May 04

# DEPARTMENT OF ECONOMICS

# MICRO QUALIFYING EXAM

# SPRING 2004

**INSTRUCTIONS: This exam consists of three parts.**

**Be sure to put your test number I.D. on each answer sheet.**

**Start each new question on a separate sheet. You will have 4 hours to complete the exam.**

# Part I

1. The government must raise taxes to generate additional revenue. It has decided to impose a 10% tax either on apples or bananas. Current data for a "typical" consumer is given in the table below.

| | Price | Quantity | $dx_i / dp_i$ | $dx_i / dw$ |
|---|---|---|---|---|
| Apples | 1 | 50 | -100 | 0 |
| Bananas | 2 | 30 | -50 | 0.5 |

Throughout the question, you may use linear approximations to the consumer's demand curves.

a) Briefly explain why it is more appropriate to use EV to determine which tax is better rather than CV.

b) Estimate the EV of the apple tax.

c) Estimate the EV of the banana tax.

d) Based on parts b and c, which tax should the government implement? Explain your answer.

2. a) The cost function for a firm is $c(w_1, w_2, y) = w_1^a w_2^b y$.

i) What do we know about a and b?

ii) Derive the conditional factor demand equation for factor $z_1$.

b) Suppose we are in a three-commodity market (i.e. L = 3). Letting $p_3 = 1$, the demand functions for goods 1 and 2 are

$$x_1(p,w) = a_1 + b_1p_1 + c_1p_2 + d_1p_1p_2$$

$$x_2(p,w) = a_2 + b_2p_1 + c_2p_2 + d_2p_1p_2$$

Assume that the demand system above is generated from utility maximization. What are the restrictions on the above coefficients implied by this assumption? Explain.

3. Consider a fully separable quasilinear model with L goods in which each consumer has preferences of the form $u_i(x_i) = x_{1i} + \Sigma_{\ell=2}^{L}\phi_{\ell i}(x_{\ell i})$ and each good 2,...,L is produced with constant returns to scale from good 1, using $c_\ell$ units of good 1 per unit of good $\ell$ produced. Assume that consumers initially hold endowments only of the numeraire, good 1. Hence, consumers are net sellers of good 1 to the firms and net purchasers of goods 2,...,L.

In this setting, consumer i's demand for each good $\ell \neq 1$ can be written in the form $x_{\ell i}(p_\ell)$, so that demand for good $\ell$ is independent of the consumer's wealth and all other prices, and welfare can be measured by the sum of the Marshallian aggregate surpluses in the L - 1 markets for nonnumeraire commodities.

Suppose that the government must raise R units of good 1 through (specific) commodity taxes. Note, in particular, that such taxes involve taxing a *transaction* of a good, *not* an individual's consumption level of that good.

Let $t_\ell$ denote the tax to be paid by a consumer in units of good 1 for each unit of good $\ell \neq 1$ purchased, and let $t_1$ be the tax in units of good 1 to be paid by consumers for each unit of good 1 sold to a firm. Normalize the price paid by firms for good 1 to equal 1. Under our assumptions, each choice of $t = (t_1,...,t_L)$ results in a consumer paying a total of $c_\ell + t_\ell$ per unit of good $\ell \neq 1$ purchased and having to part with $(1 + t_1)$ units of good 1 for each unit of good 1 sold to a firm.

Let good 1 be the untaxed good (i.e., set $t_1 = 0$). Derive conditions describing the taxes that should be set on goods 2,...,L if the government wishes to minimize the welfare loss arising from this taxation. Express this formula in terms of the elasticity of demand for each good.

4. The Bureau of Land Management rents public grazing land in the West to the same few ranchers year in and year out. The rent is low and secure–so low and secure that when a ranch with public grazing rights is sold the bureau offers the same rights to the new owner. Despite this subsidy to the lucky ranchers, an inquiry by the National Association of Feedlot Operators into alleged excess profits in ranching found no difference in return between subsidized and unsubsidized ranches. Why?

5. Consider a family with two parents and one child. What do you expect should happen to transfers to the child when the parents get divorced?

