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"Trade versus Migration, and the Role of Diversity:

A Simple Analytical Framework"

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Qui se ressemble, s'assemble

Introduction

One of the pillars of classical trade theory is the general proposition that trade in commodities is a substitute (a perfect substitute, under certain conditions) for factor movements --the so-called "factor price equalization theorem" being associated with it.[1] Given the simple assumptions of the traditional trade model, of course, there are infinite combinations of trade and factor movements which are equally "efficient" and equivalent from a welfare point of view. The purpose of this paper is to explore an extremely simple framework in which this equivalence does not hold, and in which factor movements (labor movements, more specifically) responds not only to real wage differentials, but also to what we call "diversity", for lack of a better name. Assuming that the world is populated by workers of different types, such diversity is measured by the proportion of individuals of a particular type residing in a particular country *vis-à-vis* the rest of that country's population. More specifically, we will assume that utility of individual workers of a certain type *i* will depend on consumption and such diversity coefficient, which can range from *0* (when no individual of type *i* resides in the country in question) to *1* (when all other individuals are of type *i*). This can be written as

$$U_{ij}(c, z_{ij}) \qquad U_c > 0,\ U_{cc} < 0,\ U_z > 0,\ U_{zz} < 0,\ U_{cz} > 0$$

where *c* is consumption, *i* is the individual worker's "type", *j* is the country where the individual worker resides, and

$$z_{ij} = \ell_{ij} / (\ell_j),$$

[1] One of the most elegant exposition, among many others, is probably the one in Mundell [1957]

where ℓ_{ij} is the number of workers of type *i* residing in country *j*, and ℓ_j is total labor in country *j*.

Notice that we have assumed that utility depends positively on the coefficient *z*, i.e., workers have a taste for "homogeneity" (i.e., a high *z*), rather than for "diversity".[2]

What is the rationale for such measure, what does it intend to reflect, and which is the motivation for including it in the utility function? There are two sets of possible justifications for the use of the term. The first one is the idea that different "types" generate externalities of a "cultural", social interaction nature –"atmosphere" or "cultural environment" could be appropriate words--including habits, language, religious beliefs, even institutions. These are of the nature of "public goods", in the sense that they are produced simply by the "presence" of individuals of a given type. A second possible rationale is that the term can be an imperfect but simple manner in which "network effects" can be captured. There is of course a vast literature on networks, and several empirical pieces intended to measure network effects on population location,[3] but barely any literature on a formal specification of the precise mechanism and its integration in the decision process leading to agent's decisions on where to locate. Being this the case, the inclusion of the argument in the utility function can be justified as a procedure which would summarize the benefits of networks and yield reasonable predictions.[4] Notice that "language", alluded before as a "cultural" element, could also be an important component of networking.

There is, of course, a profuse volume of literature related to some of the topics that explicitly or implicitly are touched in this paper: migration, integration, networks, and the treatment of social interaction. We will not try to survey or even refer to this literature, except for

[2] Strictly, the term "diversity" is not a good one, neither is "homogeneity". Both are too general, and are usually associated with idiosyncratic individual attributes --tastes, behavior towards risk, expectations formation and beliefs. See, for example, Fernandez and Levy [2005] for an example where the term "diversity" is used with a very different connotation than here.

[3] See, for example, Munshi, K. [2003] and references therein.

[4] This is somehow similar to the justification for introducing real money in the utility function: while not solving the problem of defining money and the reasons why it is held in rigorous terms, it yields a highly plausible reduced form for the demand for money.

mentioning that we have not found a simple framework as the one we are attempting to sketch.[5] A notable exception and the work that probably has the most in common with this paper is Schelling [1969], who analyzes the question of integration/discrimination in a context in which two different "types" (black and white) are characterized by a given "level of tolerance" to integration ratios (our *z* coefficients).

An important clarification is in order. In the analysis that follows labor is assumed to be mobile, and capital immobile. For purposes of simplicity, we will be assuming that output is the same single good everywhere, so that "trade" (in commodities) does not take place in any non-trivial sense. Within this simplified framework, we will take the case of perfect capital mobility as a proxy for the "trade" equilibrium solution --an assumption that would be valid for as long as the necessary conditions for factor price equalization are met.

