Macroeconomics Qualifier Examination

May 2007

Instructions

- **You have four (4) hours for completing this exam.**
- No material other than your pen(s), eraser, rulers and paper are allowed in the room, so please leave all other items in your office, or ask tha staff to keep it.
- The exam has two parts, each with equal weight. The total number of maximum possible points is 240, corresponding to the 240 minutes (4 hours) that you have available. **In each part you must answer all questions, and all sections of each question.**
- The weight of each question is indicated as points. The points allocated to each question are an indicative of the time you might wish to spend on the question.
- Allocate your time wisely: maximize your total score in each part, subject to the constraint of time available.
- Read each question carefully.
- Full credit will be given only to answers that explain your reasoning briefly and carefully. ***Answers should be concise and direct to the point.***
- Answers to questions other than those being asked, will be ignored.
- **Less than easily legibly answers will be ignored. You are asked to write with a dark (black or blue) ballpoint pen, and not with pencil, which appears as less legibly in copies. Thanks.**

PART I

There are 4 questions in this Part I. Answer all questions.

Question 1.- (30 points)

Consider the case of an individual born at time $t=0$ who knows with certainty will die at time $t=T$, whose income is exogenous and who can borrow or lend at a fixed interest rate r. Assume the individual maximizes the functional

$$\int_0^T U(t)\, e^{\rho t}\, dt$$

where ρ is the fixed rate of time preference, assumed to be equal to the rate of interest. Assume, further, that the utility function is of the form

$$U(c) = \frac{c^{(1-\gamma)}}{(1-\gamma)} \qquad 1 > \gamma > 0$$

The path of income is given by

[3] $$y(t) = y(0)\, e^{\alpha t} \qquad \alpha > 0, \qquad \text{for } T > R > t > 0,$$

and

$$y(t) = 0 \qquad \text{for } T > t > R$$

where you can identify R as the "retirement" date. Assume, further, that assets are zero at birth (no inheritance) and must be zero at death (no bequest, positive or negative).

(a) Derive the optimal paths of consumption and assets, and find expressions for how the initial level of consumption (i.e., at time $t = 0$) will depend on the initial level of income (at time $t = 0$);

(b) Assume now, under the same conditions as before, that the individual is subject to a proportional tax on income, at the rate v, while at the same time receiving a flow subsidy of $s(t) = s_0$ from the retirement time until death (i.e., for $T > t > R$). Derive once again the optimal paths of consumption and assets.

Question 2 (20 points)

Consider an economy populated by a large number of identical individuals who live forever, work at a fixed constant rate, and at each point in time maximize the present value of their lifetime utility, i.e.,

$$\int_0^\infty U(c(t))\, e^{\rho t}\, dt$$

where c is the consumption of the single commodity, and ρ is the fixed (positive) rate of time preference.

Assume, further, that the single commodity is produced with capital and labor, and that the production function exhibits constant returns to scale. There is zero population growth and no technical progress, and the single commodity can be either consumed or used to increase the capital stock, which depreciates at a fixed rate δ. Time is continuous. You recognized here, of course, the simple version of the Ramsey model.

(a) Specify the problem to be solved by the representative agent, and derive the necessary optimality conditions;

(b) Use a phase diagram to illustrate the nature of your answer and the dynamics of the system;

(c) Prove that there is a saddle path, and that it is unique;

(d) Assume now that at time $t = 0$, starting from an initial steady state position, a permanent proportional tax on consumption is imposed, at the rate of x per cent per period. The proceeds of the tax are to be distributed to agents as a head subsidy. Analyze the results of such a change, and explain the economic reasons for those results.

(e) Take the same problem as in (d) above, but assume now that at time $t = 0$ the tax is anticipated to be transitory, i.e., that at time $t = \tau > 0$ it will be eliminated. Assuming that, indeed, the tax is permanently eliminated at time $t = \tau$, analyze the results of such a change, and explain the economic reasons for the results. In particular, elaborate on the difference, if any, between these results and those in case (d).

Question 3 (40 points)

Consider an economy populated by a large number of identical agents who live forever, and in which the stock of capital is constant (apple trees), yielding a constant flow of perishable consumption goods (seedless apples). Population is constant, and at each initial time $t = 0$ individuals maximize the present value of their lifetime utility (which depends only on consumption), with a positive rate of time preference ρ. Individuals can borrow and lend at the rest of the world's capital market at a constant real interest rate r, assumed to be equal to the individuals' rate of time preference.

(a) Derive the typical individual's necessary conditions for optimality, and the expressions describing the overall behavior of the system --steady state and dynamic adjustment, as well as the determination of the price of capital. Construct a graphical representation.

