Macroeconomics Qualifier Examination

May 2008

Instructions

- You have four (4) hours for completing this exam.
- No material other than your pen(s), rulers and paper are allowed in the room, so please leave all other items in your office, or ask the staff to keep it.
- The exam has two parts, each with equal weight. The total number of maximum possible points is 240, corresponding to the 240 minutes (4 hours) that you have available. In each part you must answer all questions, and all sections of each question.
- The weight of each question is indicated as points. The points allocated to each question are an indicative of the time you might wish to spend on the question. Allocate your time wisely: maximize your total score in each part, subject to the constraint of time available.
- Read each question carefully.
- Credit will be given only to answers that explain your reasoning briefly and carefully. Answers should be concise and direct to the point.
- Answers to questions other than those being asked will be ignored.
- ***Less than easily legibly answers will be ignored. You are asked to write with a dark (black or blue) ballpoint pen, and not with pencil, which appears as less legibly in copies. Thanks.***

PART I

This part contains three questions. Please answer all three questions.

1.- (50 points) Consider an economy populated by a large number of identical individuals who live forever, work at a fixed constant rate, and at each point in time maximize the present value of their lifetime utility, i.e.,

$$\int_0^\infty U(c(t))\, e^{-\rho t}\, dt$$

where c is the consumption of the single commodity, and ρ is the fixed (positive) rate of time preference. The single commodity is produced with capital and labor, and the production function exhibits constant returns to scale. The capital stock depreciates at a fixed constant rate δ, time is continuous and there is zero population growth.

Assume that the single commodity can be either consumed or transformed into capital subject to a cost of transforming consumption goods into investment goods, of the form

[1] $$\textit{Total Cost of Investment} = I[1 + h(I)] \qquad h(0) = 0;\ h, h_I, h_{II} > 0$$

Assume, further, that individuals can also hold, besides capital, an alternative asset "a" (for example, an "international asset") which yields a constant real interest rate "r", and that such interest rate is equal to the rate of time preference, i.e.,

$$r = \rho$$

(a) Noticing the flow budget constraint

$$\frac{da}{dt} = f(k) + ar - c - I[1 + h(I)]$$

and the identify

$$\frac{dk}{dt} \equiv I - \delta k,$$

derive the optimality conditions for the typical individual.

(b) Describe the characteristics of this economy, i.e., the steady state solution and the dynamic behavior of all the relevant variables. In particular, express the system in the variables k (the capital stock) and p (the price of capital in terms of the final consumption good). How would this system look if the rate of depreciation is zero ($\delta = 0$) ? How does this "subsystem" in k and p interact with the variables a and c ?

(c) Starting from an initial steady state equilibrium, analyze the effects of a sudden, unanticipated one time "annihilation" (destruction) of part of the stock of the alternative asset "a". Make sure to clearly analyze the behavior of all relevant variables.

> ***Note: In parts (b) and (c), you should limit your mathematics to whatever is necessary to give a convincing and clear answer. Do not hesitate to use a graphs if you can do so convincingly.***

2.- (30 points) Consider an economy populated by a large number of identical individuals who live forever, work at a fixed constant rate, and at each point in time maximize the present value of their lifetime utility, i.e.,

$$\int_0^\infty U(c(t))\, e^{-\rho t}\, dt$$

where c is the consumption of the single commodity, and ρ is the fixed (positive) rate of time preference. The single commodity is produced with capital and labor, and the production function exhibits constant returns to scale. There is zero population growth and no technical progress, and the single commodity can be either consumed or used to increase the capital stock, which depreciates at a fixed rate δ. Time is continuous. You recognized here, of course, the simple version of the Ramsey-Cass-Koopmans (RCK) model. As you will also recognize, the rational expectations (perfect foresight) solution of the model yields the system

$$\frac{dc}{dt} = \left[\frac{U_c}{-U_{cc}}\right]\left(f_k(k) - (\delta + \rho)\right)$$

$$\frac{dk}{dt} = f(k) - \delta k - c$$

(a) Assume that starting at a full steady state equilibrium position, with $c = c^*$ and $k = k^*$, at date $t = 0$ government announces that at a future date $t = \tau$ every individual's capital stock ***in excess of*** $k = k_0 < k^*$ will be confiscated, with the proceeds of this confiscation to be thrown into the ocean --or sent to another country, which amounts to the same. Analyze what will happen.

