Aug 05

Macroeconomics Qualifier Examination

August, 2005

Instructions

This exam has two parts. You must **answer all the questions in each part.**

- The weights are indicated as points in each case. The maximum possible score in each part is 120 points. The total possible number of points is 240, which corresponds to 240 minutes of available time (4 hours). **Allocate your time wisely, so that you maximize your total score.**
- Read the questions carefully and explain your reasoning briefly but carefully.
- Less than easily legible answers, as well as answers to questions other than those being asked, will be ignored.

Part I

Question 1 (65 points)

Consider the following R&D model of growth with an expanding variety of goods. There are three types of agents: households, R&D firms and final good producers. Households maximize the following lifetime utility:

$$U = \int_0^\infty \left(\frac{c^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} dt$$

subject to the budget constraint

$$\dot{a} = ra + wL - c$$

and supply labor inelastically.

Final good producers produce the consumption good with labor and intermediate goods. Each final output producer i has access to the following technology

$$Y_i = L_i^{1-\alpha} \sum_{j=1}^{N} x_{ij}^{\alpha}$$

where L_i is the amount of labor used by firm i, x_{ij} is the amount of intermediate good j used by

firm i, and N is the number of varieties of intermediate goods available.

R&D firms invent and produce intermediate goods over which they have a perpetual monopoly. R&D firms must first decide whether or not to invent a new good, and then the price p_j at which to sell it. Once invented, an intermediate good of type j costs one unit of output to produce, and the marginal cost of production is 1. Innovation requires a fixed quantity of labor L_R such that $L_R = L - L_Y$ where $L_Y = \sum_i L_i$. In addition, assume that the interest rate r is constant. There is free entry into the R&D market.

1. Set up and solve the representative household's optimization problem.
2. Set up and solve the optimization problem of the representative final good producer.
3. In order to solve the optimization problem of the R&D firms, answer the following questions.

   (a) Consider first the pricing decision. Set up and solve the optimization problem of the representative R&D firm j. Find the price p_j charged by each intermediate firm.

   (b) Now consider the decision to enter the R&D market. What are the flow profits π_j from intermediate good production? What is the free entry condition in this case?
4. Define an equilibrium for this economy. List all equilibrium conditions.
5. What is the growth rate of N, consumption and output in the steady state? Does this model generate endogenous growth? Why or why not? Explain the intuition.

Question 2 (40 points)

1. Provide a detailed explanation for the concept of Permanent Income Hypothesis. Illustrate your answer with a simple model of consumption-saving choice under uncertainty in partial equilibrium. For simplicity, you may assume log utility over consumption and that the discount rate is equal to the real interest rate.
2. Explain the concepts of excess sensitivity and excess smoothness of consumption. Are these at odds with the Permanent Income Hypothesis? Can you suggest modifications to the model you constructed above that would reconcile your model with the observations of excess sensitivity or excess smoothness?

Question 3 (15 points)

Can a Solow-type growth model, when augmented to include the possibility of human capital accumulation, adequately explain cross country differences in the level of output per capita? Explain and refer to the relevant literature.

Part II

Question 1 (50 points)

Consider an economy with N infinitely-lived identical agents. In each period the agent is endowed with one unit of labor, which he/she supplies inelastically. Let c_t denote the

consumption of the single final good by one agent at time t. The agent's lifetime preferences are given by

$$U = \sum_{t=0}^{\infty} \beta^t \ln c_t$$

The final good is produced using capital and labor as inputs into a constant return to scale production function. Let k_t denote the capital-labor ratio at t, and let $f(k_t)$ denote the intensive production function. The function f is increasing and strictly concave, and it satisfies the Inada conditions. Finally, assume that capital depreciates completely each period.

Let w_t be the real wage at time t, r_t be the rental rate of physical capital at t, p_t be the price level at time t, and let M_t be the money supply at the end of time t. The money supply evolves according to

$$M_{t+1} = (1 + \theta)M_t \text{ for } t > 0.$$

where $\theta > 0$ is the constant rate of money growth chosen exogenously by the government. The new money created by the government is injected by giving to each agent a lump-sum transfer of $\pi_t = M_t - M_{t-1}$ at date t, in ***nominal*** terms.