(b) Assume now that the world interest rate faced by the typical individual, rather than being fixed, is a function

$$r = f(r*, b) = r* + r(b) \qquad \partial r/\partial b > 0 \quad for \quad b > 0$$

$$r = r* \qquad for \quad b \leq 0$$

where b is the *aggregate level of debt of the economy.* Perform the same analysis you did in case (a), assuming that the overall economy happens to be in the vicinity of a steady state at which there is net aggregate debt, i.e., $b > 0$. Analyze the effects of a *transitory decrease* in the "basic" world interest rate $r*$.

(c) Assume now that, in the previous model, the world interest rate faced by the typical individual is a function

$$r = f(r*, b, v) = r* + r(b, v) \qquad \partial r/\partial b > 0,\ \partial r/\partial v < 0 \quad for \quad b > 0$$

$$r = r* \qquad for \quad b \leq 0$$

where $v = k\,p$ is the value of the *aggregate capital stock* of the economy (k being the fixed aggregate physical capital, and p the price of capital). Derive the typical individual's necessary conditions for optimality, and expressions for the behavior of the three variables consumption, aggregate debt and the price of capital, as well as expressions for the steady state value of the variables.

(d) In the case assumed in the previous paragraph (c), analyze the steady state effects of a one-time exogenous increase in the level of physical capital, *k*.

Question 4 (30 points)

Calvo's celebrated analysis of price stickiness implies the following expression:

[1] $$\frac{d\pi}{dt} = -\delta^2 \beta y(t)$$

where π, δ, β are the inflation rate and two positive parameters, respectively, and y is the natural log of deviations of output from its long-run equilibrium level. A standard formulation (By Blanchard and Fischer, for example), is to add an expression such as

[2] $$y(t) = a(m(t) - p(t)) + b\,\pi(t)$$

where *m* is the log of the nominal money stock, so that *(m - p)* is the log of the real money stock, and *a, b* are positive parameters.

(a) Consider and comment on the following two statements:

> (i) "Expression [1] does not make much sense, because it states that higher levels of actual output are associated with higher levels of inflation"
>
> (ii) "Expression [2] implies a long-run naive Phillips curve, i.e., a long run relationship between inflation and output".

(b) In the system implied by expressions [1] and [2], and starting from an initial steady state equilibrium, analyze the effects of an unanticipated, permanent increase in the coefficient *b* in expression [2]. Going beyond the pure mechanics, what is your interpretation of what this coefficient represents?

Part II

There are 3 questions in this Part II. Answer all questions and all parts of each question.

Question 1 (70 points)

Consider an economy consisting of an infinite sequence of two period-lived, overlapping generations. Let N_t denote the number of households born at t, where $N_1 > 0$ is the number of initial old agents. **Population grows at the constant rate $n>0$.** At each date each young agent is endowed with a single unit of labor, which is supplied inelastically. Labor earns the prevailing real wage w_t at t and r_t denotes the real rental rate on physical capital at t. In addition, at each date there is a single final good that can either be consumed, or saved. There are no endowments of goods. If invested, one unit of the good at t becomes one unit of physical capital at $t+1$.

The monetary authority prints money at the constant rate $\sigma>-1$, so that $M_{t+1} = (1+\sigma)M_t$. Also, **the return on capital is taxed at the rate $0<\theta<1$.** This tax is paid by the owners of capital, not by firms. ***The new money and the tax on the return on capital are used each period to finance an endogenous and wasteful sequence of spending of g_t goods per young agent.*** $M_1 > 0$ is the initial money supply, which is distributed equally among the initial old.

The single good is produced each period, using the constant returns to scale technology $Y_t = K_t^{\alpha} L_t^{1-\alpha}, 0 < \alpha < 1$, i.e.: Y_t units of the final good are produced using K_t units of physical capital and L_t units of labor. Capital depreciates completely each period after production. At $t=1$, the initial old agents are also endowed with the initial capital stock of the economy $K_1 > 0$ given, which is distributed equally among them.

The lifetime preferences of an agent born at t are given by

$$u(c_{1t}, c_{2t+1}) = \ln(c_{1t} + \gamma) + \beta \ln(c_{2t+1})$$

where c_{1t} denotes consumption when young by an agent born at t, $c_{2,t+1}$ denotes consumption when old by an agent born at t, and $\gamma \geq 0$.

Also, $z_t \equiv \dfrac{M_t}{N_t p_t}$, where z_t is the real supply of money per young agent.

a) (6 points) Set up, describe briefly and solve the household's problem.
   - (4 points) Show briefly but carefully that a corner solution is possible. If needed, make a sufficient assumption to ensure an interior solution.
   - (2 points) Derive explicitly the no arbitrage condition from the household's problem and explain its meaning.

