Macroeconomic Theory III — Fall 2001

L.Auernheimer

Handout # 12

A Simple but rather Complete Model of the Small Open Economy

This handout discusses a simple model of the small open economy. Although the framework is rather complete, some assumptions allow to generate an engine of analysis that, despite being rather powerful, is quite simple (sometimes deceptively simple).

1.- **Assumptions and the basic structure**

1.1.- We consider a "small" economy: world commodity prices are constant, normalized at unity, and so is the world real rate of interest, $\boldsymbol{r}$. There are two commodities (or "bundles of commodities"): traded and home goods. The private sector produces fixed quantities of the traded and home goods ($\boldsymbol{x_T}$ and $\boldsymbol{x_H}$).

1.2.- Individuals consume the two commodities ($\boldsymbol{c_T}$ and $\boldsymbol{c_H}$) and hold net foreign assets ($\boldsymbol{a}$, denominated in terms of the world money), that yield the world real interest rate, and domestic money ($\boldsymbol{m}$,in real terms, as defined below). Domestic fiat money is held because it generates utility. Individuals live forever, and behave so as to maximize the present value of their utility.

1.3.- Government holds net foreign assets ($\boldsymbol{A}$, denominated in the same units as private foreign assets and also yielding the world interest rate), imposes *per head* taxes ($\boldsymbol{T}$, as expressed in terms of domestic goods), and spends at a rate of $\boldsymbol{g_H}$ units of the home good and $\boldsymbol{g_T}$ units of the traded good.

1.4.- There is perfect foresight, continuous costless arbitrage and no transport costs or trade taxes. Then, the domestic price of traded goods is the nominal exchange rate, i.e., $\boldsymbol{P_T = e}$, and the "real exchange rate" is defined as the ratio

[1] $$\varepsilon \equiv \frac{e}{P_H}$$

2.- **The budget constraints**

2.1.- The typical individual's budget constraint, expressed in terms of the traded good, is

[2]

where

$$\hat{e} \equiv \left(\frac{de}{dt}\right)\left(\frac{1}{e}\right)$$

is the "rate of devaluation", equal to the "rate of inflation of traded goods".

Notice that [2] is expressed in terms of traded goods, and that real cash balances, ***m***, are defined as

[3] $$m \equiv \frac{M}{e}.$$

2.1.- The government's [1] budget constraint is

[4] $$A\,r + \left(\frac{T}{\varepsilon}\right) + \left(\frac{dm}{dt}\right) + m\,\hat{e} \equiv g_T + \left(\frac{g_H}{\varepsilon}\right) + \left(\frac{dA}{dt}\right)$$

3.- **Solving the model**

3.1.- The typical individual, at any initial time ***t=0*** , maximizes

$$\int_0^\infty U(c,\ m)\ e^{-\rho t}\ dt$$

subject to its budget constraint [2]. Here, ρ is the rate of time preference, that we will assume to be equal to the world real interest rate[2] , and ***U( )*** is the usual utility function, which we will assume to be **separable** for money, so that the marginal utility of money is independent of the consumption of either of the two goods. This is an important assumption, that yields very plausible results and also simplifies a great deal. Maximization, and a few substitutions, yield the following marginal optimality conditions, that need to be fulfilled at all times:

[5] $$\frac{U_T(c_T\,,\,c_H)}{\varepsilon} = U_H(c_T\,,\,c_H)$$

[5'] $$\left(\frac{dU_T}{dt}\right) = 0$$

[5"] $$\frac{U_m(m)}{(r + \hat{e})} = U_T(c_T\,,c_H).$$

---

[1] Notice that here we are defining "government" as the central government *cum* the central bank.

[2] This is another common assumption. Its rationale is as follows: if the rest of the world is in equilibrium, then in the rest of the world the real interest rate is equal to the rate of time preference of individuals. If individuals in the small country are equal to those of the rest of the world, then the assumption follows.

Notice that [5] is the usual elementary condition that the ratio of marginal utility to prices needs to be the same for all consumed goods. [5'], in turn, is the direct result of access to a perfect capital market and the fact that the rate of time preference is equal to the real world interest rate. Finally, [5"] results from [5'] and the assumption of separability of money in the utility function, and has the same interpretation as [5]. This last expression can be appropriately called the "demand for money".

