

Price and Wage Dispersion Due to Heterogeneous Arrival of Trade Opportunities

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Abstract

In a market, agents with a richer set of opportunities to trade should be able to demand better terms of trade. For instance, workers who have more extensive social networks may be able to demand and receive higher wages than equally-qualified peers who are less well-connected, since they may hear about more job openings. When a worker hears about openings more often, the dynamic opportunity cost of foregoing a current opportunity for employment is smaller. We formalize these insights in a model of dynamic search and matching and market microstructure following Satterthwaite and Shneyrov, in which unemployed workers are matched with open jobs in discrete time, and negotiation between a firm and a set of workers is modeled as a first-price auction. We show that when workers can be ordered by a one-dimensional parameter which summarizes their frequency of hearing about job openings, which we interpret as a proxy for network connectedness, equilibrium wages, and therefore present discounted utility, is monotonically increasing in the level of connectedness. We investigate numerically the quantitative implications of the model, with a focus on the long-run value of having an extensive network as a function of the underlying economic primitives of the model.

1 Introduction

Many markets are characterized by frictions that arise from the lack of information about trading opportunities. Buyers and sellers do not know about each others ex-

istence.¹ It is possible that market participants differ in their access to information about trading opportunities. For example, it is well established that social connections that provide information about job openings play an important role in the job search process (see REES [14], GRANOVETTER [8], BEWLEY [1], and PELLIZARRI [12]). A number of recent papers have investigated the effect of social referrals on labor market outcomes (see for example CALVO-ARMENGOL AND JACKSON [4, 5]). CALVO-ARMENGOL AND ZENOU [6], IOANNIDES AND SOETEVEENT [9], FONTAINE [7], and MAYER [10] introduce job search through social networks into a labor market matching framework. In these papers it is assumed that job-seekers and firms are matched through a process that depends on the social connections of the job-seekers. Once a match is realized wages are determined through Nash bargaining. Better connected workers find jobs quicker and are unemployed for shorter periods of time. Consequently, they enter the wage bargaining process with higher outside options, which results in higher wages. In equilibrium every match between a job-seeker and vacancy leads to a filled vacancy.

In this paper we analyze a different bargaining process. We show that under certain conditions better connectedness can be associated with longer unemployment spells. We propose a framework where workers compete for job openings. In our matching technology, which is adapted from the market microstructure model of SATTERTHWAITTE AND SHNEYROV [16], in each discrete period zero, one, or more workers may be matched with each open job opportunity. The likelihood of a currently-unemployed worker hearing about some job opportunity in a given period is given by the worker's exogenously-given level of connectedness, which is a one-dimensional parameter with a specified distribution in the population. In contrast to approaches such as that of CALVO-ARMENGOL AND JACKSON [4], we abstract away from specific details of network structure in favor of the one-dimensional summary parameter.

Workers matched with a job opportunity compete in a first-price sealed-bid auction, with the worker demanding the lowest wage receiving the job and earning the per-period wage he demands. In formulating bids, workers do not know how many other workers have also been matched with the same job opening, so our bargaining technology does not require restrictive informational assumptions on the worker's side; the worker only needs to have correct beliefs regarding the steady-state of the market. Our representation of network connectness via a one-dimensional parameter allows us to characterize and compute steady-state equilibria of our model without resorting to ad-hoc behavioral rules or simulation approaches. The connectness parameter serves to differentiate workers based on their dynamic opportunity cost

¹These search or matching frictions have been studied in a vast and growing literature that mainly focuses on the labor market, see ROGERSON ET AL [15] for a survey.

implied by the expected waiting time until they next hear about a job opening. Workers therefore set their wage bids strategically, and, therefore, well-connected workers who hear about many job opportunities are able to be patient and demand higher wages. Under some conditions, this can lead to equilibria in which the length of unemployment is not monotonically related to network connectedness, as well-connected workers strategically choose to remain unemployed. Despite this, being better connected is unequivocally better for a worker in our model, as the longer unemployment times are more than compensated for by high wages while employed.

