Macro Theory Qualifying Exam July/August 2010

There are two parts. The exam is worth 240 points. Part 1 and Part 2 are each worth 120 points. Part 1 is further subdivided into Part 1 A (40 points) and Part 1 B (80 points). Weights on individual questions within each part are noted.

You have 240 minutes (4 hours) to complete this exam.

Answer all questions.

Part 1 A. (40 points)

Question 1.- (22 points)

Consider the RCK (Ramsey-Cass-Koopmans) model, and assume population is constant (and, for convenience, normalized to 1). As you know, the model can be expressed in terms of the familiar two expressions

$$\frac{dc}{dt} = \left(\frac{U_c}{-U_{cc}} \right) [f'(k) - (\delta + \rho)]$$

$$\frac{dk}{dt} \equiv f(k) - c(t) - \delta k$$

where c and k are consumption and the capital stock *per capita*, δ and ρ are the rates of depreciation and time preference, and $f'(k)$ is the marginal product of capital.

(i) Starting from the long-run equilibrium position, assume that at a time $t = 0$ the government imposes a permanent consumption tax at the rate x (i.e., 'x' per cent of each individual's consumption). Suppose that the proceeds of the tax are distributed to the public on a per-head basis, i.e., in a manner unrelated to each individual's consumption. Assume also that at time $t = 0$ when the tax is imposed, it was unanticipated by the public.

Analyze the effects of such tax, and discuss the basic economic reason for your results.

(ii) Take now the same question presented in (i) above, but assume that the tax imposed at time $t = 0$ is not permanent, but will only last until a date $t = \tau > 0$. As in (i), assume that at time $t = 0$ the policy was unanticipated, and that at that time it is announced, and believed by the public, that the policy in question will last only until $t = \tau$.

Analyze the effects of such tax, and discuss the basic economic reason for your results. In particular, compare these results with those you obtained in (i), and the reasons for any difference among them.

Question 2 (18 points) Consider an economy in which the production function, in per capita terms, is

$$y = A k,$$

and in which households maximize

$$\int_0^{\infty} U(c(t))\, e^{(n-\rho)t}\, dt$$

subject to their constraint

$$\frac{dk}{dt} = A k - \delta k - n k - c$$

where, as usual, δ is the rate of depreciation of the capital stock, n is the rate of population growth, and c, k, are consumption and capital per capita, respectively. Of course, you should recognize that this is the so-called AK model.

(i) Derive the household's conditions for optimality

(ii) Derive the equation(s) describing the overall dynamic and "balanced growth" properties of the model. Be sure to explain the economic meaning of each of the expressions you have derived, and whatever assumptions you need to make to assure that such a balanced growth path exists.

(iii) Assume that, starting from an initial balanced growth position, at time $t = 0$ there is a permanent, unanticipated decrease in the rate of depreciation of capital, δ, affecting both the existing capital stock and new capital being accumulated from $t = 0$ on. Explain what the effects of the change will be.

Part IB. (80 points)

1. (40 points) Suppose that output is given by:

$$y = \bar{y} + b(\pi - \pi^e),$$

where y is the log of real ouput, $\bar{y}$ is the natural rate of output, π is the inflation rate, and π^e is the private sector's expectation of the inflation rate.

The social welfare function is:

$$\gamma y - \frac{a\pi^2}{2},$$

where γ is a random variable with mean $\bar{\gamma}$ and variance σ_γ^2, and a is a positive constant. Assume that π^e is determined <u>before</u> γ is observed. However, the policymaker chooses π <u>after</u> γ is known.

Finally, assume policy is made by someone whose objective function differs from the social welfare function. In particular, assume the policymaker's objective function is:

$$c\gamma y - \frac{a\pi^2}{2},$$

where c is a nonrandom parameter.

a) What is the policymaker's choice of π given π^e, γ, and c?
b) Given you answer to part a, what is the private sector's value of π^e?
c) What is the expected value of the 'true' social welfare function, $\gamma y - a\pi^2/2$?
d) What value of c maximizes the expected value of 'true' social welfare?
e) Interpret your results. What does this exercise tell us about economic policy? Some claim that central bankers are unduly concerned about inflation. Does this exercise suggest a reason why that claim might be correct?

