Macroeconomics Qualifier Examination
August 2003

Instructions
Answer all questions.
Weights are indicated as points in each case. The total possible number of points is 240, which corresponds to 240 minutes of available time (4 hours). Allocate your time wisely.
Read the questions carefully and explain your reasoning. Less than easily legible answers, as well as answers to questions other than those being asked, will be ignored.

Part I

Question 1 (15 points) Growth empirics

Suppose we have cross country data on $\Delta Y/Y$, $\Delta K/K$ and $\Delta L/L$ over the period 1960-85 and we run an OLS regression on

$$(\Delta Y/Y)_i=\beta_0+\beta_1(\Delta K/K)_i+\beta_2(\Delta L/L)_i+\varepsilon_i$$

Typically, we find the estimated β_1 to be about 0.6 to 0.8. But capital share of income is about 1/3 when we use private marginal returns to capital to calculate it. Surely, there are many reasons (both theoretical and econometric) to explain such discrepancies. Could you provide <u>two</u> plausible reasons?

Question 2 (25 points) Consumption tax in a neoclassical model of economic growth

Consider a standard Ramsey-Cass-Koopmans model of economic growth with CRRA utility $u(c_t)=\frac{c_t^{1-\theta}-1}{1-\theta}$ and Cobb-Douglas production function $Y=K^{\alpha}(AL)^{1-\alpha}$. Consider the imposition of a constant proportional tax on consumption τ_c, the proceeds of which are returned to the consumers as lump-sum transfers. Show that the Euler equation and capital accumulation equation are unaffected by such a consumption tax.

Question 3 (35 points) A modified Fischer model

Consider a modified Fischer model with staggered 2-period labor contracts and non-indexed predetermined (but not fixed) wage as follows:

Nominal wage behavior : ${}_{t-i}w_t= E_{t-i}\, p_t \qquad i=1,2$
Aggregate Supply : $y_t^s=\varphi_{t-2}(p_t - {}_{t-2}w_t)+\varphi_{t-1}(p_t - {}_{t-1}w_t)+s_t$
Aggregate Demand : $y_t^d=m_t - p_t$

where ${}_{t-i}w_t$ is the nominal wage at time t for the group who sign the wage contract at the end of period t-i and E_{t-i} is expectation taking at the end of period t-i. In the original Fischer model, the fraction who sign the contract at each date is 1/2, so that $\varphi_{t-2} = \varphi_{t-1} = 1/2$. Here, consider instead that a fraction of contracts f was signed in every odd period and a faction (1-f) was signed in every even period. Derive y_t and p_t for odd and even periods respectively, in terms of the supply shock s_t and money supply m_t, as well as their expectations. Can a monetary policy feedback rule have any real effect on this economy? Explain intuitively.

Question 4 (45 points) A modified Lucas model

Consider an economy that consists of many markets (each indexed by z). The (log) supply in market z is given by

$$y_t(z) = \gamma\{p_t(z) - E[p_t \mid I_t(z)]\}$$

where $p_t(z)$ is the (log) price of good z at time t given by

$$p_t(z) = p_t + z_t$$

where p_t is the (log) general price level and z_t is (log) relative price shock due to relative demand shock. In the original Lucas model, supplier in market z could only observe its own current price $p_t(z)$ (and all past prices and quantities in all markets), but not p_t nor z_t nor $p_t(z')$ for every $z' \neq z$. That is, the info set at time t that supplier in market z has is given by $I_t(z) = \{\Omega_{t-1}, p_t(z)\}$ where Ω_{t-1} is info on all lagged prices and quantities in all markets. Now, consider instead that the supplier in market z could not only observe its own current price $p_t(z)$ but also the current price $p_t(z')$ in one other market (but still not p_t nor z_t nor z'_t nor $p_t(z'')$ $\forall z'' \neq z$ or z'.) That is, $I_t(z) = \{\Omega_{t-1}, p_t(z), p_t(z')\}$. Now, assume the general price level is given by

$$p_t = \sum_{j=0}^{\infty} v_j \varepsilon_{t-j}$$

where the v_j's are constants with $v_0=1$ and $\sum_{j=0}^{\infty} v_j^2 < \infty$. Assume the stochastic processes ε_t and z_t are serially independent processes, normally distributed with zero means and finite variances σ_ε^2 and σ_z^2 respectively. Also $Ez_s\varepsilon_t=0$ for every s and t.

a. (6 points) Express p_t in terms of $E[p_t|\Omega_{t-1}]$ and ε_t. Express $p_t(z)$ in terms of $E[p_t|\Omega_{t-1}]$ and ε_t and z_t. Show that $E[p_t(z)|\Omega_{t-1}] = E[p_t|\Omega_{t-1}]$.

b. (19 points) Now, $E[p_t|I_t(z)]=E[p_t|\Omega_{t-1},p_t(z),p_t(z')]$. Show that

$$E[p_t \mid I_t(z)] = E[p_t \mid \Omega_{t-1}] + E[\varepsilon_t \mid \varepsilon_t + z_t, \varepsilon_t + z_t']$$

Hence, show that

$$E[p_t \mid I_t(z)] = E[p_t \mid \Omega_{t-1}] + \alpha\{p_t(z) - E[p_t(z) \mid \Omega_{t-1}]\} + \alpha\{p_t(z') - E[p_t(z') \mid \Omega_{t-1}]\}$$