The paper is organized as follows. Section 2 presents our modification of the Satterthwaite and Sherstyom framework, adapted to allow matching frequencies to differ across agents, and characterizes steady-state equilibrium properties. Section 3 illustrates the equilibrium properties for selected parameter values, showing quantitatively how the wage-demand curve, unemployment time, and long-run utility depend on the thickness of the market and the distribution of connectness parameters among workers. Section 4 concludes with a discussion of possible empirical tests of the model, and of other interpretations of the model.

2 The model

The model is founded on the market microstructure model of SATTERTHWAITE AND SHNEYROV [16]. In that model, buyers are matched to sellers on a many-to-one basis, with sellers, who have idiosyncratic costs, conducting a first-price auction with reservation value. Our model simplifies the setup somewhat, in that firms, who will be the analogs of sellers, have a commonly-known maximum wage they are willing to pay, and conduct the first-price auction with that wage as the reservation value. Workers are the analogs of buyers. Workers are all identical in terms of productivity and preferences, except workers vary in the frequency with which they are matched; in the Satterthwaite and Shneyrov model, buyers are matched in every period.

In another simplification relative to the original model, in our setting there is no entry and exit. The same set of firms and workers remain in the market. Each period there is an exogenously-given probability that an existing worker-firm match will dissolve, resulting in the worker becoming unemployed, and the firm re-listing the position. Therefore, the steady-state conditions of our model are slightly different than those determined by entry and exit in Satterthwaite and Shneyrov.

2.1 Preliminaries

The model operates in discrete time $t = \{\dots, -2, -1, 0, 1, 2, \dots\}$. There is a continuum of workers with mass W . Workers are distinguished by a “connectedness” parameter c , which is distributed among the population of workers according to the cdf G with density g bounded away from zero. The support of g is $[c_L, c_H]$, with $c_L > 0$ and $\int_{c_L}^{c_H} dG(c) = W$. In each period t , a worker may be either employed or unemployed. Write g_U for the density of unemployed workers, and g_E for the density of employed workers, with $g_U(c) + g_E(c) = g(c)$ for all $c \in [c_L, c_H]$. For notational convenience, the mass of employed workers is $E = \int_{c \in [c_L, c_H]} dG_E(c)$ and the mass of unemployed workers is $U = \int_{c \in [c_L, c_H]} dG_U(c)$.

There is a total measure of J jobs available. Therefore, the number of employed workers $E \leq J$, and the number of positions available will be denoted $P \equiv J - E$. At the beginning of each period t , there is a probability λ that each existing match of worker to job dissolves. Job loss is independent of a worker’s connectedness parameter.²The connectedness parameter c determines the probability a worker will hear about a job opportunity in the period. This probability is realized independently across workers. Therefore, for any level of connectedness c , the measure of workers with connectedness less than c who hear about a job opportunity is

$$A(c) = \int_{\kappa \leq c} \kappa \cdot dG_U(\kappa). \quad (1)$$

Note that $A'(c) = cg_U(c)$.

Workers who hear about job opportunities are equally likely to hear about any job opportunity. Formally, the matching proceeds as follows. Suppose there is a measure P of jobs available, and a measure $A(c_H)$ of workers who hear about jobs, and define $\zeta = \frac{A(c_H)}{P}$ to be the workers-to-jobs ratio. Then, random matching of workers to jobs implies that the number of workers k who hear about a given job is distributed Poisson with mean ζ ,

$$\pi_k = \frac{\zeta^k}{k!e\zeta}. \quad (2)$$

Furthermore, since whether a given worker hears about a job is independent of any other worker hearing about the job, for any given job, the number of workers with parameter $c \in I_1$ and the number of workers with parameter $c \in I_2$ who hear about

²Because we are considering steady-state equilibria, and a worker’s wage is determined by his bidding optimally with respect to his dynamic opportunity cost, a worker who is currently employed will not, in equilibrium, want to leave his job.

the job are independent, and also distributed Poisson, for any intervals I_1 and I_2 . In particular, the number of workers with connectedness less than c to hear about a particular job opportunity is Poisson with mean $\frac{A(c)}{P}$.