For the rest of the question, work out the case of an interior solution, making the assumption you found in part (a).

b) (3 points) Set up, describe briefly and solve the problem of the representative firm at t.

c) (2 points) Derive the Government Budget Constraint.

d) (8 points) From your answers to (a) (b) and (c), together with market clearing and aggregate consistency conditions, derive explicitly the equilibrium laws of motion for k_t and z_t. Find all the steady state equilibria.

e) (5 points) Set up and find the first order conditions of the general planner's problem for the First Best of this economy: the planner's objective function and the feasible set for all time period t. Set up the Lagrangean. **Do not forget that there is a feasibility condition for each period t.**

f) (5 points) Set up and solve the problem solved by the planner to find the Golden Rule (First Best.) Derive explicitly the conditions for the Golden Rule and explain the meaning of each condition. Give an expression for the golden rule capital-labor ratio, k_{GR}.

g) (5 points) Find the conditions for which there is capital over accumulation in the non trivial, non monetary steady state $(k^*,0)$. Is this allocation Pareto Optimal? Explain briefly but carefully. Do not forget θ, the future generations and the initial old.

h) (7 points) Is the monetary steady state (k,m) of this economy Pareto Optimal? ***Explain carefully for all possible values of σ and θ*** Do not forget the future generations and the initial old.

i) (9 points) Is there a Mundell-Tobin effect in this economy?
- (2 points) Show briefly and explain.
- (7 points) Explain briefly how the theoretical results in monetary growth models compare to the empirical evidence. Be brief, but mention explicitly the empirical literature, and how each paper is different from each other.

j) (17 points) Dynamics and Local Stability Analysis
- (7 points) Derive the Jacobian matrix explicitly.
- (6 points) Evaluate the Jacobian matrix explicitly at the monetary steady state (k,m). Find and show the dynamic properties of this equilibrium. Explain the characteristics of dynamic equilibria in a neighborhood of this steady-state equilibrium.
- (4 points) Evaluate the Jacobian matrix explicitly at the non trivial, non monetary steady state $(k^*,0)$. Find and show the dynamic properties of

k) (3 points) Draw, BUT DO NOT DERIVE, the phase diagram for this economy on the (k_t, z_t) space. Label your diagram carefully.

Question 2 (36 points)

Consider an economy populated by infinitely-lived representative agent. Population is constant. There are N households and M firms. In each period each household is endowed with one unit of labor, which he/she supplies inelastically. Let c_t denote consumption of the single final good at date t. The agent's lifetime preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t), \text{ where } 0 < \beta < 1$$

The final good at t is produced using capital K_t and labor L_t as inputs into the constant return to scale production technology given by $F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}$, where $0 < \alpha < 1$. The households own both labor and capital, and they rent them out to the firms at the real unit prices w_t and r_t, respectively. Let k_t denote the capital-labor ratio per household at t and $\tilde{k}_t$ denote the capital-labor ratio per firm at t.

Each period, households pay a tax of $0<\theta<1$ goods per unit invested into capital. The proceedings of this tax are used to finance the wasteful spending of g_t goods per agent. Finally, assume that capital depreciates completely each period after production and there is no fiat money in this economy.

a) (5 points) Set up and solve the ***dynamic programming problem*** solved by a representative household.

b) (2 points) Set up and solve the problem solved by a representative firm each period.

c) (7 points) Find the ***competitive equilibrium for this economy***. Do not forget the market clearing conditions (you must include market clearing conditions for labor and capital.) Use the following guess for the functional form of the policy function: $k_{t+1} = xk_t^{\alpha}$ where $x>0$ is a constant. Use the method of undetermined coefficients to find the value of x and verify your guess.

d) (3 points) Find and describe briefly all the steady-state equilibria for this economy. Is the nontrivial steady-state capital increasing or decreasing in θ?

e) (7 points) Is the competitive equilibrium Pareto optimal? Show explicitly and explain. You must also check steady-state consumption.

f) (12 points) Use your answer to part (c) to explain the negative side and the positive side of the ***Lucas' Critique***. Your answer must: include what is the difference between reduced form equations and a model in its structural form; illustrate with the particular model set up in this question; and show whether the reduced-form parameters in this model depend on policy parameters or not.

Question 3 (14 points)

Describe briefly but carefully the concept of ***self-fulfilling prophecies***. Illustrate carefully with the simple example of the overlapping generations model with a constant supply of fiat money. DO NOT SOLVE THE MODEL. Use a diagram, and explain briefly but carefully the importance of the fact that money is intrinsically useless and that there is an endogenous initial condition. You must mention explicitly the tie between people's beliefs, initial prices and initial conditions.