Find the value of α in terms of σ_ε^2 and σ_z^2. (Hint: you will need to apply the econometric tool that $E[y_t|x_{1t},x_{2t}]=\beta_1 x_{1t} + \beta_2 x_{2t}$ where β_1 and β_2 are regression coefficients by regressing y_t on x_{1t} and x_{2t} using past data, assuming that the unconditional mean of y_t is zero, so the constant term in the regression is zero. Note also that in multiple regression in matrix form given by $\mathbf{Y}=\mathbf{X}\boldsymbol{\beta}$, OLS estimate gives $\boldsymbol{\beta}_{\mathbf{ols}}=(\mathbf{X}'\mathbf{X})^{-1}\,\mathbf{X}'\mathbf{Y}$.)

c. (20 points) Derive a Phillips curve of the form

$$y_t = \phi\{p_t - E[p_t \mid \Omega_{t-1}]\}$$

where y_t is real GDP from aggregating $y_t(z)$ over all z. Give a formula for ϕ in terms of γ, σ_ε^2 and σ_z^2. Is your value for ϕ larger or smaller than the value that Lucas originally derived? Can you guess what would happen to the slope ϕ if agents in market z were permitted to see current prices in n markets as n becomes larger and larger? Explain intuitively.

Part II

Question 1 (46 points) An Overlapping Generations Economy with production, Social Security, and the Ricardian Equivalence

Consider a closed economy, with overlapping generations. Agents live for two periods. N_t young agents are born each period t. Population grows at the net rate n. Time is discrete.

Each agent is endowed with 1 unit of labor when young and nothing when old. There are no endowments of goods. A single good is produced each period, using the constant returns to scale technology $Y_t = K_t^{\alpha} L_t^{1-\alpha}, 0 < \alpha < 1$, i.e.: Y_t units of the final good are produced using K_t units of physical capital and L_t units of labor. Capital depreciates at the rate δ each period. The agents' preferences are given by

$$u(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$$

where c_1 denotes consumption when young and c_2 denotes consumption when old.

Agents supply labor inelastically at the real wage rate w_t, while r_t denotes the real rental rate on physical capital.

a. (4 points) Describe briefly and solve the agent's problem and the firm's problem.

b. (7 points) Derive the equilibrium law of motion for the capital-labor ratio. Find the steady state equilibria. Is the nontrivial steady state equilibrium locally stable? Explain.

c. (7 points) Set up and solve the planner's problem for this economy: the planner's objective function and the feasible set. Find the Golden Rule allocation for this economy.

d. (4 points) Find the conditions under which there capital over accumulation in the nontrivial steady state equilibrium.

e. (14 points) Pay-as-you-go Social Security: suppose now that the government collects a lump-sum tax of τ_{1t} goods from young agents born at t. The proceedings of this tax are used to finance a lump-sum transfer of τ_{2t} goods to old agents at t. Find the Government budget constraint. Find the equilibrium law of motion for the capital-labor ratio and find the nontrivial steady state equilibrium.

f. (10 points) Now suppose that the government decides to reduce τ_{1t} and finance this tax cut by selling bonds to young households that have to be repaid next period. Will the Ricardian Equivalence hold? If your answer is yes, explain why and under which circumstances. If your answer is no, still explain why. Do not forget to explain what the Ricardian equivalence is.

Question 2 (54 points) A Representative Agent Model with a Cash-in-Advance Constraint

Consider an infinitely-lived representative agent. In each period the agent is endowed with one unit of labor, which he/she supplies inelastically. Let c_t denote consumption of the single final good at time t. Then, agent's lifetime preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

The final good is produced using capital and labor as inputs into a constant return to scale production function. Let k_t denote the capital-labor ratio at t, and let $f(k_t)$ denote the intensive production function. Assume that f is increasing and strictly concave, and that it satisfies the Inada conditions. Finally, assume that capital depreciates completely each period.

Let w_t be the real wage at time t, r_t be the rental rate of physical capital at t, p_t be the price level at time t, and let M_t be the time t money supply. The money supply evolves according to

$$M_{t+1} = (1+\sigma)M_t, \forall t > 0$$

where σ is the constant rate of money growth chosen exogenously by the government. The money the government creates is injected by giving each agent a lump-sum transfer of $\theta_t = M_t - M_{t-1}$ at date t, in **nominal** terms.

Let m_{t-1} be the amount of money carried out of t-1 (held at the beginning of period t, not inclusive of transfers) by the agent.

At each date, the agent is subject to the cash-in-advance constraint

$$p_t c_t \leq m_{t-1} + \theta_t$$

Let μ_t be the Lagrange multiplier associated with the time t cash-in-advance constraint, and let λ_t be the Lagrange multiplier associated with the agent's time t budget constraint.

a. (4 points) Formulate the agent's maximization problem (including the agent's budget constraints).

b. (5 points) Write down the first order conditions for the agent's maximization problem.

c. (20 points) Let $z_t \equiv M_t / p_t$ be the per capita supply of real balances. Assuming that the cash-in-advance constraint binds, **show step by step** that the equilibrium law of motion for the capital-labor ratio satisfies

$$\frac{f(k_t)-k_{t+1}}{f(k_{t-1})-k_t} = \beta f'(k_t) \qquad (1)$$

Also, equation (1) is a nonlinear Second Order Difference Equation. Explain, but **do not solve**, how you would analyze the dynamic properties of the steady-state equilibrium.

d. (15 points) Is the resulting equilibrium Pareto Optimal? Does optimality depends on the CIA constraint binding? Explain carefully, using the relevant Lagrange multipliers. Explain the reasons that make households hold fiat money in this model.

e. (10 points) What can you say about the Mundell-Tobin effect in this model? How do the theoretical results provided by this model relate to empirical observations on the relationship between long-run output and inflation? Describe carefully the empirical evidence.

Question 3 (20 points) Speculative Attacks in Foreign Currency Markets

What is a speculative attack on a currency? Under which types of policies are speculative attacks more likely to be observed? Please, explain carefully and give an example of policies that could prevent these speculative attacks from happening.