3.2.- The equilibrium condition in the market for home goods, which we assume is fulfilled at all times, is

[6] $$x_H = c_H + g_H.$$

For given constant levels of production and government consumption of the home good, then [6], together with [5'], imply that the private consumption of the traded good, $\boldsymbol{c_T}$, is also constant, or

[7] $$\frac{dc_T}{dt} = 0.$$ [3]

3.3.- The addition of the private sector's and the government's budget constraint yields the country's **balance of payments identity**, in which we define $\boldsymbol{w \equiv a + A}$, i.e., the country's net foreign assets:

[8] $$\frac{dw}{dt} \equiv wr + x_T - g_T - c_T.$$

Expressions [7] and [8], then, are sufficient to solve the "global" system. These expressions imply that, for a level of "total net foreign assets", say, ***w(0)***, given by past history, and in the absence of future parameter changes, the level of consumption of the traded good immediately adjusts so as to satisfy the condition

$$\frac{dw}{dt} = 0,$$

i.e., to the level $c_T(0) = w(0)r + x_T - g_T$.[4] This is indicated in the graph of Figure 1.

[3] Notice that this does not mean that the level of the traded good consumption cannot **discretely** change at some point in response to either contemporaneous or anticipated changes in policy or in other parameters.

[4] In more technical terms, the system exhibits a "saddle point" rather than a "saddle path".

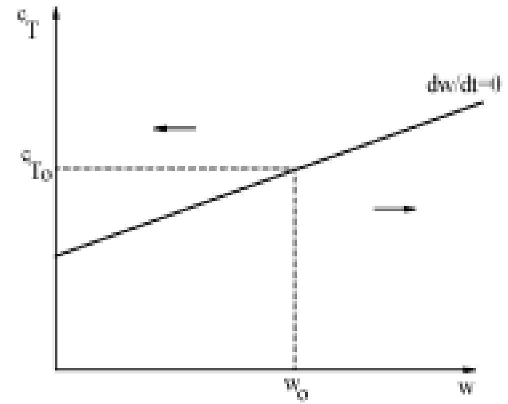

Figure 1

It should be pointed out, though, that fulfillment of the transversality condition in the aggregate system [7]-[8] is a necessary **but not a sufficient condition** for the same to happen in both the private and government sectors. Since $\boldsymbol{w} \equiv \boldsymbol{a} + \boldsymbol{A}$, it is easy to think of situations in which $\boldsymbol{dw/dt=0}$ but $\boldsymbol{da/dt \neq 0}$ and $\boldsymbol{dA/dt \neq 0}$.

4.- **Some properties of the model**

It is worthwhile to comment on some of the features of the model.

4.1.- Aggregation over all individuals, and the continuous fulfillment of the domestic market equilibrium condition [6] implies, for a given level of $\boldsymbol{x_H}$ and $\boldsymbol{g_H}$, a constant level of consumption of the home good, $\boldsymbol{c_H}$. This, together with some of the other simplifying assumptions, results in a "double dichotomy". Expressions [7] and [8] determine the level of $\boldsymbol{c_T}$, the consumption of traded goods. The system determined by these two expressions can be referred to as the "real external sector". Given this level of $\boldsymbol{c_T}$, and the fixed level of $\boldsymbol{c_H}$, expression [5] determines the real exchange rate, ε. The real exchange rate, then, should be properly seen as the relative price that "accommodates" so that agents are "persuaded" to consume the existing flow of home goods, for the choice they have made concerning their consumption of traded goods. Expression [5], then, can be referred to as the "real domestic sector". Finally, given these values, expression [5"] determines, together with the rate of devaluation, $\hat{e}$, the real money stock. Expression [5"], then, is the demand for money. Notice, incidentally, that until now we have said nothing about the "regime" chosen by the monetary authority. On this, see below, 4.3.

4.2.- Notice that, as suggested before, fulfillment of the overall transversality condition, for which $(dw/dt) \Rightarrow 0$ as $t \Rightarrow \infty$, is not sufficient for $(da/dt) \Rightarrow 0$ and $(dA/dt) \Rightarrow 0$ as $t \Rightarrow \infty$. Part of the "exercises" to be analyzed with this model have to do with the conditions under which both the private and the government sectors will fulfill their transversality conditions.

4.3.- The determination of $\boldsymbol{c_T}$ and $\boldsymbol{c_H}$ is not sufficient for the determination of the real money stock via [5"].