A worker who hears about a job opportunity does not know the number of other workers who also hear about the same opportunity, nor the connectedness parameters of those workers. She does know the distribution of connectedness parameters among the population in the steady state. Bargaining between firm and workers takes the form of a first-price auction, in which the worker who submits the lowest wage bid gets the job, at the wage he submits. All jobs and firms are identical, and the maximum salary a firm is willing to pay is normalized to one.

2.2 Wage demands are monotonic

Let $\rho(w)$ be the probability, in steady state, that a worker who makes a wage demand of w gets the job, conditional on being matched with a job opportunity. Then, the discounted ultimate probability of getting a job $P(c, w)$ if a worker of connectedness c makes a wage demand w is defined recursively as

$$P(c, w) = c\rho(w) + (1 - c\rho(w))\delta P(c, w).$$

Therefore

$$\begin{aligned} [1 - \delta + \delta c\rho(w)]P(c, w) &= c\rho(w) \\ P(c, w) &= \frac{c\rho(w)}{1 - \delta + \delta c\rho(w)}. \end{aligned}$$

Note that

$$\frac{\partial P}{\partial c} = \frac{1 - \delta}{(1 - \delta + \delta c\rho(w))^2}.$$

The discounted expected utility $V(c, w)$ of a worker of connectedness c who makes a wage demand of w is then

$$V(c, w) = c\rho(w)Mw + (1 - c\rho(w))\delta V(c, w)$$

and so

$$V(c, w) = P(c, w)Mw$$

where M is the expected length of the job. Let $w(c)$ denote the optimal wage demand as a function of c . Then, the interim utility for a worker of connectedness c is

$$V(c) = \sup_w P(c, w)Mw = P(c, w(c))Mw(c)$$

and by the Envelope Theorem

$$V'(c) = \frac{\partial P}{\partial c}(c, w(c))Mw(c).$$

Proposition. The wage demand function $w(c)$ is nondecreasing in c .

The proof is by contradiction. Suppose that $c_1 > c_2$ but $w(c_1) < w(c_2)$. Let $w_i = w(c_i)$, $i = 1, 2$. Since w_i are optimal for their respective c_i ,

$$\begin{aligned} P(c_1, w_1)Mw_1 &\geq P(c_1, w_2)Mw_2 \\ P(c_2, w_2)Mw_2 &\geq P(c_2, w_1)Mw_1. \end{aligned}$$

Then, we have

$$P(c_1, w_1)w_1 \geq P(c_1, w_2)w_2 = \frac{P(c_1, w_2)}{P(c_2, w_2)}P(c_2, w_2)w_2 \geq \frac{P(c_1, w_2)}{P(c_2, w_2)}P(c_2, w_1)w_1,$$

which implies that

$$\frac{P(c_1, w_1)}{P(c_2, w_1)} \geq \frac{P(c_1, w_2)}{P(c_2, w_2)}. \quad (3)$$

Note that

$$\frac{P(c_1, w_i)}{P(c_2, w_i)} = \frac{\frac{c_1 \rho(w_i)}{1 - \delta + \delta c_1 \rho(w_i)}}{\frac{c_2 \rho(w_i)}{1 - \delta + \delta c_2 \rho(w_i)}} = \frac{c_1}{c_2} \cdot \frac{1 - \delta + \delta c_2 \rho(w_i)}{1 - \delta + \delta c_1 \rho(w_i)}.$$

