Macroeconomics Qualifier Examination
July 2007

Instructions

• **You have four (4) hours for completing this exam.**

• No material other than your pen(s), eraser, rulers and paper are allowed in the room, so please leave all other items in your office, or ask the staff to keep it.

• The exam has two parts, each with equal weight of 120 points. The total number of maximum possible points is 240, corresponding to the 240 minutes (4 hours) that you have available. **In each part you must answer all questions, and all sections of each question**.

• The weight of each question is indicated as points. The points allocated to each question are an indicative of the time you might wish to spend on the question.

• Allocate your time wisely: maximize your total score in each part, subject to the constraint of time available.

• Read each question carefully.

• Full credit will be given only to answers that explain your reasoning briefly and carefully. ***Answers should be concise and direct to the point.***

• Answers to questions other than those being asked, will be ignored.

• **Less than easily legibly answers will be ignored. You are asked to write with a dark (black or blue) ballpoint pen, and not with pencil, which appears as less legibly in copies. Thanks.**

• **The two faculty members of the Examination Committee will be in their offices during the time of this exam. If you need a clarification please contact the staff.**

PART I

Question 1 (40 points)

Consider an economy populated by a large number (technically, a continuum) of identical individuals who live forever, and who save a constant proportion of their income. Population, equal to the labor force, increases at a constant positive rate. There is only one good that can be either consumed or used as capital, the production of which is dictated by a constant returns to scale production function in capital and labor. Capital depreciates at a constant rate. This is, of course, the simple Solow-Swan model of economic growth.

(i) Formalize and "solve" this model --i.e., carefully describe its dynamic and steady-state properties.

(ii) (a) Analyze the so-called "golden rule of accumulation", i.e., the case of a savings ratio which maximizes long run per capita consumption. (b) Show why this "golden rule" can be "dynamically inefficient", making sure you explain the economics behind it. (c) Completely specify a policy resulting in a golden rule long run equilibrium.

(iii) Suppose now that owners of labor consume all of their income (i.e., they save nothing), and that owners of capital consume nothing (i.e., they save all of their income). Notice that since this is a "representative agent" model, this means that individuals own both capital and labor, so that they save nothing of their labor income, and consume nothing of their capital income. Solve this system, and discuss its connection to the previous case of a uniform overall savings ratio.

Question 2 (40 points)

Consider an economy populated by a large number (technically, a continuum) of identical individuals who live forever, and who at any initial time $t = 0$ maximizes the functional

$$\int_0^\infty U(c(t))\, e^{-\rho t}\, dt$$

where c is consumption, $U(\ )$ is the instantaneous utility function with the usual "well behaved" characteristics (concave, twice differentiable), and ρ is a positive constant rate of time preference. There is a single commodity, which can be either consumed or used as capital, and which is produced according to the linear production function

$$Y = AK \qquad\qquad A > 0$$

where Y is total output, K is the total stock of capital, and A is a fixed coefficient. There is no population growth, and capital depreciates at a constant rate δ. Assume also that $A > \delta + \rho$. There is perfect foresight.

Assume, further, that the utility function is of the form

$$U(c) = \left\{ \frac{c^{(1-\theta)}}{(1-\theta)} \right\} \qquad 0 < \theta < 1$$

(i) Formalize and "solve" this model, and carefully describe its dynamic and balanced growth properties. Show a graphical representation of the system.

(ii) (a) Suppose now that at a date $t = 0$, at which the economy is in its balanced growth path, government announces that at a future date $t = \tau$ a one-time levy (tax) on the ***stock*** of capital will be imposed at the rate "x" --i.e., a proportion "x" of every individual's stock of capital will be confiscated. Assume, further, that the proceeds from this levy are thrown into the ocean. Analyze the consequences of such policy. Show your results analytically and in the c, k graphical space. (b) Consider the same question as in (a) above, but assuming now that the proceeds from the levy are returned to the public on a per head basis.

Question 3 (40 points)

The following two equations [1] summarize the simple Ramsey-Cass-Koopmans model of economic growth with variable labor --the basic foundation of Real Business Cycles models:

[1]

$$\frac{dc}{dt} = \left[\frac{U_c}{-U_{cc}}\right]\left(Af_k(k, \ell(c,k,A)) - (\delta + \rho)\right)$$

$$\frac{dk}{dt} = Af(k, \ell(c,k,A)) - k(\delta + n) - c$$

where c, k are consumption and the capital stock per capita, δ, n and ρ the constant positive rates of capital depreciation, population growth and time preference, $y = Af(k, \ell)$ the per capita production function, with $\ell = 1 - Leisure$ being the flow of per capita labor, which can be written as a function

[2] $$\ell = \ell(c, k.A) \qquad \ell_c < 0, \ \ell_k > 0, \ \ell_A > 0.$$

$U(c, \ell)$ is the instantaneous utility function.

(i) Elaborate on the meaning of expressions [1], including their dynamic and steady-state properties, and provide a graphical representation in the k, c space. In "elaborating on the meaning" of these two equations, be careful to explain where they come from (i.e., which are the basic building blocks), and to sketch the procedure through which those expressions, as well as equation [2] are derived.

(ii) In the context of this model, and starting from an initial steady state equilibrium, consider the effects, at an initial time $t = 0$, of the sudden anticipation that at a future date $t = \tau$ the rate of population growth, n, will permanently fall from an initial value n_0 to a lower level $n_1 < n_0$.