(b) In the same problem as in (a), assume that government announces that the proceeds of the confiscation, instead of being thrown into the ocean, will be returned to the public on a strict *per head* basis. Analyze what will happen.

3.- (40 points) The following expression is the famous "Lucas aggregate supply function" resulting from limited (imperfect) information on the general price level:

$$y_t = \beta (p_t - E(p_t)),$$

where y and p are the natural logs of aggregate output and the price level, E is the expectation operator, and

$$\beta = b\left(\frac{v_z}{v_p + v_z}\right)$$

where $b > 0$ is a coefficient, and v_z and v_p are the variances of the shocks to the money prices of individual firms and to the general price level, respectively.

(a) Discuss the basic assumptions from which such aggregate supply function is derived, sketching the fundamental mechanism behind the derivation, and the economic meaning of the function and of the coefficient β.

(b) Using a simple quantity theory of money, and assuming velocity to be normalized to unity (zero in terms of natural logs), that is,

$$m_t = p_t + y_t$$

where m is the natural log of the nominal money stock, derive expressions for the level of aggregate output and the price level as functions of the actual and the expected levels of the nominal money stock. Discuss the economic meaning of these expressions.

Part II

There are two questions in part II. Please answer all the questions.

1. (100 points) The preference of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\eta}}{1+\eta} \right]$$

where $0 < \beta < 1$ denotes the discount factor, C_t is a composite consumption good, and N_t is the number of hours worked at t. The composite consumption good consists of differentiated products produced by monopolistically competitive final goods producers. There is a continuum of such firms of measure 1, and firm z produces good $C(z)$. The composite consumption good that enters the household utility function is defined as

$$C_t = \left[\int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{1-\theta}}, \quad \theta > 1.$$

The households decision problem is a two-step procedure. First, given any level of C_t, the households minimize the cost of achieving this level of the composite good, taking as given their nominal prices $P_t(z)$. Second, given the cost of achieving any given level of C_t, the household chooses C_t and N_t optimally.

(a) (5 points) Write down the first problem of minimizing the cost of buying C_t and derive a demand curve for $C_t(z)$. What is your definition of aggregate price level P_t?

(b) (5 points) Now in order to solve the second problem of the household, a periodic budget constraint is given as

$$C_t + E_t \left[\Lambda_{t,t+1} \frac{B_{t+1}}{P_{t+1}} \right] = \frac{B_t}{P_t} + (1+\chi) \frac{W_t}{P_t} N_t + \Psi_t - T_t$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor for computing the real value at period t of one unit of consumption goods at period $t+1$, W_t is the nominal wage rate, χ is a constant rate of the employment subsidy, T_t is the lump-sum tax, and Ψ_t is the real dividend income. Assume that the labor market is perfectly competitive. Compute the first-order conditions and derive the equation for the risk-free, gross nominal interest rate R_t at period t. How is it linked to the stochastic discount factor?

Firms employ labor to produce output using a constant returns to scale technology. The production function of the firm is given by

$$Y_t(z) = A_t N_t(z)$$

where $Y_t(z)$ is the output at period t by firm z, $N_t(z)$ is the labor hired by firm z and A_t is available technology in the economy. Firms adjust their prices infrequently and define ω as the probability of keeping prices constant and $(1-\omega)$ as the probability of changing prices. Firms also have two-stage problem of decision-making. First, firms minimize their cost of purchasing input for their production and second, they set their prices given the Bernoulli distribution of price adjustment opportunity.

(c) (5 points) Write down the first problem of minimizing the cost and derive the real marginal cost of the firm.

(d) (5 points) Write down the firm's pricing decision $\{P_t^*\}$ and explain the first-order conditions.

(e) (5 points) Combine the aggregate price level P_t you defined in (a) with the sticky price assumption to write down the law of motion for the aggregate price level over time. In so doing, assume a symmetric case in which every firm changing price at t uses the same price P_t^*.

