Macroeconomics Qualifier Examination

May 2009

Instructions

- You have four (4) hours for completing this exam.
- No material other than your pen(s), rulers and paper are allowed in the room, so please leave all other items in your office, or ask the staff to keep it.
- The exam has two parts, each with equal weight. The total number of maximum possible points is 240, corresponding to the 240 minutes (4 hours) that you have available. In each part you must answer all questions, and all sections of each question.
- The weight of each question is indicated as points. The points allocated to each question are indicative of the time you might wish to spend on the question. Allocate your time wisely: maximize your total score in each part, subject to the constraint of time available.
- Read each question carefully.
- Credit will be given only to answers that explain your reasoning briefly and carefully. Answers should be concise and direct to the point.
- Answers to questions other than those being asked will be ignored.
- ***Less than easily legibly answers will be ignored. You are asked to write with a dark (black or blue) ballpoint pen, and not with pencil, which appears as less legibly in copies. Thanks.***

PART I

This part contains 3 questions. Please answer all xxx questions.

Question 1 (30 points)

Consider the basic Ramsey-Cass-Koopmans (RCK) growth model, with identical individuals who maximize the present value of their flow of utility and a constant returns to scale production function in capital and labor (equal to population). Call $\rho>0$ the rate of time preference, $\delta>0$ the rate of physical capital depreciation, and $n > 0$ the rate of population growth.

Take the typical individual's budget constraint to be

[1] $$\frac{dk}{dt} \equiv f(k) - c(t) - (\delta + n)k$$

where, as usual, k and c are capital and consumption per unit of labor, and consider the two alternative instantaneous utility functions

[2] $$U(c(t))$$

and

[2'] $$U(c(t))N(t),$$

where $N(t)$ is the measure of population,

$$N(t) = N(0)e^{nt} = e^{nt}$$

for an initial population normalized at $N(0) = 1$.

(a) Find the necessary conditions for optimality for each of the two cases [2] and [2'], and describe the behavior (long-run steady state equilibrium, and the adjustment path of capital and consumption *per capita)*

(b) Discuss the economic meaning of **both** the two alternative specifications of the instantaneous utility functions [2] and [2'] **and** of their implications.

Question 2 (40 points)

Consider the case of an extremely simple economy, in which both output and the real interest rate are fixed, and in which the demand for money can be written as

[1] $$m^d = \ell(\pi)$$

where m^d is the demand for real money (in natural logs) and π is the inflation rate. Assuming that at all times the real money stock is equal to the demand for real money are equal, then

[2] $$m = \ell(\pi)$$

where $m = M - P$ is the real money stock (again, in natural logs), and M, P are the natural logs of the nominal money stock and the price level. From this last expression, of course, we can write

[3] $$\frac{dm(t)}{dt} = \mu(t) - \pi(t)$$

where μ is the (proportional) rate of nominal money growth.
Prices are perfectly flexible, there are rational expectations (perfect foresight), and time is continuous.

(a) Assuming that the monetary authority (central bank) fixes the path of the money supply with a constant rate of nominal money growth (μ), "solve" the model, by expressing it in the form of one differential equation for the real money stock. Discuss your result, and the implications of the model for the long-run and the dynamics of adjustment.

(b) Starting from an initial equilibrium, with a rate of monetary growth $\mu = \mu_0$, assume that at an initial time $t=0$, the central bank increases the rate of monetary growth to a level $\mu = \mu_1 > \mu_0$. Analyze what will happen.

(c) Starting from the same equilibrium position as in (b) above, assume now that the same change (increase) in the rate of monetary growth is announced to be transitory, with the rate of money growth to return to its initial level $\mu = \mu_0$ at a future time $t = \tau$. Analyze what will happen, under the following two possible alternatives:

(i) At time $t = \tau$., as anticipated, the rate of money growth is lowered back to its initial level, and
(ii) At time $t = \tau$., and contrary to anticipations, the rate of money growth remains forever at its higher level $\mu = \mu_1 > \mu_0$.

Question 3 (50 points)

Consider a model with the same characteristics as the Ramsey model concerning agents and the production technology (assuming for simplicity that population, and hence the labor force, is constant and normalized to unity), but with two important differences: (i) individuals can hold two assets: real capital (k) and a foreign asset (a) which yields a constant interest rate r, assumed to be equal to the rate of time preference, and (ii) there are adjustment costs of investment, in the form of a cost of transforming output (consumption goods) into investment goods. More specifically, the generation of *one unit* of gross investment per unit of time (I) that can be added to the stock of capital, requires

[1] $$1 + h(I) \qquad h_{II},\, h_{II} > 0, \quad h(0) = 0$$

units of output (consumption goods). Notice that the typical individual's flow budget constraints are, then, given by

[2] $$\frac{da}{dt} = f(k) + ar - c - I[1 + h(I)]$$

[3] $$\frac{dk}{dt} = I - \delta k,$$

where all the terms are in per capita basis.