## Part IIA

1. Consider a pure exchange economy with free disposal, with $L$ goods and $n$ traders. Let $p^l$ be the price of the $l$-th good, let $p = (p^1, \ldots, p^L)$, and let $S^{L-1}$ be the $(L-1)$-dimensional unit simplex; i.e.,

$$S^{L-1} = \{p \in \Re^L_+ : \sum_{l=1}^{L} p^l = 1\}$$

Assume that on $S^{L-1}$ there is a (single-valued) aggregate excess demand function $\hat{z}(p) : S^{L-1} \to \Re^L$ with $\hat{z}^l(p) : S^{L-1} \to \Re$ is the aggregate excess demand function for good $i$.

An 'individual characteristic' of consumer $i$ is given by $e_i = (X_i, \succeq_i, w_i)$, where the first component is the consumer's consumption set, the second is the consumer's preference relation, and the third is the consumer's initial endowment.

For this economy, a non-zero vector $p^* \in S^{L-1}$ is called an "equilibrium price vector" if and only if: for each $i = 1, \ldots, L$:

$$\hat{z}^l(p^*) \leqq 0 \tag{1}$$

$$\hat{z}^l(p^*) < 0 \text{ if and only if } p^{*l} = 0 \tag{2}$$

(i) Specify a set of conditions on $\hat{z}$ implying the existence of an equilibrium price vector.

(ii) Sketch a proof that the set of conditions you have specified under (i) does imply the existence of an equilibrium price vector. Indicate where and how the conditions are used in your proof.

(iii) Could the conditions specified under (i) be satisfied if, for all consumers, preference relations are continuous and strictly monotone, and their endowments are strictly positive? Justify your answer.

2. Consider a pure exchange economy with $n = 2, X_i = R^2_+$, $w_1 = (4, 0)$, $w_2 = (2, 4)$. Suppose the consumers have the following utility functions

$$u_1(x_1^1, x_1^2) = \frac{x_1^1}{2} + x_1^2$$

$$u_2(x_2^1, x_2^2) = x_2^1 + x_2^2$$

(a) Draw the Edgeworth Box with indifference curves and the initial endowment point clearly marked.

(b) Find the set of Pareto efficient allocations in the Edgeworth Box.

(c) Find the demand functions of the two agents.

(d) Draw the offer curves in the Edgeworth Box.

(e) Find all the Walrasian equilibria. Is it unique?

(f) Find the set of core allocations in the Edgeworth Box.

(g) Find the set of fair allocations in the Edgeworth Box.

3. Consider an economy with two goods, a private (rivalrous) good $x$, say leisure, and a public (non-rivalrous) good $y$, say radio broadcast music. Both goods are measured in hours per day. There are two consumers, 1 and 2, and one firm. The firm produces $y$, using labor $v$ as input. (Labor is leisure given by a consumer to the firm; hence if consumer $i$ supplies $v_i$ units of labor to the firm, then the amount of leisure left to $i$ is $x_i = w_i - v_i$, where $w_i$ is $i$'s initial endowment of leisure.) Let the production function of the firm be linear (constant returns to scale), with $k$ units of $v$ needed to produce one unit of $y$ at any scale of output ($k > 0$). There is no free disposal. The initial endowments $w_i$ of $x$ are positive, but the initial endowment of $y$ is zero.

Assume that each consumption set consists of all points $(x_i, y)$ such that $x_i \geqq 0$ and $y \geqq 0$,for $i = 1, 2$. Furthermore, for any $x_i \geqq 0$ and $y \geqq 0$, the utility function of consumer $i$ is given by:

$$u_i = z_i + \phi_i(y), \tag{3}$$

where the valuation function $\phi_i(y)$ is twice differentiable, and has a positive derivative $\phi_i'(y) > 0$, and a negative second derivative $\phi_i''(y) < 0$ for all $y \geqq 0$, for $i = 1, 2$. (Remember it is assumed $w_i > 0$ for $i = 1, 2$.)

Suppose that each consumer $i$ voluntarily chooses to contribute an amount of labor $v_i \geqq 0$ toward the production of the public good $y$, with $v_i < w_i$. By definition, at a Nash equilibrium allocation, written $(\bar{x}_1, \bar{x}_2, \bar{v}_1, \bar{v}_2, \bar{y})$, each consumer $i$ maximizes $u_i$, treating $v_j$ as given (for $j \neq i$)), and taking into account the equality

$$ky = v_1 + v_2. \tag{4}$$

(a) Find the conditions that characterize Pareto efficient allocations. (These will be equations in $x_1$, $x_2$, $y$ and the original endowments.)