So (3) is equivalent to

$$\begin{aligned} \frac{1 - \delta + \delta c_2 \rho(w_1)}{1 - \delta + \delta c_1 \rho(w_1)} &\geq \frac{1 - \delta + \delta c_2 \rho(w_2)}{1 - \delta + \delta c_1 \rho(w_2)} \\ 1 - \frac{1 - \delta + \delta c_2 \rho(w_1)}{1 - \delta + \delta c_1 \rho(w_1)} &\leq 1 - \frac{1 - \delta + \delta c_2 \rho(w_2)}{1 - \delta + \delta c_1 \rho(w_2)} \\ \frac{\delta(c_1 - c_2)\rho(w_1)}{1 - \delta + \delta c_1 \rho(w_1)} &\leq \frac{\delta(c_1 - c_2)\rho(w_2)}{1 - \delta + \delta c_1 \rho(w_2)} \\ \frac{1 - \delta + \delta c_1 \rho(w_1)}{\rho(w_1)} &\geq \frac{1 - \delta + \delta c_1 \rho(w_2)}{\rho(w_2)} \\ \frac{1 - \delta}{\rho(w_1)} &\geq \frac{1 - \delta}{\rho(w_2)} \\ \rho(w_1) &\leq \rho(w_2). \end{aligned}$$

But, since $w_2 > w_1, \rho(w_2) < \rho(w_1)$, which completes the contradiction. **End of proof.**

2.3 Steady-state distributions of workers and jobs

The monotonicity of wage demands implies that the least-connected worker who hears about a job is the one to get it. Therefore, the probability $\eta(c)$ that a worker of type c gets a job in a given period is

$$\eta(c) = c \cdot \Pr(\text{no worker less connected hears about this job}) \quad (4)$$

$$= c \cdot \exp\left[-\frac{A(c)}{P}\right] \quad (5)$$

So, we can write the conditions required for a distribution g_U and an unfilled measure of jobs P to form a steady-state equilibrium. For any $c \in [c_L, c_H]$, the density of workers of type c who get jobs must be equal to the density of workers of type c whose jobs disappear:

$$g_U(c)\eta(c) = \lambda g_E(c) \quad (6)$$

$$c g_U(c) \exp\left[-\frac{A(c)}{P}\right] = \lambda(g(c) - g_U(c)) \quad (7)$$

$$c A'(c) \exp\left[-\frac{A(c)}{P}\right] = \lambda(c g(c) - A'(c)). \quad (8)$$

In the last step, we convert the differential equation into one in $A(c)$, the measure of workers with connectedness less than c who hear about a job opportunity. We will solve this differential equation, and then recover the density of unemployed workers via the identity $g_U(c) = \frac{A'(c)}{c}$. Rearranging this into the standard form for a first-order differential equation,

$$A'(c) = \frac{\lambda c g(c)}{c \exp\left[-\frac{A(c)}{P}\right] + \lambda} \quad (9)$$

The boundary condition for the differential equation is $A(c_L) = 0$, because we assume $G(c_L) = 0$.

Therefore, the distribution of unemployed workers $g_U(c)$ and the measure P of positions available in steady state are not yet determined. To nail this down, we also need to have that the total flow of workers in and out of jobs must be equal. We assume that hiring and termination decisions are simultaneous, in that workers leaving their positions do not participate in the search process, and firms do not attempt to fill those positions until the next period.

We entertain the possibility that no worker hears about some jobs. In this case, that job remains vacant and available in subsequent periods (“the position will remain

open until filled"). Steady-state occurs when the number of positions filled exactly equals the turnover rate:

$$(1 - \pi_0) \cdot P = \lambda E \quad (10)$$

$$\left(1 - \exp\left[-\frac{A(c_H)}{P}\right]\right) \cdot P = \lambda E \quad (11)$$

$$\exp\left[-\frac{A(c_H)}{P}\right] = 1 - \frac{\lambda E}{P} \quad (12)$$

$$A(c_H) = -P \ln\left[1 - \frac{\lambda(J - P)}{P}\right] \quad (13)$$

This last equation directly relates the total mass of workers hearing about jobs in a period, $A(c_H)$, to the total number of unfilled positions P . This therefore serves as an upper boundary condition on the differential equation for $A(c)$, which makes this a shooting problem. The solution method is thus to search over P for a value such that the upper boundary condition is satisfied.

