Macroeconomics Qualifier Examination

July 2008

Instructions

- You have four (4) hours for completing this exam.
- No material other than your pen(s), rulers and paper are allowed in the room, so please leave all other items in your office, or ask the staff to keep it.
- The exam has two parts, each with equal weight. The total number of maximum possible points is 240, corresponding to the 240 minutes (4 hours) that you have available. In each part you must answer all questions, and all sections of each question.
- The weight of each question is indicated as points. The points allocated to each question are an indicative of the time you might wish to spend on the question. Allocate your time wisely: maximize your total score in each part, subject to the constraint of time available.
- Read each question carefully.
- Credit will be given only to answers that explain your reasoning briefly and carefully. Answers should be concise and direct to the point.
- Answers to questions other than those being asked will be ignored.
- ***Less than easily legibly answers will be ignored. You are asked to write with a dark (black or blue) ballpoint pen, and not with pencil, which appears as less legibly in copies. Thanks.***

PART I

This part contains three questions. Please answer all three questions.

Question 1.- (30 points) An individual is born at time $t = 0$ and dies at time $t = T$. It receives a constant flow of income y_1 between dates *0* and *R*, and a fixed flow of income y_2 between dates *R* and *T*, where

$$0 < R < T$$

and

$$y_1 > y_2 > 0.$$

He or she can borrow or lend at a fixed interest rate r, by holding a real asset "a", which can be positive or negative. Assume, further, that the fixed interest rate is equal to the individual's rate of time preference. The individual receives no inheritance and leaves no bequest (which means that its level of assets at death ($t=T$) must be zero.

Making sure that you explain your answers,

(a) Assuming that the (rational) individual maximizes the present value of its utility at birth, solve the individual's optimization problem, and show the characteristics of the individual's path of consumption and assets;

(b) Assume that at time $t = R$ government imposes an **unexpected, one time** tax of 10 per cent on the individual's assets, without any return of the tax proceeds. Analyze what the result of this policy will be on the path of consumption and assets;

(c) Assume now that the imposition of the same tax as in (b) above is announced to individuals at the time of their birth ($t = 0$). Analyze what the result of this policy will now be on the path of consumption and assets;

(d) Compare the effects of alternatives (b) and (c) on the individual's welfare.

Question 2.- (50 points) Consider the standard Ramsey-Cass-Koopmans model with fixed labor. For simplicity, assume population is constant and normalized to one.

Assume that, starting at an initial equilibrium at the long run steady state, government introduces an ***unanticipated permanent subsidy*** of *"x"* per cent on gross investment.

(a) Assume, first, that government finances the subsidy from "external sources" (maybe a permanent flow gift from the World Bank). Derive the optimality conditions for the typical individual (household), and analyze the response of the economy in the short and the long run;

(b) Assume now that, instead, the same subsidy is financed via a *per head* tax. Perform the same analysis as in part (a), and compare the two outcomes

(c) Assume now that, instead, the same subsidy is financed via a tax on consumption. Perform the same analysis as in part (b) and compare the two outcomes.

Question 3.- (40 points) ***Notice that this question is rather long, in order to be clear and explicit, but that this does not necessarily mean that the answer should be extremely long. Read the question carefully.***

The following are two important macroeconomic "theories" of the possible effects of monetary policy on output and prices.

One is the Blanchard-Fischer specification based on Calvo's mechanism of price determination. As you know, this model incorporates the Calvo price equations of lagged price adjustment which result in the expression

[1] $$\frac{d\pi}{dt} = -\delta^2 \beta y(t)$$

where π is the inflation rate and $\delta, \beta > 0$. The Blanchard-Fischer version is completed by specifying

[2] $$y(t) = a(m(t) - p(t)) + b\,\pi(t)$$

where *m*, *p* are the logs of the nominal money stock and the price level, respectively, so that $(m - p)$ is the log of the real money stock, and *a, b* are positive parameters.

Differentiation [2] yields

[3] $$\frac{dy}{dt} = a(\mu - \pi) - b\,\delta^2 \beta y(t)$$

where μ is the rate of growth of the nominal money stock, so that now expressions [1] and [3] are a dynamic system in the variables output and the inflation rate.