Although our analysis could be generalized to "n" countries and individual types, in what follows we consider the simple two-country model, with two types of individuals. The analysis will not be symmetric: after the initial general presentation, we will elaborate only on the case of unidirectional or "one way" migration, i.e., on the case in which only one of the two types of labor is mobile (type *a*), with individuals of type *b* remain in country *2* throughout.

A Simple 2- Country Model

Consider the case of two countries, *1* and *2*, with fixed endowments of capital k_1 and k_2, respectively. There are two continuous of workers, each in fixed quantities, one of type *a*, with mass a^{o}, and another of type *b*, with mass b^{o}, which are assumed to be initially located in countries *1* and *2,* respectively.

The two different types of agents are defined by their preferences, which are assumed to be

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[5] To mention a few pieces, see, for example, Manski [2000], Kreps [1997], Lindbeck [1997] on social interaction, Munshi [2003] on networking, Boeri and Brücker [2005] on coordination failure and Bolt and Permentier [2004] on integration.

[1] $U_{ij}(c, z_{ij}) \quad i = a, b;\ j = 1, 2 \qquad U_c > 0,\ U_{cc} < 0,\ U_z > 0,\ U_{zz} < 0,\ \ U_{cz} > 0$

where c is consumption, and

[2.1] $$z_{aj} = a_j / (a_j + b_j)$$

[2.2] $$z_{bj} = b_j / (a_j + b_j).$$

Each country is endowed with a fixed stock of capital, k_j, which is immobile, while labor can freely migrate. Assume also that there is a single good being produced by either of the two countries, according to the same constant returns to scale production technology

[3] $$y_j = F(k_j, \ell_j)$$

where y_j is output in country j, and $\ell_j = a_j + b_j$ is total labor in each of the two countries. Throughout this work we will be concerned only with the welfare (utility) of the single representative worker of each type, and we will refer to those workers as "populations". Owners of capital "have no soul" --their utility depends only on their consumption (wages of capital), equal to whatever happens to be the marginal product of capital.

Labor wages in each of the two countries are given, of course, by the marginal product of labor, i.e.,

[4.1] $$w_1 = \partial y_1 / \partial \ell_1 = F_{\ell_1}(k_1/\ell_1)$$

[4.2] $$w_2 = \partial y_2 / \partial \ell_2 = F_{\ell_2}(k_2/\ell_2)$$

which, given the constant returns to scale assumption, depends only on the capital-labor ratios.

We further assume, as customary in the traditional international trade model, that output is instantaneously perishable and hence non-storable. Then, there are no savings, and consumption equals the real wage. Utility functions for type *a* and type *b* agents, located in countries *1* and *2*, then, can be written as [6]

[5.1] $$U_{a1} = U(w_1, z_{a1})$$

[5.2] $$U_{a2} = U(w_2, z_{a2})$$

[5.3] $$U_{b1} = U(w_1, z_{b1})$$

[5.4] $$U_{b2} = U(w_2, z_{b2})$$

Long Run Equilibrium

Labor mobility will ***allow the possibility*** of a long-run equilibrium at which utility for each type will be the same in the two countries, i.e.,

$$U_{a1}(w_1, z_{a1}) = U_{a2}(w_2, z_{a2})$$
$$U_{b1}(w_1, z_{b1}) = U_{b2}(w_2, z_{b2})$$

We call this an "interior solution", which may exist for either one or both types, or for none of the two. More specifically, a "full interior solution", for which both equations are satisfied, will be associated with the case where $a^o > a_2 > 0$ and $b^o > b_1 > 0$, while "partial interior solutions" (when only one of the equations is satisfied) with the case in which only one of these inequalities holds. Obviously, the case in which none of the two inequalities holds will be associated with $a_2 = b_1 = 0$ (autarky).

Replacing real wages by their values as given by [4.1] and [4.2], yields

[6.1] $$U_{a1}(F_{\ell_1}(k_1/\ell_1), z_{a1}) = U_{a2}(F_{\ell_2}(k_2/\ell_2), z_{a2})$$

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[6] Since we are dealing with only 2 countries and 2 types, in what follows we write all relevant expressions in a detailed rather than a compact form.

