Microeconomics Qualifier Examination
August 2003

Instructions
Answer all questions in Part I and Part II; questions in each part are equally weighted.

Part I

1. Consider an economic agent with preferences represented by the utility function, $u(x) = \sqrt{x_1} + \sqrt{x_2}$ in answering the following:

   (a) Are these preferences homothetic? strictly convex? strictly monotonic? Include in your answer definitions of these fundamental characterizations of preferences.
   (b) What is meant by "representability"?
   (c) Obtain the Hicksian compensated demands and the expenditure function.
   (d) Find the Marshallian demands and indirect utility function.
   (e) Compute the Slutsky derivatives, $S_{11}(p,m)$ and $S_{22}(p,m)$, and show:

   $p_1^2 S_{11} = p_2^2 S_{22}$.
   (f) Are these two goods gross (net) substitutes or complements?
   (g) Obtain the inverse Marshallian demands and provide an intuitive explanation of what they represent. How would one relate the signing of cross-price effects with cross-quantity effects to explain the substitute-complement classification?

(e) What is meant by "bundle A is revealed preferred to bundle B"? State the weak axiom of revealed preference. Consider the following set of observations of price-quantity data:

$$p^1 = (5,5), p^2 = (6,4), x^1 = (100,100), x^2 = (80,\theta)$$

   (a) For what range of values on the parameter θ, the amount of the second commodity purchased in the second observation, would the weak axiom be violated?
   (b) For what range of values on θ will bundle one be revealed preferred to bundle two, consistent with the weak axiom?
   (c) For what range of values on θ will bundle two be revealed preferred to bundle one, consistent with the weak axiom?
   (d) What can be said about the range of values on θ that is not accounted for by (i)-(iii)?

3. What are lexicographic preferences? Are they rational? continuous? monotonic? Include formal definitions in your answer. Determine the Marshallian demands and justify.

4. Consider a C.R.S. technology characterized by the production function, q=F(L,K). In short run, when capital is fixed at K=1, the variable profit function is $\pi(p)=p^a$; what restriction must be placed on the parameter a and why? Why might one define $\pi(p)$ as producer surplus? Find the short-run supply and derived demand for labor functions, then find the short-run production function, Φ (L)≡F(L,1). Define the elasticity of substitution,

then compute for this technology from the production function, F(L,K), found from $\Phi(L)$. Do both factors exhibit the law of diminishing returns?

5. **(a)** Graph the short-run supply curve for a competitive firm with the cost function, $C(q)=q^2+1$, when (a) $C(0)=1$, fixed costs are sunk, and (b) $c(0)=0$, fixed costs are not sunk.

   (b) What is the linkage, if any, between the law of supply and the law of diminishing returns?

   (c) Why is the short-run marginal cost curve steeper than the long-run marginal cost curve?

Part II

1. Answer both parts of this question.

   (a) Prove that if preferences $\geq$ defined on the space of lotteries Δ^{N-1} can be presented by the von Neumann-Morgenstern utility function then preferences must satisfy the independence axiom.

   (b) Consider the "quadratic Bernoulli utility function": $u(x) = \alpha x + \beta x^2$.

      i. Assuming $x \geq 0$, what restrictions need to be placed on α and β for the quadratic Bernoulli utility function defined over wealth to display risk aversion? Are any restrictions required on the domain (of wealth, x)?

      ii. Show that under the restrictions derived in part (i), the function cannot exhibit decreasing absolute risk aversion.

      iii. What is the individual's certainty equivalent for the lottery $l =$ (-1, 0.9; 100, 0.1).

2. Consider the Rubinstein-Stahl model of sequential bargaining. In period 1 player 1 makes a proposal. Both players discount the future at rate σ.

   (a) Suppose the bargaining lasts only one period. If no decision is reached then both players receive a payoff of 0. What are the rationalizable strategies for player 1 in this game? What are the Nash equilibria of this game? What are the subgame perfect Nash equilibria of this game?

   (b) Now suppose that the game lasts three periods. (Hence, the last person who may get to make a proposal is player 1). What are the subgame perfect Nash equilibria of this game?

(c) Suppose the bargaining may continue infinitely many periods. Provide a sketch of an argument of the equilibrium outcome and strategies in this game.

3. Consider the game between Bill and John. This game takes place every day (after Bill has had breakfast). John can do two things: fight or not. He likes to fight with people who are feeling cowardly and gets a payoff of 1 if he does (and gets nothing if he doesn't fight). John, however, doesn't like to fight with a person who is feeling brave. He gets a payoff of 0 from fighting with a brave person and a payoff of 0 from not fighting. The problem is John does not know whether Bill is feeling cowardly of if he is feeling brave today. Bill, of course, knows how he's feeling in the morning. John knows that with probability .75 Bill is feeling cowardly and with probability .25 that he is feeling brave (and Bill knows this). John observes what Bill has had for breakfast. And Bill can, hence, use what he has for breakfast as a signal to John (regarding whether Bill is feeling brave or cowardly). Bill can have Pop for breakfast or he can have Beer. Bill prefers to have Pop for breakfast. And Bill prefers not to fight. He gets a payoff of 2 by having Pop and not fighting, and a payoff of 0 from having Beer and not fighting and a payoff of -1 from having Pop and fighting and a payoff of -2 from having beer and fighting.

(a) Represent the above game in extensive form and find all the pooling and separating equilibria for this game (use the notion of equilibria that you think is appropriate for this game).

(b) How would the equilibria change if the game was between Bob and John. In contrast to Bill, Bob likes beer. He gets a payoff of 2 by having beer and not fighting, and a payoff of 0 from having Pop and not fighting and a payoff of -1 from having beer and fighting and a payoff of -2 from having Pop and fighting. And, Bob feels cowardly with probability .25 and brave with probability .75 (and John knows this).

4. Two armies can either "attack" or "not attack" a country. Each army is either "strong" or "weak" with equal probability (and independently of each other). The army knows its type (but the other army does not). The country is worth M if captured. An army can capture the country either by attacking when its opponent does not or by attacking when it rival does and it is strong and the rival is weak. If two armies of equal strength attack then neither captures the country. The cost to the army of attacking is "s" when it is strong and "w" when it is weak, with $s<w$. There is no cost of attacking when its rival does not. Find all the pure strategy Bayesian Nash equilibria of this game.