(a) "Solve" the model, i.e., derive the necessary conditions for optimality, and the reduced forms for the behavior of consumption, the levels of capital and foreign assets, and the price of capital in terms of consumption goods (p). Notice that in this economy you can derive a system in the variables k and p.

(b) Provide a graphical representation (a phase diagram) of the system in the variables k and p.

(c) Starting from an initial long-run equilibrium position for the variables k and p, discuss the effects of a permanent, unanticipated change in the function [1] --more specifically, in the function $h(I)$-- so that now it takes less units of the consumption good to generate the same level of investment.

Part II

There are two questions in part II. Please answer all the questions.

1. (60 points) Consider the following Ramsey model with two periods, $t = 0, 1$. There is one good used for both private and government consumption. Government consumption (g_t) is exogenous and random in period 1. Specifically, in period 0, $g_0 = 0$, and in period 1, $g_1 = G$ with probability p (State 1) and $g_1 = \phi G$ with probability $1 - p$ (State 2), where $0 < \phi < 1$. There is no government debt outstanding at the beginning of period 0. Output is produced by a linear technology as a product of output productivity shock (A_t) and labor. In period 0, $A_0 = 1$, and in period 1, $A_1 = (1 - \delta)$ with probability p and $A_1 = (1 + \delta)$ with probability $(1 - p)$, where $0 < \delta < 1$. Thus the technology is given as

$$\begin{aligned} c_0 + x_0 &\leq 1, \\ c_{11} + G &\leq (1 - \delta)(1 - x_{11}) \text{ with } p, \\ c_{12} + \phi G &\leq (1 + \delta)(1 - x_{12}) \text{ with } 1 - p, \end{aligned}$$

where c is private consumption, x is leisure, and the first subscripts denote periods, while the second ones denote states. The preferences of the representative household are given as

$$u(c_0) + v(x_0) + \beta E\left[u(c_1) + v(x_1)\right],$$

where u and v are strictly increasing, twice continuously differentiable, and strictly concave with $\beta \in (0, 1)$, and $E(\cdot)$ is the expectation operator. To finance its spending, the government can levy a flat-rate tax (τ_t) on labor income in each period. Assume that the government can levy the tax differently for each state in period 1.

(a) (10 points) Write down the consumer's problem in the competitive economy with distortionary taxes and solve the problem.

(b) (5 points) Define the competitive equilibrium with distorting taxes.

(c) (10 points) Write down the implementability constraint for the benevolent government.

(d) (15 points) Formulate the Ramsey problem with distortionary taxes and derive the conditions for the government to achieve the maximum welfare in the second best sense.

(e) (15 points) In this economy, by construction, the government spends more when the low productivity shock is realized. How about the tax policy according to the Ramsey problem? Is it low when the economy is in bad shape? Why or why not? Justify your answer.

(f) (5 points) If the labor income tax is not state-contingent. That is, if the government has to commit to one tax rate for the period 2, how are your results from (d) and (e) different?

2. (60 points) Consider an economy in which there is a representative household with preferences of

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[\frac{\left(c_t^{\theta} m_t^{1-\theta}\right)^{1-\gamma}}{1-\gamma} + \frac{x_t^{1+\eta}}{1+\eta} \right],$$

where c_t is goods consumption, m_t, real balance holdings (M_t/P_t), and leisure (x_t). The household is endowed with one unit of time, and spends $1 - x_t$ amount of time producing $(1 - x_t)A_t$ units of the consumption good (A_t is a stochastic process). The budget constraint of this household at time t is then written as

$$M_{t+1} = M_t + T_t + P_t A_t (1 - x_t) - P_t c_t, \quad t = 0, 1, 2, ...,$$

where T_t is the government transfer in a lump-sum fashion. For future references, let's further define some notations, $1 + \pi_t = P_{t+1}/P_t$, and $\tau_t = T_t/P_t$. Assume that the fiscal transfers are the only way the government can change money supply in this economy. The government transfer is exogenous and follows a (Markovian) stochastic process.

(a) (5 points) Rewrite the budget constraint in real terms.

(b) (10 points) Define a recursive competitive equilibrium in this economy. In so doing, make sure that you state the equilibrium conditions for (π_t, c_t, x_t, m_t).

(c) (10 points) Solve the model and define the nominal interest rate R_t and calculate it.

(d) (10 points) If there is an asset which pays one unit of the consumption good as dividend, can you write down the price of this asset?

(e) (25 points) Extend this model so that there exist two agents (1 and 2) who differ from each other in terms of productivity, A_{1t}, and A_{2t} respectively. Assume further that $A_{1t} = A_1$, a constant, while A_{2t} follows a Markov process. Repeat the question (c) and (d) and compare your results as sharply as you can.