(f) (15 points) Note that firms in this model may have different relative prices because of staggered price setting and it is possible that this can generate distortions to the economy. When $N_t = \int_0^1 N_t(z)dz$ represents aggregate labor and the aggregate output Y_t is defined as $\left(\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}}dz\right)^{\frac{\theta}{1-\theta}}$, determine this distortion denoted by Δ_t, satisfying

$$Y_t = \frac{A_t}{\Delta_t}N_t.$$

(g) (15 points) Show that the measure of relative price distortion can be re-expressed as

$$\Delta_t = (1-\omega)\left[\frac{1-\omega(1+\pi_t)^{\theta-1}}{1-\omega}\right]^{\frac{\theta}{\theta-1}} + \omega(1+\pi_t)^{\theta}\Delta_{t-1}$$

where $\pi_t = (P_t - P_{t-1})/P_{t-1}$ is the inflation between $t-1$ and t.

(h) (15 points) Suppose that government expenditure G_t is exogenously given and assume that the employment subsidy χ is set $1/(\theta-1)$ so that the distortion from the imperfect competition in goods market is offset. Write down the Ramsey problem using a Bellman equation to study optimal monetary policy. Is there an *implementability* constraint in your Ramsey problem? Why or why not?

(i) (5 points) Compute the first-order conditions.

(j) (15 points) Show that

$$\begin{aligned}\pi_t &= \frac{\Delta_t - \Delta_{t-1}}{\Delta_{t-1}}\\ \Delta_t &= \Delta_{t-1}\left[\omega + (1-\omega)\Delta_{t-1}^{\theta-1}\right]^{-1/(\theta-1)}\end{aligned}$$

How does optimal monetary policy prescribe if there is no initial price dispersion, i.e. $\Delta_{-1} = 1$? What if there exists initial price dispersion?

(k) (10 points) Is there a time-consistency issue for the Ramsey problem that you tackled in (h)-(j)? Why or why not?

2. (20 points) There exists a representative household with preferences over consumption and labor supply $\{C(z^t), N(z^t)\}$ over history $z^t := \{z_t, z_{t-1}, ...\}$ defined as

$$U = \sum_{t=0}^{\infty} \beta^t \left[\sum_{z^t} \pi(z^t) \left\{ \log C(z^t) - \frac{\zeta\eta}{1+\eta} N(z^t)^{\frac{1+\eta}{\eta}} \right\} \right]$$

where $0 < \beta < 1$ and $\pi(z^t)$ is the probability measure for the history z^t. The household budget constraint is written as

$$(1 + \tau_c(z^t))C(z^t) + \sum_{z_{t+1}} (1 + \tau_k(z^{t+1}))\, q(z^{t+1})a(z^{t+1}) \leq (1 - \tau_h(z^t))w(z^t)N(z^t) + a(z^t) + T(z^t).$$

$a(z^t)$ refers to the units of a real contingent bond with price q, τ's measure taxes, w is the wage per hour, and T is the lump-sum transfer. There exists a representative firm with a Cobb-Douglas technology, producing net output

$$y(z^t) - \delta k(z^t)$$

where $y(z^t) = A(z^t)k(z^t)^{\alpha}n(z^t)^{1-\alpha}$, k and n are the capita and labor it employs, $A(z^t)$ is history dependent total factor productivity, and δ is the depreciation rate. It rents capital at the interest rate $r(z^t)$ and pays wage $w(z^t)$.

(a) (10 points) Derive the labor supply N as a function of combination of taxes, and consumption-output ratio. If future taxes are expected to increase for financing a very futile project, how does the labor supply vary? What if this increase in future taxes can be given back in a lump-sum fashion?

(b) (10 points) Say that you re-align the equation you derived in (a) such that the left hand side has the tax combination term and all others in the right-hand-side. At business cycle frequencies, especially around recessions, how does the right-hand-side term fluctuate? Can you justify this fluctuation in terms of tax movement? Can you suggest a better explanation for the fluctuation of the right-hand-side by modifying the model? Make sense of your answer.