Another, which follows from a model of lack of perfect information due to Lucas (the famous "islands" model), results in the expressions

[4] $$y_t = \left(\frac{\beta}{1+\beta}\right)(m_t - E(m_t))$$

[5] $$p_t = \left(\frac{1}{1+\beta}\right)[\beta E(m_t) + m_t]$$

where *y*, *p* and *m* are again the natural logs of real income, the price level and the nominal money stock, $\beta > 0$ is a constant coefficient and $E(m_t)$ is the expected level of the nominal money stock.

Notice that both models rely on rational expectations. The first model is a continuous time formulation, with perfect foresight, and the second is a discrete time formulation, with uncertainty.

> Starting from an initial long run equilibrium in the economy with a constant nominal money stock (i.e., $\mu = 0$ in the first model), use these two models to analyze an unanticipated one time increase in the nominal money stock. Compare your answers for the two models and evaluate the results. Which "theory" do you like best, if any, and why?

Part II

There are two questions in part II. Please answer all the questions.

1. (60 points) Suppose that there exists an economy in which government spending (g_t) is the only exogenous, aggregate shock and its evolution over time is identified through a state variable (s_t). s^t denotes the history of events, i.e. $\{s_0, s_1, ..., s_t\}$. Assume that s_t follows a Markov process. Consider a stochastic economy with an infinitely lived representative agent whose preference is given as

$$\sum_{t=0}^{\infty} \sum_{s^t \in S} \beta^t u(c_t(s^t), l_t(s^t)) \pi_t(s^t),$$

where u is increasing, strictly concave, and three times continuously differentiable in consumption c and leisure l, $0 < \beta < 1$ and $\pi_t(s^t)$ is the probability measure over a finite set S. The household is endowed with one unit of time each period and the resource constraint of the economy is given by

$$c_t(s^t) + g_t(s^t) \leq 1 - l_t(s^t)$$

Assume that there is a one-period real government bond whose price is denoted as $q_t(s^t)$. This is measurable with respect to time t information. Government also levies a proportional tax $\tau_t(s^t)$ on the income $(1 - l_t(s^t))$ and it gives back later as a lump-sum transfer $Z_t(s^t)$.

(a) (5 points) Define a competitive equilibrium for this economy. Assume additional conditions if necessary.

(b) (10 points) Write down (an) implementability constraint(s) for this economy.

(c) (10 points) How is your result in (b) different from the case with a complete market case such as contigent government debt model? Can you relate your result to Barro's result based on an ad-hoc loss function?

(d) (15 points) Define a Ramsey equilibrium for this economy as a sequence problem, and compute the first-order conditions. Interpret your results.

(e) (20 points) Write down the Ramsey problem using Bellman equation approach and check if you obtain the same conditions as in (d). In so doing, mention clearly what the state variables are.

2. (60 points) Suppose that there exist two types of households in an economy indexed by $[0, 1]$ interval. They are labeled as patient household (P) and impatient household (I) with the fraction of λ and $(1 - \lambda)$ respectively. Both types have the periodic felicity function of $\ln(C_t)$ and the preference function of each type (z) is defined by

$$\sum_{t=0}^{T} \beta(z)^t \ln(C_t(z)),$$

where $z = P, I$ and $\beta(P) > \beta(I) > 0$. We consider an exchange economy with the endowment process y_t for each type and $y_t/y_{t-1} = g > 1$, a constant. In addition, we assume that there exist a set of risk-free bonds with different maturities (1 to T periods), the time 0 prices of which are denoted as $q^{(1)}$, $q^{(2)}$,...$q^{(T)}$.

(a) (5 points) Define a competitive equilibrium for this economy and compute the first-order conditions for both types of the households.

(b) (15 points) Compute the equilibrium one-period real interest rate for this economy.

(c) (15 points) For this question only, assume that $T = 2$. Then, compare the result you obtained in (b) with the real interest rate with the representative agent version of this economy. Which economy has higher real interest rate and why? Justify your answer.

(d) (15 points) Compute the time 0 price of the $n-$period bond. In so doing, decompose the bond price into two parts, the homogeneous part (representative agent case), and the patience heterogeneity part. (*hint*: In fact, the second part can be further expressed into two parts, one of which is closely related to Jensen's inequality.)

(e) (10 points) How is the average shape of the real yield curve? What kind of implication does this result have in light of modeling a macroeconomic model using the representative agent?