[6.2] $$U_{b1}(F_{\ell_1}(k_1/\ell_1), z_{b1}) = U_{b2}(F_{\ell_2}(k_2/\ell_2), z_{b2})$$

These two expressions contain the six variables: $\ell_1, \ell_2, z_{a1}, z_{a2}, z_{b1}, z_{b2}$. By construction, $z_{b1} = 1 - z_{a1}$ and $z_{b2} = 1 - z_{a2}$. It is also easy to show that $z_{a2} = (a - \ell_1 z_{a1})/(\ell - \ell_1)$.. Then, by appropriate substitutions, the two expressions [6-1]-[6.2] can be reduced to functions of the two variables ℓ_1 and z_{a1}. The system is "solvable", although this does not guarantee a unique solution, or even a solution at all. Since the attainment of an equilibrium at which these expressions are satisfied will in general require movement of labor starting from autarky, it is clear that it would be easy to specify basic parameters (such as the capital stocks and total labor of the two types), and/or production and preference functional forms, for which labor has no initial incentive to move in either direction.

Interesting cases, though, are those in which there is a gain from migration. Although, despite its non-linearities, the system is amenable to an analytical solution, at this stage we chose to specify some particular functional forms, a simpler procedure that will allow us to generate some preliminary results.

Consider the following particular forms for the production and utility functions. Assume the Cobb-Douglas production function

$$y = k^{\alpha} \ell^{(1-\alpha)}$$

and the utility functions for type *a* and type *b* workers

$$U_{aj} = c^{\gamma_a} z_{aj}^{\delta_a}$$
$$U_{bj} = c^{\gamma_b} z_{bj}^{\delta_b}.$$

Labor wages in countries *1* and *2* are, then, [4.1.1] $$w_1 = \partial y/\partial \ell_1 = (1-\alpha)\ (k_1/\ell_1)^{\alpha}$$

[4.2.1] $$w_2 = \partial y/\partial \ell_2 = (1-\alpha)\ (k_2/\ell_2)^{\alpha}$$

Then, equating consumption to wages, equilibrium requires

$$w_1^{\gamma_a}\, z_{a1}^{\delta_a} = w_2^{\gamma_a}\, z_{a2}^{\delta_a}$$
$$w_1^{\gamma_b}\, z_{b1}^{\delta_a} = w_2^{\gamma_b}\, z_{b2}^{\delta_a}$$

Substitution for the level of wages given in [4.1.1] and [4.2.1], yields

[6.1.1] $$((1-\alpha)\ (k_1/\ell_1)^{\alpha})^{\gamma_a}\, z_{a1}^{\delta_a} = ((1-\alpha)\ (k_2/\ell_2)^{\alpha})^{\gamma_a}\, z_{a2}^{\delta_a}$$

[6.2.1] $$((1-\alpha)\ (k_1/\ell_1)^{\alpha})^{\gamma_b}\, z_{b1}^{\delta_b} = ((1-\alpha)\ (k_2/\ell_2)^{\alpha})^{\gamma_b}\, z_{b2}^{\delta_b}$$

For the same reasons explained before, this system can be expressed in the two variables ℓ_1 and z_{a1}.

A special (but reasonable) case is when preferences of type *a* and type *b* agents are of an identical functional form, i.e., when $\gamma_a = \gamma_b = \gamma$ and $\delta_a = \delta_b = \delta$. In this case, it's straightforward to show that, if a full interior solution exists, it will satisfy

$$a_1/a_2 = a_2/b_2 = a^o/b^o$$

what also implies

$$z_{a1} = z_{a2}$$
$$z_{b1} = z_{b2}$$
$$w_1 = w_2$$

Since labor wages depend only on the capital/labor ratio and therefore those ratios will be the same in both countries (and equal to the overall ratio) this implies that, in this case, wages will be the same as in the "trade" or perfect mobility of capital solution, but utility will be lower for both types, since all $z_{ij} < 1$. Notice that this is a "stable" equilibrium, in which labor would have no incentive to move, but neither would capital, if capital mobility were allowed starting from this equilibrium.

The basic conclusion is that migration, even when a full interior solution is attained, is welfare inferior to the capital mobility ("trade" solution). This is almost an obvious conclusion once we introduce the *z* factor in the worker's preferences.

The Simplest Case of One-way Migration

A particular and simpler instance of the 2-country case results if we assume that only one of the two types of workers (say, type *a*) is mobile, while workers of the other type (*b*) remain at their initial location (say, country *2*).

In this case,

$$b_1 = z_{b1} = 0, \quad b_2 = b^o, \quad z_{a1} = 1, \quad z_{a2} = a_2/(a_2 + b^o)$$

Utility of the type *a* individual worker is therefore given by

[5.1.1] $$U_{a1} = U_a(w_1, 1)$$

if the individual resides at country *1*, and by expression [5.2] if residing at country *2*, with expression [5.4] describing utility of the immobile type *b* workers at country *2*.

