Quantifying the Effects of Job Matching through Social Networks

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The recent literature has helped us to understand the theoretical implications of the matching of workers to jobs through social networks. However, these insights are obtained for extremely simplified economies or they are based on strong simplifying assumptions about the structure of social networks. Therefore, it is difficult to obtain a sense for the quantitative importance of the effects generated by real life social networks. In this paper, I augment a labor market matching model to allow for information transmission through social networks. I illustrate the effects of social networks on the labor market and I use simulations to quantify the predictions of the model. This approach makes it possible to consider complex and realistic social networks. I find that information transmission through social contacts reduces the steady state unemployment rate from a hypothetical 6.5% to 5%. Social referrals can explain about 1/5th of the observed duration dependence of unemployment. They cannot explain much of the variation in wages of otherwise homogeneous workers and do not substantially influence aggregate outcomes over the business cycle.
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1. Introduction

Many workers use the help of friends or relatives when searching for jobs. Survey evidence suggests that half of all employed workers found their jobs through social contacts (Rees 1966, Granovetter 1973, Bewley, 1999, and Pellizarri 2004). The recent literature has improved our understanding of the theoretical implications of social referrals for labor market outcomes.[1] These advances rely on simplifications to reduce the complexity generated by the introduction of information transmission though social networks. One approach (see Calvo-Armengol and Jackson, 2004 and 2007, or Fontain, 2006) is to assume a very simple structure of the networks. An alternative is to assume that networks are constantly dissolved and reformed randomly (see Calvo-Armengol and Zenou, 2005, or Ioannides and Soetevent, 2006). The need to rely on these assumptions makes it difficult to obtain a sense for the magnitude of the effects generated by real life social networks.

In this paper, I try to asses these magnitudes by simulating a labor market matching model with social networks. I augment a simple matching model similar to the urn-ball model of Blanchard and Diamond (1994) to allow for information transmission through social networks. I use numerical simulations to solve the model. This makes it possible to consider more realistic – and complex – network structures and quantify the effects of information transmission through these social networks on labor market outcomes.[2]

I consider a social network that exhibits well documented, common characteristics of social networks, like clusteredness and a right skewed distribution of the number of connections.[3] To illustrate

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[1] Calvo-Armengol and Jackson (2004 and 2007) show that information transmission through social networks can result in a positive correlation of employment status and wages across agents and time. It can generate duration dependence of the probability of leaving unemployment, as well as, wage inequality between different sub-groups. This last point is also made by Arrow and Borzekowski (2004), who provide some quantitative results by based on simulations. Calvo-Armengol and Jackson (2008) show that intergenerational persistence in labor market outcomes can be generated though inheritance of social ties. Buhai and van der Leij (2006) present a model of where social ties lead to inbreeding in occupational categories. Fontaine (2006) studies effects of differences in unemployment rates between otherwise identical social networks. Calvo-Armengol and Zenou (2005) and Ioannides and Soetevent (2006) analyze the properties of a job matching function with social referrals.

[2] This paper (and most of the papers cited above) studies the spread of information about vacancies among workers. I do not consider the effects of the transmission of information about worker characteristics to potential employers. Montgomery (1991) and Arrow and Borzekowski (2004) investigate this mechanism.

[3] See Jackson 2006, Newman 2003, and Mayer and Puller 2008.

the mechanisms through which social networks affect labor market outcomes I also consider simpler structures of social networks.

In my model, information transmission through social networks increases the probability that a worker finds a vacancy and thus improves the matching of workers to vacancies. My simulation results suggest that this reduces the unemployment rate from a hypothetical 6.5% without information from social contacts to an observed 5%. The presence of social networks does not substantially affect differences of the steady state values of aggregate variables, such as unemployment rate or movements in and out of unemployment, between different points in the business cycle.

I find that differences in the social networks of individuals can generate substantial variation in the length of unemployment spells and the probability of being employed. The probability of employment is increasing at a decreasing rate in the number of social connections of a worker. Social referrals can generate some (up to 20%) of the observed duration dependence of unemployment, but other sources – such as heterogeneity in worker characteristics – seem to be more important.

Differences in the expected duration of unemployment can affect wages through the bargaining process. Using a standard Nash bargaining framework, I find that the resulting wage differences are only minor. Information transmission through social networks cannot explain the variation in wages of otherwise homogeneous workers.