2.4 Equilibrium bidding strategies

We can now turn to the characterization of the wage demand function. The first-order necessary condition for w to be the optimal wage demand for c is

$$\begin{aligned} \frac{\partial P}{\partial w}(c, w)w + P(c, w) &= 0 \\ \frac{(1 - \delta)c\rho'(w)}{(1 - \delta + \delta c\rho(w))}w + \frac{c\rho(w)}{(1 - \delta + \delta c\rho(w))} &= 0 \\ (1 - \delta)\rho'(w)w + (1 - \delta + \delta c\rho(w))\rho(w) &= 0. \end{aligned}$$

Let $\tau(w)$ be the inverse wage demand function. Since the wage demand function is increasing, so is its inverse, and so we can write, based on earlier calculations,

$$\begin{aligned} \rho(w) &= \exp\left[-\frac{A(\tau(w))}{P}\right] \\ \rho'(w) &= -\frac{A'(\tau(w))\tau'(w)}{P}\exp\left[-\frac{A(\tau(w))}{P}\right]. \end{aligned}$$

Combining this with the first order condition,

$$-(1 - \delta)w\frac{A'(\tau(w))\tau'(w)}{P}\exp\left[-\frac{A(\tau(w))}{P}\right] + \left(1 - \delta + \delta c\exp\left[-\frac{A(\tau(w))}{P}\right]\right)\exp\left[-\frac{A(\tau(w))}{P}\right] = 0.$$

$$(1 - \delta)wA'(\tau(w))\tau'(w) = P \left(1 - \delta + \delta c \exp \left[-\frac{A(\tau(w))}{P} \right] \right).$$

$$\tau'(w) = \frac{P \left(1 - \delta + \delta \tau(w) \exp \left[-\frac{A(\tau(w))}{P} \right] \right)}{wA'(\tau(w))(1 - \delta)}.$$

The boundary condition on the differential equation is determined by observing that a worker of type c_H only gets a job if she is the only unemployed worker to hear about the job. Therefore, in equilibrium, $w(c_H) = 1$, the maximum willingness to pay of the firm, and so $\tau(1) = c_H$.

3 Numerical examples

We explore properties of the steady-state equilibria as a function of parameter values. In all cases, the distribution $g(\cdot)$ is uniform on the interval of connectedness parameters, the mass of workers W is set to 1, and the discount factor is $\delta = 0.99$.

3.1 Baseline case

As a baseline case, consider $[c_L, c_H] = [0.1, 0.4]$, with $J = 0.99$, $\lambda = 0.01$. The equilibrium functions are plotted in Figure 1. In this case, the number of jobs available is close to the number of workers, and job turnover is slow. Therefore, equilibrium unemployment $U \approx 0.049$ and the stock of unfilled positions $P \approx 0.038$ are low, and the probability that a well-connected unemployed worker is the only one to hear about a particular job opportunity is high. Therefore, the probability of getting a job in a period is an increasing function of c , and unemployed workers tend to be less connected. The wage-demand function is close to linear.

Figure 2 depicts an intermediate case of natural unemployment, where the mass of jobs available is $J = 0.95$. The mass of job openings in steady state decreases to $P = 0.138$, and the mass of unemployed workers increases to $U = 0.064$. Therefore, it becomes less likely for a well-connected worker to be the only one to hear about a position, so the probability of getting a job in a period initially increases in c , then decreases in c , with a corresponding U-shaped distribution of unemployed workers: both well-connected and ill-connected workers tend to be unemployed, while those in the middle are least likely to be unemployed. Nevertheless, the wage-demand function is still increasing in c , as is the value function; well-connected workers make up for longer stints out of work by higher wages when working.