Let m_{t-1} be the amount of money carried out from $t - 1$ by an agent (held at the beginning of period t, not inclusive of transfers.)

At each date, the agent is subject to the cash-in-advance (CIA) constraint

$$p_t c_t \leq m_{t-1} + \pi_t$$

Let μ_t be the Lagrange multiplier associated with the time t cash-in-advance constraint, and let λ_t be the Lagrange multiplier associated with an agent's time t budget constraint.

1. Formulate the agent's maximization problem (including the agent's budget constraints). Derive and explain the first order conditions for the agent's maximization problem.
2. Formulate the representative firm's maximization problem. Derive and explain the first order conditions for the firms's maximization problem.
3. Let $z_t \equiv \frac{M_t}{N \cdot p_t}$ be the per capita supply of real balances. Assuming that the cash-in-advance constraint binds, ***show step by step*** that the equilibrium law of motion for the capital-labor ratio satisfies
$$\frac{f(k_t)-k_{t+1}}{f(k_{t-1})-k_t} = \beta f'(k_t) \qquad (1)$$
4. Equation (1) is a nonlinear Second Order Difference Equation. Derive, explain and solve for the dynamic properties of the steady-state equilibrium.
5. Is the resulting equilibrium Pareto Optimal? Does optimality depend on the CIA constraint binding? Explain carefully, using the relevant Lagrange multipliers and Kuhn-Tucker conditions. Explain the reasons that make agents hold fiat money in this model.
6. Define the Mundell-Tobin effect. What can you say about the Mundell-Tobin effect in this model? How do the theoretical results provided by this model relate to empirical observations on the relationship between long-run output and inflation? Describe carefully the empirical evidence. You must cite the results ***of at least two papers*** studied in class.

Question 2 (35 points)

Consider an economy consisting of an infinite sequence of two period lived, overlapping generations. There is a ***fixed*** stock of A acres of productive land. There are $N_0 > 0$ agents in

the initial old generation who each posses an equal amount of land. Population is constant. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. There is a single consumption good in the economy each period. Each agent is endowed with w units of the good when young and nothing when old. Each acre of land produces d units of the consumption good every period.

By tradition, land is always sold after the crop is harvested. There are no other assets in the economy.

The lifetime utility of an agent born at t is given by

$$u(c_{1,t}, c_{2,t+1}) = \ln c_{1,t} + \ln c_{2,t+1}$$

Let v_t denote the price of an acre of land in terms of the consumption good, and a_t denote the number of acres of land purchased by a young agent at t.

1. Set up and solve the problem of an individual born at t. Explain.
2. Derive and explain the market clearing condition for land. What is the equilibrium price of land? Is this equilibrium stationary?
3. Derive the equilibrium rate of return on land as a function of the equilibrium price of land.
4. Set up and solve explicitly the social planner's problem that derives the golden rule. Compare the competitive equilibrium in the long run to the golden rule. Explain.
5. Suppose that the government finances a lump-sum subsidy of σ goods to each of the young agents through a tax of τ goods per acre of land purchased, payable when young.
   (a) Find the effect of an increase in τ on the stationary equilibrium price level of land v. Explain.
   (b) What value of τ is consistent with the golden rule? Explain.

Question 3 (20 points)

(20 points) Consider a single-period economy in which a social planner must allocate consumption among T (an integer) identical agents in order to maximize their utility. The planner weights equally the utility of all agents. There exist Y units of a perfectly divisible good. There are also q units of a good that can be consumed only in integer amounts. The integer q is between T and 2T. The utility function of each agent i is given by:

$$u(y_i, z_i) = \ln y_i + \ln z_i ,$$

where y_i denotes the agent i's consumption of the divisible good, and z_i is her consumption of the indivisible good.

1. Set up and solve the planner's problem, and show, briefly but accurately, that maximizing the expected utility of the people DOES require an equal allocation of the divisible good. Explain also how the indivisible good is allocated. Explain this result.
2. Set up an optimal lottery that is an actuarially fair gamble and explicitly show that it replicates the allocation that you obtained in (1). Solve and explain.

Question 4 (15 points)

Explain in detail and carefully the Ricardian Equivalence. Does the Ricardian Equivalence always hold? Explain briefly but accurately.