With immobility of type *b* workers, wages in countries *1* and *2* can be written as a function of the number of type *a* workers in each of the two countries,

[7.1] $$w_1 = w_1(a_1)$$

[7.2] $$w_2 = w_2(a_2 + b^o)$$

Substitution of these last two expressions into [5.1.1] and [5.2] yields

[8.1] $$U_{a1} = U_a(w_1(a^o - a_2), 1)$$

[8.2] $$U_{a2} = U_a(w_2(a_2 + b^o), a_2/(a_2 + b^o))$$

which are functions of a_2 .

Long Run Equilibrium

In the long run, unidirectional movement of type *a* workers may (or may not) results in

[9] $$U_{a1} = U_{a2} ,$$

i.e., an equilibrium at which utility of type *a* is the same in both countries --the "interior solution" we defined before, now restricted, of course, to the case of type *a* workers. If we use the same functional forms for the production and utility functions that we specified in the general 2-country case, expressions [8.1]-[8.2] become

[8.1'] $$U_{a1} = (1-\alpha)\ (k_1/(a^o - a_2))^{\alpha\gamma_a}$$

[8.2'] $$U_{a2} = (1-\alpha)\ (k_2/(b^o + a_2))^{\alpha\gamma_a}\ (a_2/(b^o + a_2))^{\delta_a}$$

It is easy to verify that under certain conditions

$$\partial U_{a1}/\partial a_2 > 0,\ \partial^2 U_{a1}/\partial^2 a_2 > 0$$
$$\partial U_{a2}/\partial a_2 > 0,\ \partial^2 U_{a2}/\partial^2 a_2 < 0$$

i.e., U_{a1} is convex and U_{a2} is concave. In economic terms, as a_2 increases (and consequently a_1

decreases), the real wage in country 1 rises at an increasing rate, while the corresponding z coefficient remains constant at unit, so that utility of the representative worker remaining in country 1 increases at an increase rate --despite decreasing marginal utility, for "reasonable" parameter values. As a_2 increases, wages fall in country 2, but at a decreasing rate, while the coefficient z_{a1} increases at a decreasing rate. For U_{a2} to be rising as a_2 increases all what is needed is a sufficiently high coefficient δ_a, i.e., for the "diversity" coefficient z_{a1} to be "sufficiently important" --a condition that we assume obtains.

Notice also that utility of the immobile workers of type *b* decreases as a_2 increases, since wages in country *2* fall, and so does the z_b coefficient.

The equilibrium condition [9] results in

[9.1] $$(k_1/(a^o - a_2))^{\alpha\gamma_a} = (k_2/(b^o + a_2))^{\alpha\gamma_a} (a_2/(b^o + a_2))^{\delta_a}$$

Examination of [9.1] reveals that there are four possible interesting outcomes, depending on the exact form of the production and preference functions and the magnitude of the various parameters involved:

(i) $U_{a1} > U_{a2}$ for all and any a_2, and no equilibrium exists at which [9.1] is satisfied;

(ii) $U_{a1} > U_{a2}$ for all except one value of a_2, at which $U_{a1} = U_{a2}$, and a unique equilibrium exists at which [9.1] is satisfied ;

(iii) $U_{a1} < U_{a2}$ for any $a_2 < a_2*$ and $U_{a1} > U_{a2}$ for any $a_2 > a_2*$, with $U_{a1} = U_{a2}$ for $a_2 = a_2*$, and a unique equilibrium exists at which [9.1] is satisfied;

(iv) $U_{a1} = U_{a2}$ for $a_2 = a_2*$ and for $a_2 = a_2** > a*$, with $U_{a1} > U_{a2}$ for $a_2 < a_2*$ and for $a_2 > a_2**$, and $U_{a1} < U_{a2}$ for $a_2* < a_2 < a_2**$; in this case two equilibria exist.

The graphs in Figures 1 to 4 depict the possible configurations of the left and right hand sides of expression [9.1], corresponding to these four possible outcomes. These graphs measure the terms U_{a1} and U_{a2} as functions of a_2.

Consider now each of the four possible outcomes. In the first case (i), depicted in Figure 1, there is no equilibrium satisfying [9,1], simply because at no level of migration (i.e., a_2) the gains in real wages from migrating are sufficient to compensate for the lower utility resulting from a lower z_{a2} coefficient. This is clearly a case in which the result may be due exclusively to the impact of "diversity".