I present the matching model in Section 2 and discuss the characteristics of social networks in Section 3. In section 4, I characterize the resulting economy and illustrate the effects of social networks. The quantitative results of simulations of the model are presented in Section 5. The effect of social referrals on wages is analyzed in Section 6. Section 7 concludes.

2. Matching model

I use a labor market matching model similar to Blanchard and Diamond (1994). My model differs from the common modeling approach exemplified by Pissarides (2000) in two ways. First, the matching function is not a black box. The matching of job seekers and vacancies is modeled explicitly. This makes it possible to also explicitly model the effects of social referrals. The second difference is that the number of productive jobs is exogenous. This feature greatly simplifies the simulation of the model.

There are *N* workers in the economy. Workers can be employed or unemployed. The stocks of employed and unemployed workers are denoted by *E* and *U*. Consequently, *N*=*E*+*U*. There are *K* jobs in the economy. A job can be productive or unproductive. If a productive job is matched to a worker it produces an output of y. The number of filled jobs equals the number of employed workers and is given by *E*. The number of vacancies, productive but unfilled jobs, is given by *V*. *W* gives the number of unproductive jobs. An unproductive job does not produce any output and will not be matched to a worker. The total number jobs is given by:

(1) $K = E + V + W$.

Time is discrete. Each period is divided into K sub-periods, one for each job.[4] In its sub-period a job can receive a productivity shock. The probability that a previously productive job stays productive is given by π_{11} . A productive job turns unproductive with probability $1-\pi_{11}$. If a filled job becomes unproductive the match is destroyed, the respective worker looses her job and is unemployed. The probability that a previously unproductive job becomes productive is given by π_{01}. If a job becomes productive a vacancy appears. A vacancy is filled when it is seen by an unemployed worker. An unfilled vacancy reappears in the next period.

Workers are connected through an exogenously given social network. I refer to connected workers as 'friends'. The vacancy-worker matching mechanism allows for the referral of vacancies by employed workers to their unemployed friends. In the next section, I describe these social networks.

Each worker hears with probability *a* about each vacancy. Both employed and unemployed workers hear about a vacancy with the same probability. If an unemployed worker hears about a vacancy she applies for the job. If an employed worker hears about a vacancy she tells one of her unemployed friends about it. The unemployed friend applies to the job. If an employed worker has no unemployed friends she does not transmit information about the vacancy. The vacancy is filled by

[4] All jobs are identical and the order of the sub-periods does not matter.

randomly picking one of the applicants. If no worker applies for the vacancy the vacancy remains unfilled and the sub-period ends. An unemployed worker always accepts the job offer and is immediately employed.

The filling of vacancies, the opening of new vacancies and the disappearance of productive jobs is characterized by the following system of equations:

(2) $$E_t = \pi_{11}E_{t-1} + \theta_t \pi_{11} V_{t-1} + \theta_t \pi_{01} W_{t-1}$$

(3) $$V_t = (1-\theta_t)\pi_{11}V_{t-1} + (1-\theta_t)\pi_{01}W_{t-1}$$

(4) $$W_t = (1-\pi_{11})E_{t-1} + (1-\pi_{11})V_{t-1} + (1-\pi_{01})W_{t-1}$$

The matching rate θ_t is the probability that a vacancy is filled in a given period. It depends on the probability that information about the vacancy reaches at least one unemployed worker and hence on information transmission through social networks.

3. Networks

To characterize the structure of social networks, I need to introduce some notation.[5] As mentioned above, I consider an economy with *N* workers - or in the terminology of network analysis, a network with *N* nodes. Workers *i* and *j* can be friends with each other, in this case the nodes *i* and *j* are linked - or connected. This relationship is symmetric; if worker *i* is a friend of worker *j*, then worker *j* is also a friend of worker *i*. The friendships between workers are recorded in the symmetric *n*n* matrix ***g***. If worker *i* and worker *j* are friends, the corresponding elements of the friendship-matrix ***g*** are equal to one, $g(i,j)=1$ and $g(j,i)=1$. Otherwise, the elements of ***g*** are equal to zero.