Moving on to a higher natural unemployment case, Figure 3 shows the equilibrium characteristics when the number of jobs available is $J = 0.90$. In equilibrium, the

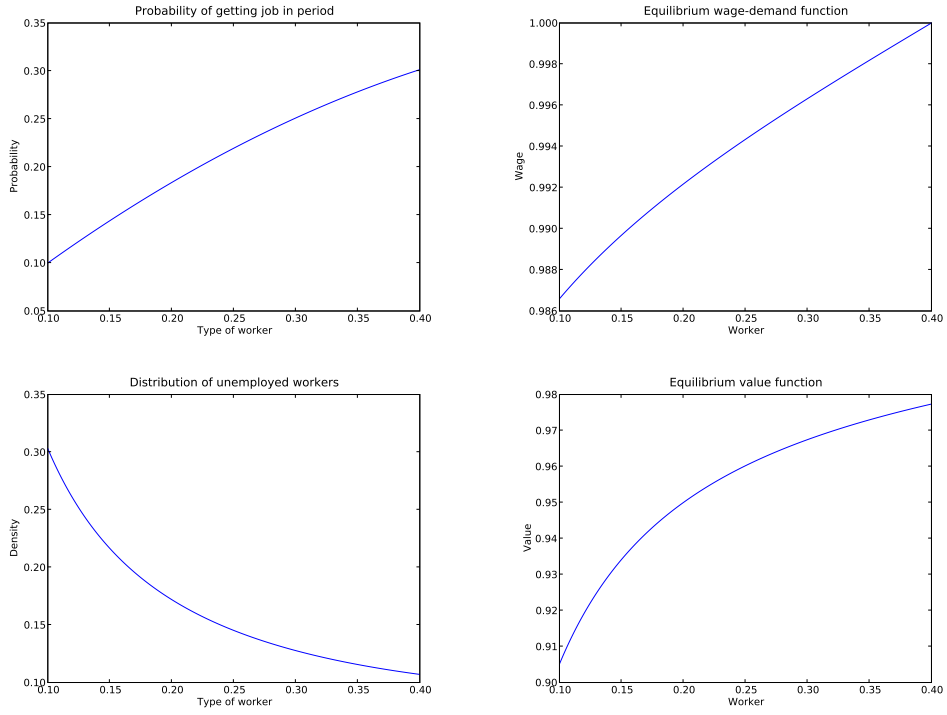


Figure 1: Steady state equilibrium functions for baseline case.

