Macroeconomics Qualifier Examination

July 2009

Instructions

- You have four (4) hours for completing this exam.
- No material other than your pen(s), rulers and paper are allowed in the room, so please leave all other items in your office, or ask the staff to keep it.
- The exam has two parts, each with equal weight. The total number of maximum possible points is 240, corresponding to the 240 minutes (4 hours) that you have available. In each part you must answer all questions, and all sections of each question.
- The weight of each question is indicated as points. The points allocated to each question are indicative of the time you might wish to spend on the question. Allocate your time wisely: maximize your total score in each part, subject to the constraint of time available.
- Read each question carefully.
- Credit will be given only to answers that explain your reasoning briefly and carefully. Answers should be concise and direct to the point.
- Answers to questions other than those being asked will be ignored.
- ***Less than easily legibly answers will be ignored. You are asked to write with a dark (black or blue) ballpoint pen, and not with pencil, which appears as less legibly in copies. Thanks.***

PART I

This part contains 4 questions. Please answer all questions.

Question 1 (30 points)

Consider the simple (Solow) neoclassical growth model, with identical individuals who save a fixed proportion "s" of their income. The single output, which can be consumed or invested, is produced with capital and labor, with a production function subject to constant returns to scale, and capital depreciates at a fixed constant rate δ. For simplicity, assume that there is no technical change, and that there is a constant population (labor force), normalized to unity.

(a) (8 points) Briefly analyze the "solution" to this model --i.e., the characteristics of the steady state and of its dynamics

(b) (8 points) What is the so-called "Golden Rule" in this model? Briefly discuss the concept and its implications.

(c) (14 points) Assume now, in the same model, that owners of labor consume all of their income (i.e., they save nothing), and that owners of capital consume nothing (i.e., they save all of their income). You can, of course, assume that since this is a "representative agent" model, individuals own both capital and labor, so that they save nothing of their labor income, and consume nothing of their capital income. Find the steady state solution of the model, and carefully discuss its characteristics. In particular, compare it to the standard version of the model with an overall common savings ratio

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Question 2 (30 points)

Take the case of a simple economy in which the only existing asset is capital, which is constant (depreciation is zero, and output cannot be invested). Each unit of capital produces v units of output, which is a perishable consumption good. This is, as you will recognize it, the Lucas "seedless apples" model. Population is constant, and made up of identical individuals who live forever and maximize the present value of their flow of utility, which depends on consumption. Individuals have a fixed positive rate of time preference. Notice that although the aggregate level of the capital stock (trees) is constant, individuals can trade capital (apple trees) in exchange for output (apples). Expectations are rational (perfect foresight) and time is continuous.

(a) (10 points) Briefly analyze the model which describes such an economy --that is, the derivation of expressions describing the behavior of consumption and the price of capital.

(b) (20 points) Consider now the following problem. Suppose that, starting from an initial equilibrium, at an initial time $t=0$, agents anticipate that at some future time $t = \tau > 0$ the flow of production will permanently fall, say, from an initial level v_0 to a level $v_1 < v_0$. As precisely as possible, describe what will happen.

Question 3 (30 points)

Consider the case of an extremely simple economy, in which both output and the real interest rate are fixed, and in which the demand for money can be written as

[1] $$m^d = \ell(\pi)$$

where m^d is the demand for real money (in natural logs) and π is the inflation rate. Assuming that at all times the real money stock is equal to the demand for real money are equal, then

[2] $$m = \ell(\pi)$$

where $m = M - P$ is the real money stock (again, in natural logs), and M, P are the natural logs of the nominal money stock and the price level.

Prices are perfectly flexible, there are rational expectations (perfect foresight), time is continuous and the monetary authority (central bank) fixes the path of the nominal money stock with a constant rate of nominal money growth (μ).