The number of connections (friends) of an individual is referred to as the number of *degrees* of a node. The *cluster coefficient* measures the cliquishness of a network. It captures the fraction of the

[5] The presentation here is based on Jackson (2006). For other ways to characterize networks, see Newman (2003) and Wasserman and Faust (1994).

friends of a given individual who are friends with each other. The literature considers different ways of calculating this measure. I follow Jackson and Rogers (2005) and define the *cluster coefficient* as:

$$C = \frac{\sum_{i:j\neq i,k\neq j,i} g_{ij} g_{jk} g_{ik}}{\sum_{i:j\neq i,k\neq j,i} g_{ij} g_{jk}}.$$

According to Jackson (2006) and Newman (2003), social networks are characterized by a number of common characteristics. The degree distribution (the distribution of the number of friends) is right skewed and has fat tails. Social networks tend to be cliquish and exhibit a cluster coefficient that cannot be explained by random formation of links. In a randomly generated network with many nodes and few connections, the cluster coefficient equals the probability that two nodes are connected and is close to zero. Mayer and Puller (2008) find cluster coefficients between .17 and .27 for large friendship networks on university campuses. Newman (2003) and Jackson (2006) report cluster coefficients ranging from .09 to .45 for co-authorship networks in different academic disciplines; Goyal et al. (2006) report cluster coefficients from .16 to .20 among co-authors in economics. Newman also reports a cluster coefficient of .2 for a network of actors, where a link is established when 2 actors co-star in the same movie.

To study the role of the network structure in the job search process, I consider 4 different kinds of networks. First, as a benchmark, I examine an economy without referrals through social connections. Second, I consider a network that consists out of pairs. Each worker shares information with exactly one other worker. I use this simple case to illustrate the effect of social referrals. The third case is a network where each pair of workers has an equal probability of being linked. This network results in a unique social environment for each worker, but does not exhibit the classic characteristics of social networks, like clusteredness. The number of links of individuals is binomially distributed. This kind of network is also called ‘random graph’. The forth – and most realistic case – is a network with the classic characteristics of social networks. I simulate a network that exhibits characteristics like clusteredness and skewness of the degree distribution. Comparing the results to the results for the random network reveals the importance of the network structure.

4. Effects of Social networks

In this section, I describe the steady state equilibrium of the economy.[6] I illustrate effects of referrals through social networks on aggregate and individual level effects. In Section 5, I use simulations to quantify the magnitude of these effects.

4.1 Aggregate Effects

I denote steady state outcomes with subscript *s*. The steady state level of employment depends on the exogenously determined parameters (π_{11}, π_{01}, and *a*) and on the matching rate (θ_S), which is determined endogenously and depends on the structure of the referral networks. Social referrals influence the steady state through their effect on the matching rate, θ_S. θ_S depends on the number of job seekers and how they obtain information. The transition probabilities π_{11} and π_{01} determine the steady state level of productive jobs:

(5) $$E_s + V_s = \frac{\pi_{01}}{1-\pi_{11}+\pi_{01}} K .$$

The total number of new job-worker matches in each period is given by:

$$M\left(U_s, V_s\right) = \theta_s\left[\pi_{11} V_s + \pi_{01} W_s\right].$$

In the steady state, the total matches per period equals the total number of destroyed matches, $M\left(U_s, V_s\right) = \left(1-\pi_{11}\right)E$. The steady state employment level is determined by θ_S, π_{11}, and π_{01}:

(6) $$E_s = \frac{\theta_S}{\left(1-\pi_{11}\right)}\left(\pi_{11} V_s + \pi_{01} W\right).$$

[6] Note that the steady state applies for each sub-period. The sequence in which each job is hit by the productivity job and is filled does not matter.

No network

As a benchmark, I derive the matching rate θ_S^{NO} without information transmission through social networks.[7] The superscript *NO* indicates the absence of social referrals. The probability that a vacancy receives no application is given by: $(1-a)^{U_S}$. The steady state matching probability is:

$$(7) \qquad \theta_S^{NO} = \left[1-(1-a)^{U_S}\right].$$

Given *a*, π_{11}, and π_{01} equations (5), (6), and (7) describe the steady state level of employment. It is also possible to derive the effects of changes in the exogenous variables on the steady state outcome. The reaction of θ_S^{NO} to a change in the employment level is given by:

$$(8) \qquad \frac{d\theta_S^{NO}}{dE_S} = (1-a)^{(1-E_S)}\ln(1-a) < 0.$$

An increase in the employment rate decreases the matching rate.