The second case, shown in Figure 2, reflects the (unlikely) case in which there is a unique value of a_2 for which [9,1] obtains (point A in Figure 2), but $U_{a1} > U_{a2}$ for all other values. Notice the stability properties of this case: if $a_2 > a_2^*$ then type *a* workers in country *2* would have an incentive to return to country *1*, so that the equilibrium is locally stable ; if $a_2 < a_2^*$, the same incentive would operate, so that, to the left, the equilibrium is unstable.

The third case, depicted in Figure 3, is one in which also there is a unique value a_2^* for which equilibrium [9,1] obtains, but with all $a_2 < a_2^*$ yielding $U_{a1} < U_{a2}$, and with all $a_2 > a_2^*$ yielding $U_{a1} > U_{a2}$. This is clearly a case in which the wage differential dominates, and migration always takes place. Notice also that the unique equilibrium (point A in Figure 3) is stable.

The most interesting case is case (iv), depicted in Figure 4. There are two values of a_2 at which equilibrium [9,1] obtains (points A and B in Figure 4). For easiness of reference, call a_2^A and a_2^B the values corresponding to these points A and B, respectively. For values $a_2 < a_2^A$, $U_{a1} > U_{a2}$, and there will be no incentive for type *a* workers to migrate. In fact, at any point in this range any type *a* worker located at country *2* would return to country *1*. The equilibrium at A is "unstable" on its left-hand-side. In the range $a_2^A < a_2 < a_2^B$, $U_{a1} < U_{a2}$, and there will be an incentive for type *a* workers to migrate to country *2*. Equilibrium at point A is also "unstable" on its right-hand-side. Finally, for values $a_2 > a_2^B$, $U_{a1} > U_{a2}$ and, once again, type *a* workers, if residing in country 2, will have an incentive to return to country *1*. Notice that therefore, equilibrium B will be "stable".

There are a couple of comments to be made concerning the last case. The first, and obvious, is that in an initial position at which $a_2 = 0$, there will be no incentive for any type *a* worker to migrate, as the gains in real wages are not sufficient to compensate for the fall in utility generated by the lower homogeneity coefficient z_{a2} –in fact, $z_{a2} = 0$ for an individual worker when $a_2 = 0$. Notice that this happens despite the fact that, from the viewpoint of type *a* workers, the equilibrium at $a_2 = a_2{}^A$ is clearly welfare superior than at .$a_2 = 0$ The "coordination problem" can only be resolved with a minimum initial migration $a_2 < a_2{}^A$. The second comment is with respect to the characteristics of the "stable" equilibrium at point B, with $a_2 = a_2{}^B$. Notice that, in this case, further migration (i.e., an increase in a_2) would benefit both those who have already migrated to country *2* and those remaining in country *1*; yet, no single type *a* worker will have an incentive to migrate.

Also with reference to case (iv), it is interesting to note what happens to workers of type *b* in country *2*. Figure 5 reproduces Figure 4, with the addition of the behavior of the utility level of those workers. Obviously, as the level of type *a* workers residing in country *2* increases, type *b*'s workers utility decreases for two reasons: the z_{b2} decreases and so does the capital/labor ratio and hence labor wages. Figure 5 also shows what is labeled as "Utility with 'Trade' ", as a horizontal line. This is the level of utility for workers of type *b* that would obtain under perfect mobility of capital --which, as mentioned before, we take as a proxy for "trade".in commodities. Note that in the graph of Figure 5 the levels for $a_2 = 0$ are those that obtain in "autarky", i.e., before any migration takes place. It is clear that type *b* workers are better off in autarky than under trade --they would oppose trade, and in fact they would prefer "some" migration to the trade solution. As migration proceeds and the number of type *a* workers in country *2* increases, their utility will fall to the free trade level, and still beyond, so that after that point they will prefer free trade to migration. Notice how, despite its simplicity, the model suggests some propositions that are both interesting and testable.

Concluding Remarks

We have presented a "minimalist", very simple model that hopes to provide with an initial framework for the analysis of migration as being influenced not only by wage differentials but also by social interaction factors. At the same time, despite its simplicity, the framework has a few testable implications. One should also note that immigration issues related to the social interactions that we attempted to model are pervasive, and appear in such diverse contexts as within Latin America, Latin America *vis-à-vis* the USA, as well as within the European Union -- witness the recent "no" vote in France and the Netherlands.

