Aug 05

Micro Qualifier Exam

Part I.

Do all three of the following problems, which are equally weighted.

1. A consumer buys many goods, among which are x_1 and x_2.
Prices are initially $\overline{p}_1, \overline{p}_2$.
The demand functions for these goods are:

$$x_1 = \alpha_1 - a(p_1 - \overline{p}_1) + b(p_2 - \overline{p}_2)$$

$$x_2 = \alpha_2 + b(p_1 - \overline{p}_1) - c(p_2 - \overline{p}_2)$$

Assume that a, b, c > 0.
Note that there are no income effects in these demand functions.

a) What other restrictions on a, b, c, if any, do we know, based on the theory of demand?

b) The government implements a tax per unit of x_1 purchased in the amount of t_1. What change in the consumer's initial wealth would have had the same effect on the consumer's welfare as this tax does?

c) How much revenue does this tax raise?

d) Define the deadweight loss of a tax change. How much is the deadweight loss of the tax t_1?

2. At a certain university the number of students wishing to attend substantially exceeds the available number of places. The most important criteria used to allocate places are money (high tuition fees) and intellectual ability (high-school records, admission test scores). A philanthropist offers a sum of money to the university, the yield on which, at the market rate of interest, will equal total annual tuition revenue, if the university agrees to eliminate money as a criterion for admission.

Assume that, at a tuition price of zero, 100,000 students would apply for 1,300 places. The university then considers three schemes for admitting students (assume initially that admission rights are not transferable):

i. Admission based strictly on test scores;
ii. Admission based on a lottery;
iii. Potential students must form a line in front of the Admissions Office. At 9:00 a.m. on April 1, each year, the first 1,300 students are admitted.

a) Would you recommend i or iii over ii on grounds of economic efficiency? Explain your answer carefully.

b) Now assume admission rights can be transferred. How would this change your answer to part a)?

3. Suppose that there is a road from A to B. The demand for trips from A to B depends only on the time taken, according to the function $p = 20 - 0.001x$, where x is trips per day and p is the time per trip in hours. The more trips are made in total, the slower each trip becomes because one person's extra journey slows down the other drivers. The relation of time taken to trips made is given by $p = 2 + 0.001x$. There are no other costs of travel, and the value of time is $1 per hour for all trips.

   a) What is the optimal number of trips?

   b) What money tax should be levied on drivers for a trip in order to support optimal utilization?

Part II.

Do all three of the following problems, which are equally weighted.

1. Consider the distribution function

$$F(x) = \begin{cases} 0 & if \quad x < 0 \\ x & if \quad 0 \le x < 1 \\ 2 - x & if \quad 1 \le x < 2 \\ 0 & if \quad x \ge 2 \end{cases}$$

Identify a distribution function G that first-order stochastically dominates F, and a distribution function H that differs from F by a mean-preserving increase in risk. Prove that your distributions possess the desired properties.

2. Prove that expected utility maximizer's preferences satisfy the independence axiom.

3. Draw an Edgeworth box example showing a failure of the Second Fundamental Theorem of Welfare Economics when preferences are not convex.

Part III

1. Consider a duopoly operating in a market with (inverse) demand function

$$p(q) = a - q,$$

where $q = q_1 + q_2$ is the aggregate quantity in the market and q_i is the output of firm i. Firm 1 has costs

$$c_1(q_1) = cq_1.$$

Costs of Firm 2 are $c_2(q_2) = c_l q_2$ with probability μ and $c_2(q_2) = c_h q_2$ with probability $(1-\mu)$. Information is asymmetric. Firm 2 knows its cost function, but firm 1 only knows the probability with which firm 2 cost function is of either type. Both firm know the demand function. All this is common knowledge.

   (a) Suppose that the two firms simultaneously choose quantities. What is the pure strategy Bayes-Nash equilibrium of this game (make assumptions concerning a, c_h, c_l, μ, and c such that all equilibrium quantities are positive) when $\mu = 0$? Do these coincide with the (appropriate) Nash equilibria?

   (b) Suppose that the two firms simultaneously choose quantities. What is the pure strategy Bayes-Nash equilibrium of this game (make assumptions concerning a, c_h, c_l, μ, and c such that all equilibrium quantities are positive) when $\mu \in (0, 1)$.

   (c) Suppose that Firm 1 chooses first (and, so, is the Stackelberg leader). Suppose $\mu \in (0, 1)$. Find all the pure strategy Bayes-Nash equilibria. (make assumptions concerning a, c_h, c_l, μ, and c such that all equilibrium quantities are positive)

2. Consider the Rubinstein-Stahl model of sequential bargaining. In period 1 player 1 makes a proposal. Both players discount the future at rate δ.

(a) Suppose the bargaining lasts only one period. If no decision is reached then both players receive a payoff of 0. What are the rationalizable strategies for player 1 in this game? What are the Nash equilibria of this game? What are the subgame perfect Nash equilibria of this game?

(b) Now suppose that the game lasts two periods. (Hence, the last person who may get to make a proposal is player 2). What are the subgame perfect Nash equilibria of this game? Comment on the change in "bargaining power" between this game and game studied in Part (a). How does the bargaining power depend upon δ?

(c) Now suppose that the game lasts three periods. (Hence, the last person who may get to make a proposal is player 1). What are the subgame perfect Nash equilibria of this game?