Here, we need to specify the regime selected by the central bank. The first possibility is that the monetary authority follows an ***exchange rate rule***, i.e., the exogenous determination of the path of the exchange rate. A particular case of interest is when such exogenous preannounced path involves a ***constant devaluation rate***.[5] In this case, of course, the exogenous rate of devaluation completes the system. The second possibility is for the monetary authority to follow a ***monetary rule***, i.e., to determine an exogenous path of the nominal money supply; again, here, a particular form of this rule is the setting of a constant proportional rate of nominal monetary growth, or rate of monetary expansion μ. In this case, of course, the path of the nominal exchange rate is endogenous, and the solution for the monetary sector requires the use of the identity

[9] $$\frac{dm}{dt} \equiv m\mu - m\hat{e}.$$

5.- **The Consequences of Relaxing some Assumptions**

There are three assumptions that play an important role in the simplicity of the results. Here, we comment on the restrictiveness of those assumptions, and the results of relaxing them.

5.1.- *Separability*

We have assumed that the utility function is separable in money (i.e., the marginal utility of money does not depend on the levels of consumption, and *vice versa*). This assumption is less restrictive than it may appear, and the result is still a demand for money that depends on the levels of consumption of the two commodities. A more general form of the utility function would eliminate the "dichotomy" between the real international sector and the monetary sector, and greatly complicate the solution of the model, and take account of cross effects that, in general, would be of the second order of smalls.

5.2.- *The "deflator" for money*

We have defined the real money stock entering into the utility function as nominal money over the nominal exchange rate (or price of traded goods). We have assumed, then, that the "real money" agents care about is in terms of the traded good. The result of this definition is that the "monetary sector", or demand for money, does not depend on the real exchange rate. Two other alternatives would have been (i) to use as the deflator the domestic money price of home goods, or (ii) to use a general price index of the form

---

[5] I.e., a path of the form

$$e(t) = e(0)\exp(\gamma t),$$

where $\gamma = \hat{e}$ is the rate of devaluation. Then, the particular case $\gamma = \hat{e} = 0$ is the traditional "fixed exchange rate" system.

$$\tilde{P} = e^{\gamma} P_H^{(1-\gamma)}$$

where γ is the proportion of expenditures in traded goods out of total expenditures.

The complication, **which is not substantial**, is that in both these cases the monetary sector depends also on the real exchange rate. The latter is the most general (and strictly the most correct) way to proceed.

5.3.- *Constant production of commodities*

We have assumed that the production of both traded and home goods is constant. A relaxation of this assumption (still within the framework of a "constant level of resources", or "no growth" model) would be to assume a standard transformation function linking the production of the two. In this case, it would be appropriate to write

$$x_T = x_T(\varepsilon), \; x_H = x_H(\varepsilon).$$

The complication in this procedure is that the dichotomy between the international and the domestic real sectors is broken. In evaluating the possible relaxation of this assumption, of course, one needs to consider the problem at hand. If the model is to be used for discussing short and medium-term phenomena, one should remember that movements along the transformation curve are not costless, and tend to be slow.

Appendix

1.- Consider the Cobb-Douglas form of the separable utility function,

[A.1] $$U(c_T, c_H, m) = c_T^{\alpha}\, c_H^{\beta} + V(m) \qquad (\alpha + \beta) < 1,$$

which conforms to the separability assumption. In this case, expression [5] reduces to the simple form

[A.2] $$c_T\, \varepsilon = \left(\frac{\alpha}{\beta}\right) c_H$$

Replacing the equilibrium condition [6], yields

[A.3] $$c_T\, \varepsilon = \left(\frac{\alpha}{\beta}\right) (x_H - g_H),$$

which makes very clear the inverse relationship between the level of government expenditures in home goods and the real exchange rate --a relationship which is found in most empirical studies. It also shows the negative association between consumption of traded goods (and hence, the trade deficit) and the real exchange rate, for a given level of $\boldsymbol{x_H}$ and $\boldsymbol{g_H}$. This association is very often described as implying causality going from the real exchange rate to the balance of trade; such a description, that unfortunately seems to confirm the naive partial equilibrium analysis of the "exchange rate market", is wrong. If there is causality is precisely in the opposite direction: from the level of $\boldsymbol{c_T}$ (and hence the balance of trade) to the real exchange rate. The mistake in identifying the correct causality goes beyond an analytical mistake: it is at the root of the furious efforts by most countries to "generate a high real exchange rate", a policy that more often than not is useless, wrong and detrimental.