At the theoretical level, a more detailed analysis of the full 2-country model (with by-directional rather than uni-directional labor mobility) needs to be put in place, perhaps with a more complete specification in which two commodities exist so that the trade solution can be analyzed, even at the risk of some computational complications.

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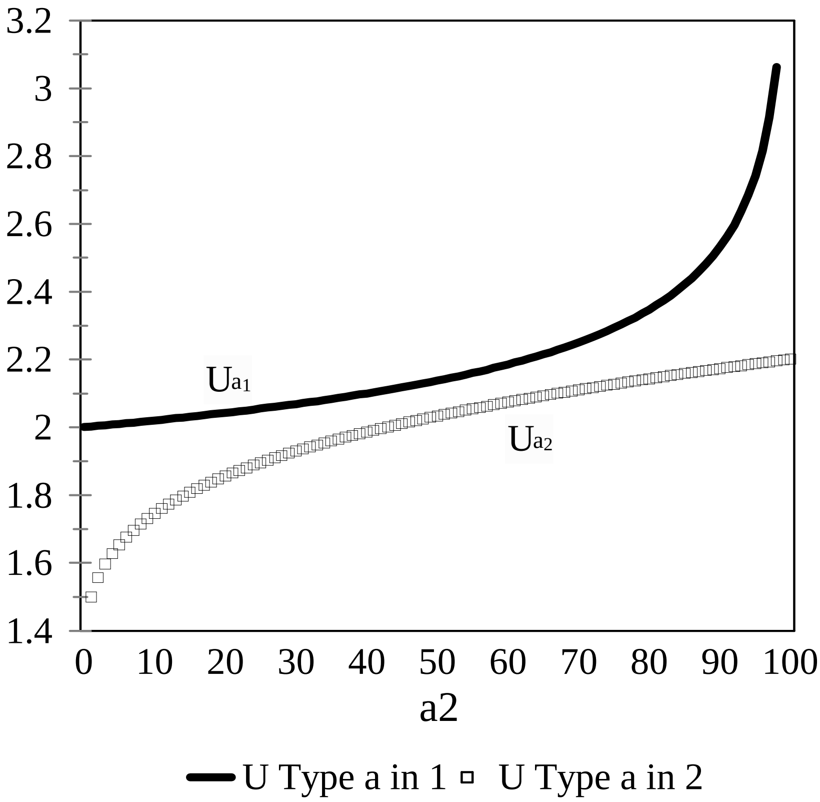
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Figure 1

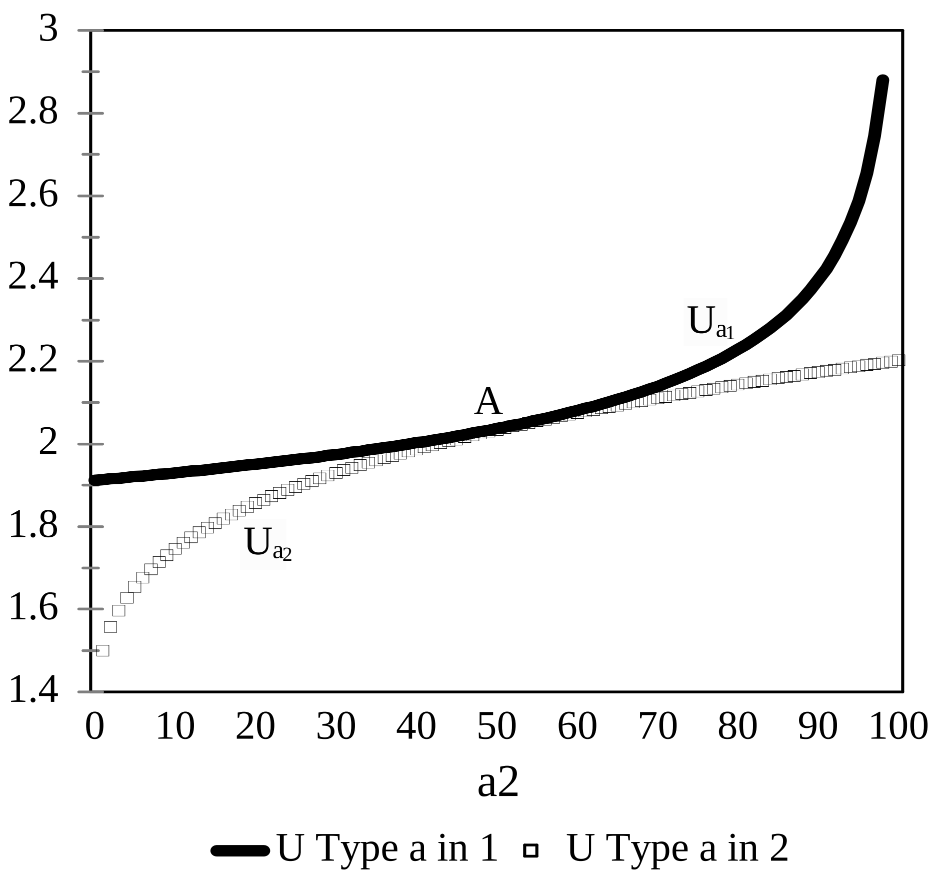
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Figure 2