Part II

This part of exam has two (2) questions, and each question has different parts. **You must answer all the parts in all the questions.**

Question 1 (70 points)

Consider an infinitely-lived representative agent. In each period the agent is endowed with one unit of labor, which he/she supplies inelastically. Let c_t denote consumption of the single final good at time t. Then, agent's lifetime preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t),\ 0<\beta<1$$

Let K_t and L_t denote, respectively, the capital stock per firm at time t and the labor per firm at time t. The final good is produced using capital and labor as inputs into the constant return to scale aggregate production function given by $F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}$, where $0<\alpha<1$. Finally, assume that capital depreciates completely each period after production.

Let w_t be the real wage at time t, r_t be the rental rate of physical capital at t, p_t be the price level (in terms of fiat money per good) at time t, and let M_t be the time t nominal money supply. The money supply evolves according to

$$M_{t+1} = (1+\sigma)M_t, \forall t>0$$

where σ is the constant rate of money growth chosen exogenously by the government. The money the government creates is injected by giving each agent a lump-sum transfer of $\varepsilon_t = M_t - M_{t-1}$ at date t, in **nominal** terms.

Let m_{t-1} be the amount of money carried out from period $t-1$ (held at the beginning of period t, not inclusive of transfers) by the agent.

At each date, the agent is subject to the cash-in-advance constraint

$$p_t c_t \le m_{t-1} + \varepsilon_t$$

Let μ_t be the Lagrange multiplier associated with the time t cash-in-advance constraint, and let λ_t be the Lagrange multiplier associated with the agent's time t budget constraint at time t.

a) (4 points) Formulate carefully the agent's maximization problem (including the agent's budget constraints).
b) (8 points) Write down the first order conditions for the agent's maximization problem.
c) (17 points) Let $z_t \equiv M_t / p_t$ be the per capita supply of real balances. Assuming that the cash-in-advance constraint binds, **show step by step** that the equilibrium law of motion for the capital-labor ratio satisfies

$$\frac{f(k_t)-k_{t+1}}{f(k_{t-1})-k_t} = \beta f'(k_t) \qquad (1)$$

d) (3 points) Find and describe briefly all the steady-state equilibria for this economy. Explain.

e) (13 points) Note that equation (1) is a nonlinear Second Order Difference Equation. **Use the method of augmenting the state-space** to:

  e.1) (6 points) Derive the Jacobian matrix explicitly.

  e.2) (7 points) Evaluate the Jacobian matrix explicitly at the steady-state equilibrium. Find and show the dynamic properties of this equilibrium. Explain the characteristics of dynamic equilibria in a neighborhood of this steady-state equilibrium.

f) (15 points) Derive the Golden Rule allocation for this economy, and explain whether or not the competitive equilibria are Pareto Optimal? Does optimality depends on the CIA constraint binding? Explain carefully, using the relevant Lagrange multipliers from the agent's First Order Conditions. Explain the reasons that make households hold fiat money in this model.

g) (10 points) What can you say about monetary superneutrality in this model? How does the result in this model compare to other monetary growth models? Explain.

Question 2 (50 points)

Consider a 3-period model of N identical agents. Each agent is endowed with one unit of time (labor) in her second period of life (middle aged), and with y_1 and y_2 units of the single good in their first period of life (young) and second period of life (middle age), respectively. Agents have no endowments of any type in their last period of live (old).

Young agents can choose how much labor to supply (l). Each unit of labor pays the fixed real wage of w goods ***when young***.

When ***middle aged***, an agent may create physical capital: one unit of the good invested (not consumed) yields one unit of capital. Physical capital $\boldsymbol{k}$ pays the fixed real gross rate of return R in the ***third period of life***.

The utility of an agent is given by:

$$u(c_1,c_2,l) = \ln(c_1) + \beta \ln(c_2) + \beta^2 \ln(c_3) + \ln(1-l),$$

where c_1, c_2, c_3 and l denote, respectively, consumption when young, consumption when middle-aged, consumption when old and labor supplied.

At the beginning of the first period, before any choices are made by agents, the government announces that physical capital (not the return on physical capital) and the return on labor will be taxed at the rates $\boldsymbol{\delta}$ and $\boldsymbol{\tau}$, respectively, and agents take this into account to make their decisions. Notice that $0 < \delta \leq 1$ and $0 < \tau \leq 1$.

At the beginning of the second period the government chooses the tax rates τ and δ on the returns from labor and capital respectively. Notice that the maximum tax rate is 100%. The government wishes to leave its citizens with the highest possible utility, but needs to raise G goods worth of revenue. Notice that $G > Nw$, i.e.: G cannot be levied by only taxing the return on labor; ***capital must be taxed too.***

Taxes are collected at the end of the second period.

a) (10 points) Set up ***and solve explicitly*** the individual's problem (**forward solution**.) Explain.

b) (25 points) Suppose now that households use **backward induction**: agents make their choices starting from the last period, taking previous decisions as given, and they solve backwards.

  b.1) (7 points) Show that when households choose $\boldsymbol{k}$ at the beginning of the second period, taking $\boldsymbol{l}$ as given, this choice is elastic: There is an inverse relationship between $\boldsymbol{k}$ and $\boldsymbol{\delta}$ (so, the choice of capital is elastic but the choice of labor is inelastic.)

  b.2) (18 points) Show that when a benevolent government chooses $\boldsymbol{\tau}$ and $\boldsymbol{\delta}$ to maximize the households' utility, incorporating the reaction function for capital and taking labor constant, and subject to the government budget constraint, they will ***choose a corner solution for one of the tax rates*** . Set up and evaluate explicitly the Kuhn-Tucker conditions if necessary. Use the envelope theorem and look for corner solutions

c) (15 points) Define carefully the concept of **time inconsistency of economic policy**. Also, given the results you found in part (b), illustrate and explain CAREFULLY and IN DETAIL why there is (or there is not) a time consistency problem from the government's side in this economy.