We assume that every $u_i(\cdot)$ is strictly concave and differentiable.

The total initial endowments of the two contingent commodities are $\bar{w} = (\bar{w}_1, \bar{w}_2) > 0$. We assume that every consumer gets half of the random variable $\bar{w}$, that is $(w_{11}, w_{21}) = \frac{1}{2}\bar{w}$ and $(w_{12}, w_{22}) = \frac{1}{2}\bar{w}$.

a. Suppose that consumer 1 is risk neutral, consumer 2 is not, and both consumers have the same subjective probabilities. Show that at an interior Arrow-Debreu equilibrium consumer 2 insures completely.

b. Suppose now that consumer 1 is risk neutral, consumer 2 is not, the subjective probabilities of the two consumers are not the same.. Show that at an interior Arrow-Debreu equilibrium consumer 2 will not insure completely.

c. Which is the direction of the bias in terms of the difference in subjective probabilities? Argues also that consumer 1 (the risk-neutral) will not gain from trade.