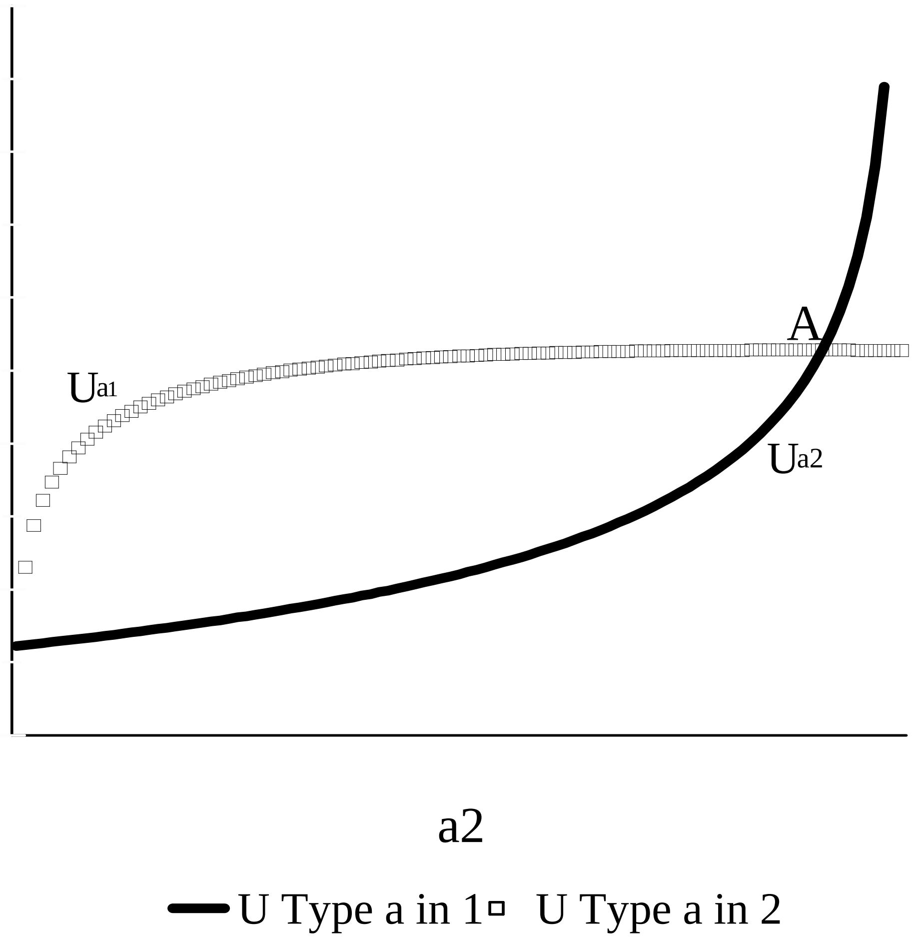

Figure 3

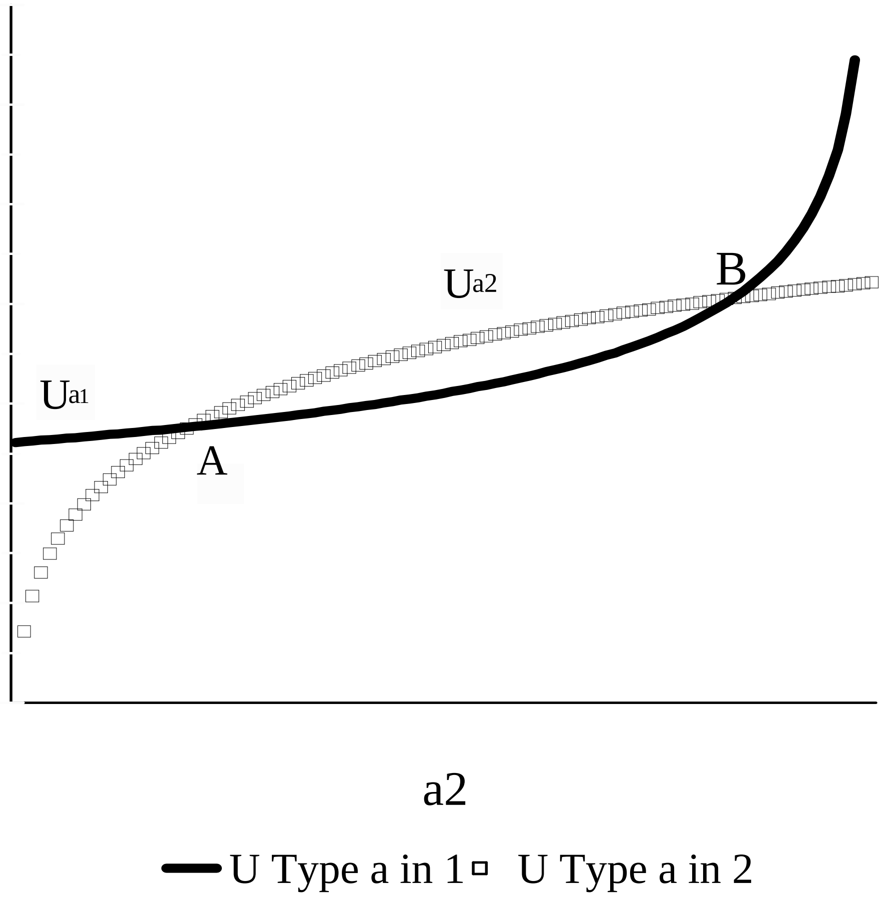