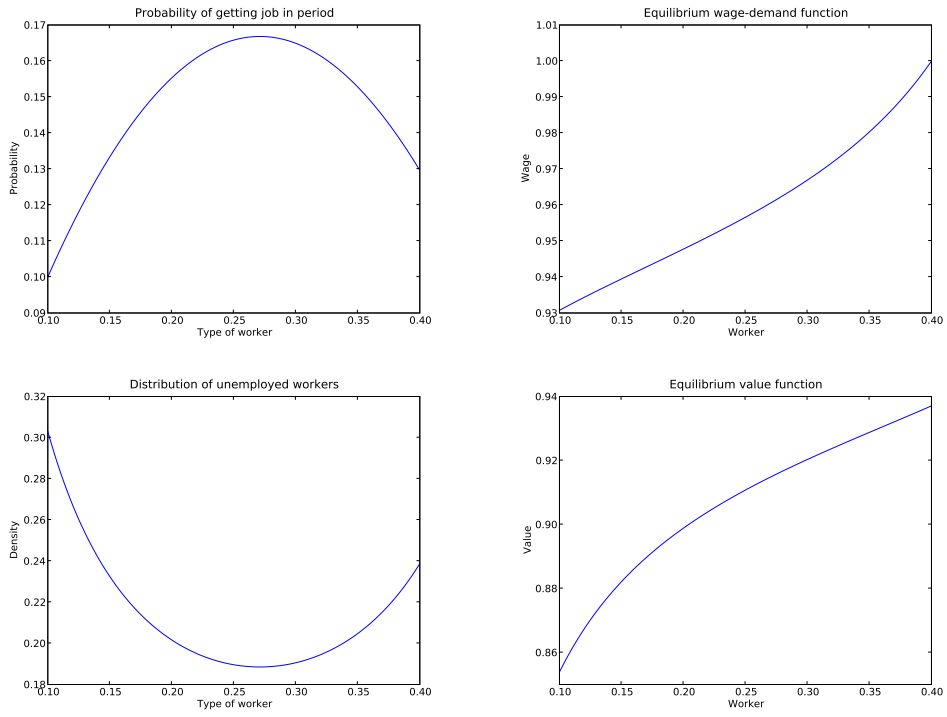


Figure 2: Steady state equilibrium functions for medium natural unemployment case.

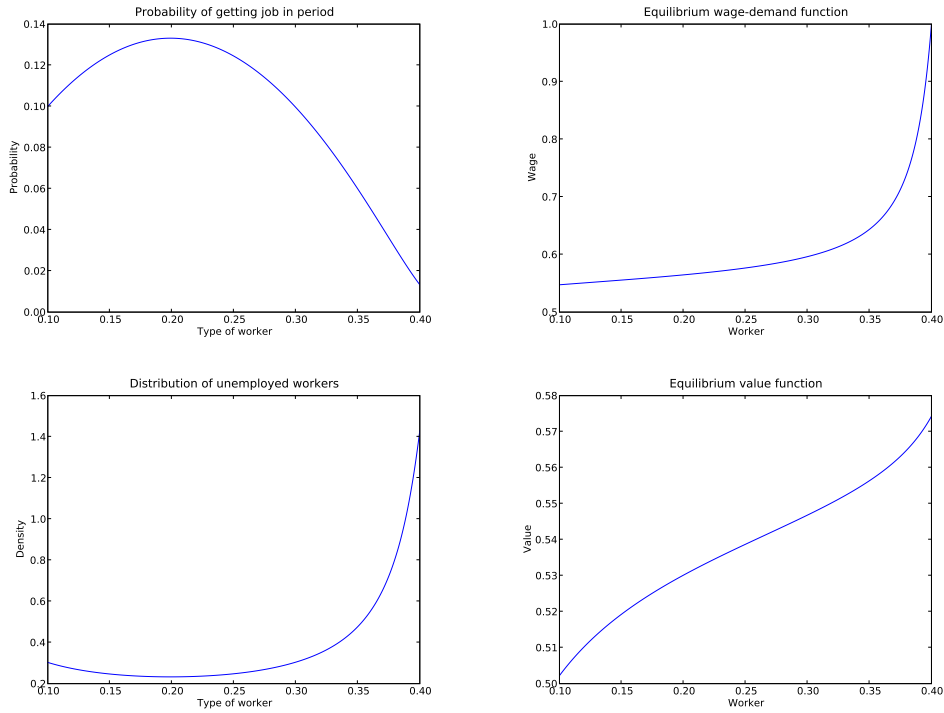


Figure 3: Steady state equilibrium functions for high natural unemployment case.

mass of unemployed workers is $U \approx 0.109$ and the number of open jobs is $P \approx 0.009$. Therefore, well-connected workers go a long time before obtaining a position, but, when they do obtain one, they do so at significantly higher wages than their less-connected counterparts.

The results for the high natural unemployment case depend on the relatively small turnover rate. Increasing the turnover rate to $\lambda = 0.05$ while maintaining the other parameters of the high unemployment case gives equilibrium values displayed in Figure 4. In steady state, the mass of open jobs is $P = 0.110$ and the mass of unemployed workers is $U = 0.210$. With the smaller workers to jobs ratio, well-connected workers do not go long before being the only worker to hear about a job.