> Starting from an initial equilibrium, with a rate of monetary growth $\mu = \mu_0$, assume that at an initial time $t=0$, the central bank announces that at a future date $t = \tau$. the nominal money ***stock*** will be increased by 10 per cent, with no change in the rate of nominal money growth. Assume, further, that such change is to take place via a one time head subsidy, unrelated to each individual's holdings of money. Describe what will happen, assuming that either
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> (i) the announced change takes place, or
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> (ii) contrary to the announcement, at date $t = \tau$ the change does not take place

Question 4 (30 points)

What follows is the Blanchard-Fischer specification of macroeconomic model based on Calvo's mechanism of price determination. As you know, this model incorporates the Calvo price equations of lagged price adjustment which result in the expression

[1] $$\frac{d\pi}{dt} = -\delta^2 \beta y(t)$$

where π is the inflation rate and $\delta, \beta > 0$. The Blanchard-Fischer version is completed by specifying

[2] $$y(t) = a(m(t) - p(t)) + b\,\pi(t)$$

where *m*, *p* are the logs of the nominal money stock and the price level, respectively, so that $(m - p)$ is the log of the real money stock, and *a, b* are positive parameters.

Differentiation [2] yields

[3] $$\frac{dy}{dt} = a(\mu - \pi) - b\,\delta^2 \beta y(t)$$

where μ is the rate of growth of the nominal money stock, so that now expressions [1] and [3] are a dynamic system in the variables output and the inflation rate.

> Starting from an initial long-run equilibrium with a rate of monetary growth μ_0, assume that at time $t=0$ there is a one time, unanticipated discrete increase in the level of the nominal money stock, with no change in the rate of monetary expansion. Will anything happen, and if so, what? If not, why?

Part II

There are two questions in part II. Please answer all the questions.

1. (80 points) The preference of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\eta}}{1+\eta} \right]$$

where $0 < \beta < 1$ denotes the discount factor, C_t is a composite consumption good, and N_t is the number of hours worked at t. Assume that the budget constraint of the household at time t as

$$C_t + q_t B_{t+1} \leq (1-\tau_t)N_t + B_t + T_t,$$

where q_t is the price of the one-period government debt, B_{t+1} is the amount of government debt issued at time t, τ_t is the tax rate imposed on labor income, and T_t is the lump-sum transfer/tax. It is further assumed that B_t is constrained by the following debt limits:

$$\underline{B} \leq B_t \leq \bar{B}$$

(a) (5 points) Write down the government budget constraint when it meets the budget balance every period. G_t is the government expenditure at time t.

(b) (10 points) Express the price of the government bond and labor income tax using the first-order conditions of the household problem.

(c) (10 points) Define the competitive equilibrium of this economy.

(d) (10 points) Write down the Ramsey problem of this economy.

(e) (10 points) Suppose that the debt limits do not bind only for the part (e). Solve the Ramsey problem you set up in (d).

(f) (20 points) Can you show when debts and taxes will follow Random walk processes whether or not the government expenditures have serial correlations? If it is not possible, explain your results.

(g) (15 points) How is your answer in part (f) changed if the representative household is equipped with a full set of one-period contingent bonds? Characterize your answer as sharply as possible.

2. (40 points) Consider an economy in which the representative agent has preferences of

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \delta C_{t-1})^{1-\gamma}}{1-\gamma},$$

where $0 < \delta < 1$ and $\gamma > 1$.

(a) (10 points) Derive the formula for the price ($Q(t,n)$) of a **nominal** bond at time t with the remaining maturity of n periods using the intertemporal marginal rate of substitution for the household. Denote P_t as the level of the goods (C_t) price at time t.

(b) (10 points) Suppose that the aggregate consumption process follows

$$\ln C_t - \ln C_{t-1} = g + \sigma \varepsilon_t,$$

where g and σ are positive constants and ε_t follows an *i.i.d.* normal distribution $N(0,1)$. Solve for the short-term nominal interest rate (i.e., one-period nominal interest rate). Assume that the correlation between consumption growth and inflation is denoted by $\rho_{c\pi}$.

(c) (15 points) If the n-period nominal interest rate $y(t,n)$ is defined by $-\ln Q(t,n)/n$, can you show $E(y(t,n)) \geq E(y(t,1))$ if correlation between consumption growth and inflation is negative?

(d) (5 points) Compare your results in (c) with a case in which $\delta = 0$ holds.