Figure 4

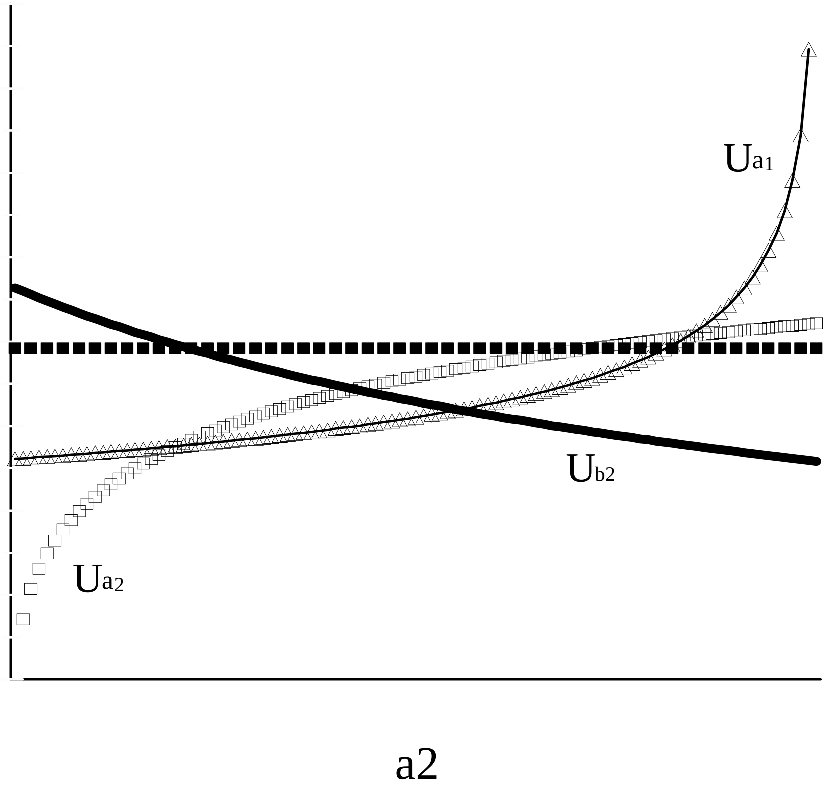

Figure 5