Microeconomic Theory
Qualifier Exam
July 2006

Answer all ten questions on the exam. The questions are each worth 25 points.

Part I.

1. Consider a monopolist whose total cost function is $C = kx^2$ and who faces the demand curve $x = a - bp$. A per-unit tax of t is applied to the firm's output.

   a) What restrictions on the values of the parameters a, b, and k would you be inclined to assert, a priori?

   b) Find the explicit function $x = x^*(t)$. Confirm that for the restricted values of a, b, and k placed in part a) that $x^{*\prime}(t) < 0$, i.e., output decreases as the tax increases.

   c) What restrictions does the hypothesis of profit maximization place on the parameters a, b, and k? Are these weaker or stronger than your a priori restrictions?

   d) Confirm that, for this specification of the model, profit maximization alone implies $x^{*\prime}(t) < 0$.

   e) What is the effect on output and output price of a parallel shift in the demand curve?

2. In an economy there are two firms each producing a single output from a single non-produced resource according to

$$q_1 = \sqrt{z}$$

$$q_2 = \max(\sqrt{R - z} - aq_1, 0)$$

   where q_i is the amount produced of good i, z is the amount of the resource used in the production of good 1, R is the total stock of the resource, and a is a parameter.

   a) What phenomenon does this model represent?

   b) Draw the production-possibility set.

   c) Assuming that all consumers are identical, sketch a set of indifference curves for which (a) an efficient allocation may be supported by a pseudo market in externalities; (b) a pseudo market fails.

d) What role does the parameter *a* play in the answer to the previous part?

3. A classic study of US airlines (Eads et al. 1969) modeled long run costs as

$$C(w,q) = C_f + kq^{\frac{1}{\gamma}} w_1^{\frac{a_1}{\gamma}} w_2^{\frac{a_2}{\gamma}}$$

where q is an index of airline output, C_f is the cost of fuel (separately estimated), w_1 is the price of labor other than pilots and co-pilots and w_2 is the price of pilots and co-pilots: a_1, a_2 and γ are parameters to be estimated econometrically, and k is also a function of the parameters.

a) Show that the conditional demand for labor of type 1 is given by the log-linear equation:

$$\log(z_1^*) = \beta_0 + \beta_1 \log(w_1) + \beta_2 \log(w_2) + \beta_3 \log(q).$$

b) What restrictions on the coefficients in the conditional demand for labor of type 1 are implied by the theory of cost-minimization?

c) How would you interpret the parameter γ?

4. Let an individual's utility level U depend on his consumption of a composite commodity Y, whose price is equal to unity, and F, the favored commodity. Suppose the utility function is of the constant elasticity of substitution form:

$$U = [aF^{-b} + (1 - a)Y^{-b}]^{-1/b}, \; b > -1,$$

where a and b are taste parameters, and the elasticity of substitution is given by $\sigma = 1/(1 + b)$. Let consumption of the favored commodity, F, be subsidized at rate s.

a) Derive the demand functions for F and Y.

b) The indirect utility function is of the form:

$$V = [(M + Z)/Q(s)][A(s)]^{-1/b},$$

where $A(s) = a + (1 - a)[K(1 - s)^{\sigma} P_F^{\sigma}]^{-b}$

and $Q(s) = (1 - s)P_F + K[(1 - s)P_F]^{\sigma}.$

How was this derived? (Don't do it, just say how.)

c) Suppose the deadweight loss or excess burden of the subsidy is defined as the monetary value of the difference between the utility level achieved with the subsidy and the level that would have been achieved had the subsidy been given as a lump sum. Derive a closed-form solution for the deadweight loss/excess burden, B, as a function of the model parameters.

d) Now consider a scheme which awards a lump sum subsidy of G to individuals who consume at least some critical amount F*, and zero subsidy otherwise. Discuss how you would analyze the deadweight loss of this scheme, highlighting differences between this scheme and the standard subsidy on parts a) - c). (You don't need to solve everything here, just outline your approach.)