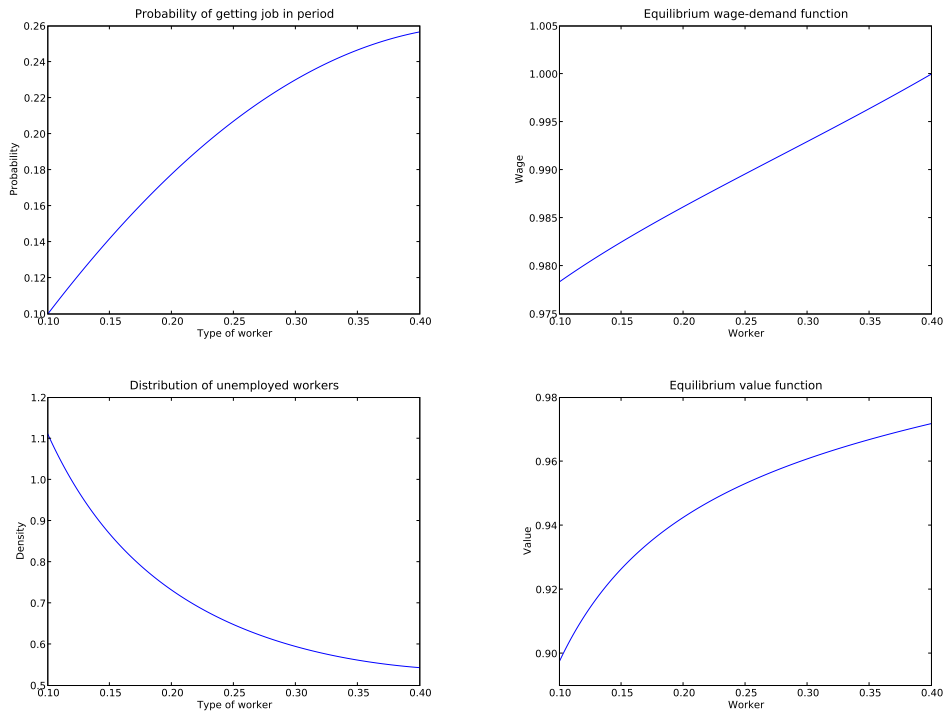


Figure 4: Steady state equilibrium functions for high natural unemployment, high turnover case.

4 Conclusion

Based on SATTERTHWAITE AND SHNEYROV [16] we develop a dynamic search and matching model that allows for heterogeneous arrival rates of trade opportunities. We phrase our model in terms of a labor market where better connected workers are more likely to hear about job-openings. We show that wages are increasing in connectedness. Depending on the parameterization of the model unemployment duration can be increasing or decreasing in connectedness.

The fundamental challenge of any empirical analysis of the role of information is that information flows (or connectedness) is usually not observed. However, the framework presented here provides testable implications. In the low unemployment case wages and unemployment duration are negatively associated. In the high unemployment case wages and unemployment duration are positively associated. Matching on an individual basis and subsequent wage determination through Nash bargaining always predicts a negative association between unemployment duration and wages.

We have focused so far on investigating the quantitative effects on wages and long-run welfare that better social networks can provide. We have set market participation costs to zero and ignored entry and exit possibilities for workers. With zero participation costs, workers would not want to exit. We also assume that firms have a degenerate distribution of reservation wages which is commonly known. This could be generalized within the Satterthwaite-Sherstyom model, which does contemplate private information on both sides of the market, but adding firm-specific reservation wages would not change the qualitative conclusions. Because we investigate steady-state equilibria, workers who are currently matched with a job would not be made better off by voluntarily terminating their employment and returning to the market to search again; however, we do rule out the possibility of on-the-job search in the current model.

We have motivated the modification of the Satterthwaite-Shneyrov model as a labor market, in which the connectedness parameter is interpreted as the effectiveness of a worker's social network in funneling news of job opportunities to the worker. The theoretical contribution is to show that the connectedness parameter as we have defined it functions to order agents in equilibrium by their dynamic opportunity cost. Other interpretations of the model are possible. For instance, in a real estate market, the analog of job openings would be available properties, and workers would map into potential buyers.

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