2. (40 points) Consider the following economy. There is a representative agent who has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where the function u is differentiable, increasing, and strictly concave, and c is real consumption. The technology in this economy is given by

$$c_t + x_t + g_t \leq f(k_t, g_t),$$

$$k_{t+1} \leq (1-\delta)k_t + x_t,$$

$$(c_t, k_{t+1}, x_t) \geq (0,0,0),$$

and the initial condition $k_0 > 0$, given. Here k_t and g_t are capital per worker and government spending per worker. Investment, consumption, and government spending per worker are given by x_t, c_t, and g_t, respectively. The function f is assumed to be strictly concave, increasing in each argument, twice differentiable, and such that the partial derivative with respect to both arguments converge to zero as the quantity of them grows without bound.

a) Describe a set of equations that characterize an interior solution to the planner's problem when the planner can choose the sequence of government spending.

b) Assume that the technology level, represented by z, can vary. More precisely, assume that the production function is given by $f(k, g, z) = zk^{\alpha}g^{\eta}$, where $0 < \alpha < 1$, $0 < \eta < 1$, and $\alpha + \eta < 1$. Go as far as you can describing how the investment/GDP ratio and the government spending/GDP ratio vary with the technology level z at the steady state.

Part II

There are two questions in part II. Please answer all the questions.

1. (60 points) Consider an economy in which the representative consumer has preferences of

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\begin{aligned}
U &= E\left[\sum_{t=0}^{\infty}\beta^t \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}\right], \\
X_t &= \bar{C}_{t-1}^{\theta} N_t^{1-\theta}
\end{aligned}
$$

where $0 < \beta < 1$, $0 < \theta < 1$, $\gamma > 1$, and $E[\cdot]$ is the expectation operator. $\bar{C}_t$ is the aggregate consumption and N_t is the labor supplied for production with the $[0,1]$ interval. Production technology for this economy is given by

$$Y_t = A_t N_t,$$

where A_t is a Markov process. In addition, this economy features a Cash-In-advance constraint meaning that consumer has to rely on cash for transactions. Specifically, money (M) is held because consumers cannot consume their own endowment. Time line for this economy is as follows: Asset market opens up first, followed by goods market. Assume that one-period nominal bond ($B^{\$}$) is traded in the asset market (Define $Q^{\$}$ as the price of this bond). Assume further that gross money growth ($\bar{M}_t/\bar{M}_{t-1}$) is exogenously given by a Markov process ϖ_t.

(a) (10 points) Write down the budget constraint(s) and the Cash-In-Advance constraint(s) for this economy.

(b) (15 points) Write down the household maximization problem and solve it as much as you can.

(c) (5 points) Define a recursive competitive equilibrium in this economy and characterize it as much as you can.

(d) (10 points) Can you write down the formula for the price of a bond that matures in two-period? Explain your formula.

(e) (20 points) Yield to remaining maturity, say n at time t for a bond is defined as $y_t^{(n)} = -\log Q_t^{(n)}/n$. To distinguish nominal bond from its real counterpart, nominal yield is denoted as $y_t^{\$(n)}$, while the real yield is $y_t^{(n)}$. Can you describe the signs of the two measures, $y_t^{\$(2)} - y_t^{\$(1)}$ and $y_t^{(2)} - y_t^{\$(1)}$ implied by this model?

2. (60 points) Consider an economy in which the representative agent has the preferences

$$\sum_{t=0}^{\infty} \beta^t \frac{\{c_t^\theta (1-n_t)^{1-\theta}\}^{1-\gamma} - 1}{1-\gamma}, \gamma \neq 1, \gamma > 0,$$

where c_t is private consumption and $0 < \beta < 1$, $0 < \theta < 1$. Each person has one unit of time, of which the fraction n_t is devoted to producing the consumption good and $1 - n_t$ is devoted to accumulating human capital h_t. The technology is described by the two equations:

$$c_t = (1-\tau_t)\alpha h_t u_t, \alpha > 0,$$

$$h_{t+1} = \delta h_t (1-u_t), \delta > 0,$$

Assume that the proceeds of the tax are rebated to consumers as a lump sum supplement to consumption.

(a) (20 points) Suppose that $\tau_t = 0$ for all t for this question only. Can you compute the first-best allocation of this economy?

(b) (10 points) If $\tau_t = \tau$ a constant, how does this tax affect the equilibrium time allocation compared with a case with no tax?

(c) (10 points) If τ_t is an increasing function of the existing human capital (h_t), say $\tau_t = v(h_t)$, $v' > 0, v'' < 0$. how does this tax affect the equilibrium time allocation compared with a case with no tax?

(d) (20 points) When τ_t is a policy variable for the fiscal authorities, can you solve and describe the second-best allocation of this economy à la Ramsey?