Part II

1. Answer both parts:

   (a) Prove that any strictly dominant strategy in the game $G = \left(I,\ (\Sigma_i)_{i \in I},\ (u_i(\cdot))_{i \in I}\right)$ must be a pure strategy.

   (b) Consider the following game:

	h	t
h	4,0	0,1
t	0,1	1,0

      i. Find the Nash equilibrium of this game.

      ii. Argue that each player is playing a rationalizable strategy in the Nash equilibrium

2. Answer both parts:

   (a) Suppose the following normal for game is played twice. Players observe the actions chosen in the first play of the game and prior to the second play.

	l	c	r
u	10,10	2,12	0,13
m	12,2	5,5	0,0
d	13,0	0,0	1,1

   Find *all* the pure strategy subgame perfert equilibria of this game.

   (b) In the first period of a game player 1 can exert some effort to increase the size of the pie. If the effort exerted is c_i, $i \in \{L, H\}$, then the pie size if π_i, $i \in \{L, H\}$, where $c_L < c_H$ and $\pi_L < \pi_H$. In period 2, player 1 and player 2 bargain over the pie (when both know the size of the pie). Bargaining proceeds as follows. Player 2 makes player 1 an offer. If player 1 accepts the offer then the game ends and this offer is implemented. If player 1 rejects the offer then each player gets nothing. Find a pure strategy subgame perfect equilibrium of this game.

3. Consider the following two normal form games:

	l	m	r
u	1,2∈	1,0	1,3∈
d	2,2	0,0	0,3

and,

	l	m	r
u	1,2∈	1,3∈	1,0
d	2,2	0,3	0,0

where $0 < \in < 1/2$.

(a) Suppose each player knows that each game is equally likely (and that is all). Find the Bayes Nash equilibrium of the game.

(b) Suppose the column player is informed about the game she is playing but the Row player only knows that each game is equally likely. Find the Bayes Nash equilibrium of the game.

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Part III

III.(1) **(25 points)** Consider a two-agent two-good exchange economy with no free disposal. Agent 1 has a lexicographic preference relation (with the first commodity primary) on $\mathbb{R}^2_+$. Agent 2 also has a lexicographic preference relation (with the first commodity primary) on $\mathbb{R}^2_+$. The agents endowments are:

$$\begin{aligned} w_1 &= (20, 10) \quad \text{for Agent 1} \\ w_2 &= (10, 20) \quad \text{for Agent 2.} \end{aligned} \tag{1}$$

(a) Construct an Edgeworth box diagram showing and labeling the endowment allocation and typical indifference sets and directions of increasing preference for each consumer.

(b) If this economy has Pareto optimal allocations, indicate them clearly on the diagram. If there are none, state that.

(c) If this economy has competitive (Walras) equilibria, show them clearly on the diagram, indicating price ratios, budget lines, and consumptions. If there are none, state that. No justifications are required. However, you should make sure that your graph is clearly labeled, and the requested sets are clearly indicated.

III. 2 **(25 points)** Under what circumstances can it happen that, in a Walrasian tatonnement process, the price vector does not converge to an equilibrium value? (Assume smooth aggregate demand functions having positive homogeneity of degree zero and satisfying the Walras Law. Also, assume that at least one equilibrium vector exists).

Sketch a proof of non-convergence. (If you use any concept of stability, given a precise definition).

III. 3 **(25 points)** Consider an economy with two goods x and y and an arbitrary number of agents. Assume preferences are strictly increasing in each good. Consider two cases:

(a) x is perfectly divisible but y comes only in integer units.

(b) Both x and y come only in integer units.

Could we have a competitive equilibrium allocation that is not Pareto optimal? (Answer separately for case (a) and case (b)). If your answer is "yes," give an example with a competitive equilibrium allocation that is not Pareto optimal. (Justify both facts: (i) that the allocation in your example is a competitive equilibrium allocation, and (ii) that it is not Pareto optimal.). If your answer is "no," sketch a proof to justify it.