Micro Qualifying Exam:

Part I.

1. Suppose an individual has quasi-linear preferences

$$u(x) = f(x_1) + x_2$$

where f( ) is strictly concave. The budget constraint is $px_1 + x_2 = w$.

a) Find the *inverse* Walrasian demand for good 1, $p(x_1)$.

b) Consider a quantity change from x_1' to x_1''. Show that in this case the Consumer's Surplus is an *exact* measure of the change in utility.

c) Give an intuitive explanation for your answer in (b).

2. Suppose that the long-run total cost curve of a representative firm in a competitive industry where an infinite number of potential firms have access to a common cost function c(q) given by the formula:

$$c(q) = 1200q - 60q^2 + q^3.$$

a) Show that when the industry reaches long-run equilibrium the price will be \$300 per unit, and each firm will produce 30 units, if the industry demand curve is $P^D = 375 - .025Q$. Show that there will be 100 active firms in long-run equilibrium.

b) The government wishes to raise \$45,000 by taxing this industry. Two different kinds of taxes are under consideration:

i) An excise tax of \$15 per unit;

ii) A lump-sum tax of \$450 per firm, regardless of how much the firm produces.

Will either of these taxes generate \$45,000 in revenue for the government? Will the new long-run equilibrium price be the same for both tax schemes? If not, will the price increase be greater under the sales tax or the lump-sum tax? Draw a diagram summarizing your answer.

3. Robinson Crusoe is stranded on a deserted island. He has preferences over fish (F) and leisure (L). He begins with no fish, but has available 24 hours of leisure that he can either consume or spend working.

Model 1: Robinson cares only about fish, so his preferences can be represented by the utility function $u = F$. Robinson can obtain fish by working for the native Friday at the rate of w fish per hour $H = 24 - L$ of labor supplied. However, Robinson is already malnourished, and requires calories to be able to work, at the rate of 1 fish per hour of labor.

a) Define Robinson's budget set and his consumption set.

b) Describe the solution to Robinson's utility maximization problem.

Model 2: Robinson's preferences can be represented by the utility function $u = F - [24 - L]$. Again, he can obtain fish by working for Friday at the rate of w fish per hour. Now, Robinson is healthy and does not need to consume any fish in order to be able to work any number of hours between 0 and 24.

c) Describe the solution to Robinson's utility maximizing problem.

d) Is there any empirical test that would allow you to distinguish between Model 1 and Model 2 on the basis of observations on F, L, and W, without knowing the state of Robinson's health? Explain.

Part II.

Do Problem 1 and *<u>either</u>* *Problem 2* *<u>or</u>* *Problem 3.*

1. Consider an exchange economy with goods x and y and individuals A and B. A has utility function $u_A(x,y)$ and B has utility function $u_B(x,y)$. A has endowment (x^A,y^A) and B has endowment (x^B,y^B).

    (a) What conditions on preferences and endowments are required for the existence of a Walrasian equilibrium?

    (b) Find the slopes of the indifference curves at the endowment point.

    (c) Describe the steps one would take to find the Walrasian equilibrium price vector and allocations.

    (d) Will the equilibrium allocations be Pareto efficient? Why or why not?

2. Let the vector (p_0, p_{100}, p_{160}) denote a lottery which pays \$0 with probability p_0, \$100 with probability p_{100}, and \$160 with probability p_{160}. An expected utility maximizer strictly prefers the lottery (0.6, 0.4, 0) to the lottery (0.75, 0, 0.25).

    (a) If the same individual is given a choice between (0.5, 0, 0.5) and (0.2, 0.8, 0), which one would he choose? Justify your answer.

    (b) If the same individual is given a choice between (0.34, 0.11, 0.55) and (0.04, 0.91, 0.05), which one would he choose? Justify your answer.

3. Consider the functions $u(x) = 3 + 2x^{1/2}$ and $v(x) = 4 + 9\ ln\ (x)$, both defined over the domain $[0, M]$.

    (a) Are the two functions risk averse? Justify your answer.

    (b) Is one more risk averse than the other? Justify your answer.

Part III

1. Consider the following two players game of incomplete information. The R(ow) player knows her type but the C(olumn) player only knows that with probability μ that R is as in the top matrix. All of this is common knowledge. Players choose simulataneously.

	l	r
u	0,-1	2,0
d	2,1	3,0

	l	r
u	1.5,-1	3.5,0
d	2,1	3,0

   (a) Find all pure strategy Bayesian Nash equilibrium.

   (b) Find a mixed strategy Bayesian Nash equilibrium.

2. Consider a Cournot duopoly operating in a market with (inverse) demand function

$$p(q) = a - q,$$

where $q = q_1 + q_2$ is the aggregate quantity in the market and q_i is the output of firm i. Both firms have costs

$$c_i(q_i) = cq_i.$$

Demand, is uncertain: It is high (a_h) with probability μ and low (a_l) with probability $(1 - \mu)$. Information is asymmetric. Firm 1 knows whether demand is high or low, but firm 2 does not. All this is common knowledge. The two firm simultaneously choose quantities.

   (a) What are the strategy spaces of the two firms.

(b) What is the Bayes-Nash equilibrium of this game (make assumptions concerning a_h, a_l, μ, and c such that all equilibrium quantities are positive).

3. Consider the following normal form game

	a	b	c
a	4,4	0,5	0,0
b	5,0	2,2	0,0
c	0,0	0,0	3,3

(a) Suppose the game is played twice. Players discount future payoffs at rate δ. Find a subgame perfect Nash equilibrium in which the strategy profile (a,a) is played in the first period (for a suitable δ). Verify your claim. What restrictions does it impose of δ?

(b) Suppose this game is repeated $T > 2$ times. Both players discount future payoffs at rate δ. Propose a subgame perfect equilibrium in which (a,a) is played $T-1$ times (for suitable δ). Verify your claim. What is the payoff of each player with this strategy? What restrictions does it impose on δ?

(c) Suppose the game is played infinitely often. Both players discount future payoffs at rate δ. EITHER propose a subgame perfect equilibrium (for suitable δ) in which (a,a) is played in every period OR prove that such a subgame perfect equilibrium cannot exist. Verify your claim. What is the payoff of each player with this strategy (if it exists)? What restrictions does it impose on δ?