(b) Suppose that, at a Nash equilibrium, consumer 2 contributes a positive amount of labor, but is still left with positive amounts of leisure, i.e., $w_2 > \bar{v}_2 > 0$, while consumer 1 contributes nothing, i.e., $\bar{v}_1 = 0$. Could such an equilibrium be Pareto optimal?

(c) Suppose that, at a Nash equiibrium, both consumers contribute positive amounts of labor, but are still left with positive amounts of leisure. Could such an equilibrium be Pareto optimal?

(d) Suppose that, a Nash equilibrium, neither consumer contributes any labor. Could such an equilibrium be Pareto optimal?

Explain your answers to questions (a), (d) (c), and (d) as fully as possible.

Part IIB

1. Ms. A has Bernoulli utility function defined over dollars given by $u^A(x) = \sqrt{x} - 2$. Assume that $x \geq 0$. She has available three lotteries: $l_1$ gives \$1 for sure, $l_2$ gives \$4 with 0.5 probability and \$0 with 0.5 probability and $l_3$ gives \$3 with 0.25 probability, \$2 with 0.25 probability, \$1 with 0.5 probability.

   (a) Calculate the certainty equivalent for Ms A for each of the three lotteries. Which one will she choose?

   (b) Mr B has a Bernoulli utility function over dollars given by $u^B(x) = 2x + 5$. Which lottery would he choose.

   (c) Compare the risk attitudes of Ms A and Mr B. Who is more absolutely risk averse? Who is more relatively risk averse?

2. Consider the game between Mr. B and Mr. J. This game takes place every day (after Mr. B has had breakfast). Mr. J can do two things: fight or not. He likes to fight with people who are feeling cowardly and gets a payoff of 1 if he does (and gets nothing if he doesn't fight). Mr J, however, doesn't like to fight with a person who is feeling brave. He gets a payoff of -1 from fighting with a brave person and a payoff of 0 from not fighting. The problem is Mr. J does not know whether Mr. B is feeling cowardly or if he is feeling brave today. Mr B, of course, knows how he's feeling in the morning. Mr. J knows that with probability .75 Mr. B is feeling cowardly and with probability .25 that he is feeling brave (and Mr. B knows this). Mr. J observes what Mr. B has had for breakfast. And Mr. B can, hence, use what he has for breakfast as a signal to Mr. J (regarding whether Mr. B is feeling brave or cowardly). Mr. B can have Coke for breakfast or he can have Pepsi. Mr. B prefers to have Coke for breakfast. And Mr. B prefers not to fight. He gets a payoff of 2 by having Coke and not fighting, and a payoff of 0 from having Pepsi and not fighting and a payoff of -1 from having Coke and fighting and a payoff of -2 from having Pepsi and fighting.

   (a) Represent the above game in extensive form.

   (b) Find all the pooling and separating weak perfect Bayesian equilibria for this game.

3. Consider the following normal form game

| | a | b |
|---|---|---|
| a | 7,7 | 7,5 |
| b | 5,7 | 9,9 |

(a) Suppose the game is played twice. Players discount future payoffs at rate $\delta$. Find a subgame perfect Nash equilibrium in which the strategy profile (a,a) is played in the first period (for a suitable $\delta$). Verify your claim. What restrictions does it impose of $\delta$?

(b) Suppose this game is repeated $T > 2$ times. Both players discount future payoffs at rate $\delta$. Propose a subgame perfect equilibrium in which (a,a) is played $T-1$ times (for suitable $\delta$). Verify your claim. What is the payoff of each player with this strategy? What restrictions does it impose on $\delta$?

(c) Suppose the game is played infinitely often. Both players discount future payoffs at rate $\delta$. EITHER propose a subgame perfect equilibrium (for suitable $\delta$) in which (a,a) is played in the first period, followed by (b,b) in the second period, (a,a) in the third period, (b,b) in the fourth period, and so on, alternating deterministically between (a,a) and (b,b) forever thereafter OR prove that such a subgame perfect equilibrium cannot exist. Verify your claim. What is the payoff of each player with this strategy (if it exists)? What restrictions does it impose on $\delta$?

4. Consider a game in which the following simultaneous move game is played twice and players do not discount future payoffs. Prior to the second play of the game players observe the actions chosen in the first play.

| | l | c | r |
|---|---|---|---|
| u | 10,10 | 2,12 | 0,13 |
| m | 12,2 | 5,5 | 0,0 |
| d | 13,0 | 0,0 | 1,1 |

Find all the pure strategy subgame perfect Nash equilibria of this game.