Network of Pairs

Now, I consider a very simple network structure to highlight some of the mechanisms of information transmission through social contacts. I use the superscript *PAIR* to indicate social referrals through a network of pairs. Each worker is connected to one other worker. Workers can be part of a fully employed couple, a partially employed couple, or an unemployed couple. The stock of these couples is given by Q_s^2, Q_s^1, and Q_s^0. The total number of workers is $N = 2\left(Q_s^0 + Q_s^1 + Q_s^2\right)$. The number of employed workers is given by $E = Q_S^1 + 2Q_S^2$. A vacancy is filled if it is seen by an unemployed worker or an employed worker with an unemployed partner. Holding the value of *a* fixed, social referrals increase the probability that a vacancy is filled from

$$\theta_S{}^{NO} = \left[1-(1-a)^{2Q_S^0+Q_S^1}\right]$$

to

[7] The resulting matching process is equivalent to an urn-ball matching process, see Blanchard and Diamond,1994.

$$\theta_S^{\ PAIR} = \left[1-(1-a)^{2Q_S^0+2Q_S^1}\right].$$

The model with social referrals that generates the matching probability implied by the data with a lower value of *a*, the probability that a given workers sees a vacancy,

$$\overline{\theta}_S = \theta_s^{NO} = \theta_S^{PAIR} \Rightarrow \ a^{PAIR} < a^{NO}.$$

The reaction of θ_S^{PAIR} to a change in the aggregate employment level is more complex than without social referrals. Similar to the case without referrals, θ_S^{PAIR} decreases, if a worker whose partner was employed finds a job:

$$\left.\frac{d\theta_S}{dQ_S^2}\right|_{\overline{Q_S^0}} = 2(1-a)^{2\left(Q_S^0+Q_S^1\right)}\ln(1-a) < 0.$$

However, if a worker from a pair with two unemployed workers finds a job θ_S^{PAIR} does not change:

$$\left.\frac{d\theta_S}{dQ_S^1}\right|_{\overline{Q_S^2}} = 0.$$

This illustrates that, the aggregate unemployment rate is not sufficient to determine the matching probability. The position of the employed and unemployed workers in the social network can affect the probability that a vacancy is matched to a worker. This suggests that the presence of social referrals can potentially affect differences between steady state values at different points in the business cycle.

Complex networks

It is not possible to derive a closed form expression for θ_S for more realistic and complex network structures. Workers have different numbers of friends and friends of friends. Each individual has a unique position in the network associated with a unique state dependent probability of seeing a job.

4.2 Individual level effects

The probability that an unemployed worker, *i*, finds a job during a given time period, p_i, depends on the social network of worker *i*. The probability that she is hired depends on the number of other workers applying for the same job. It is given by:

$$p_i = \Pr\left(\text{Hired}\middle|\text{See\_Vacancy}\right)\Pr\left(\text{See\_Vacancy}\right)$$

$$= \Pr\left(\text{See\_Vacancy}\right)\left(\Pr\left(other = 0\right) + \Pr\left(other = 1\right)\frac{1}{2} + ... + \Pr\left(other = l\right)\frac{1}{l+1}\right),$$

where $\Pr\left(other = l\right)$ gives the probability that *l* other unemployed workers hear about the vacancy.

Without social referrals this probability is identical for all workers $p_i = \overline{p}$. The probability that a worker hears about a specific vacancy is given by *a* and the number of workers that hear about a vacancy is distributed binomial. Therefore,

$$p_i = \overline{p} = \left[1 - \left(1-\phi\right)^{\pi_{11}V_s + \pi_{01}W_s}\right], \quad \text{with} \quad \phi = a\sum_{k=0}^{U-1}\frac{1}{1+k}\binom{U-1}{k}a^k\left(1-a\right)^{U-k}.$$

When considering a network of pairs, the probability that an unemployed worker with an unemployed partner hears about a given vacancy is *a.* The probability that a worker with an employed partner hears about a vacancy is *2a*. This introduces heterogeneity in employment probabilities. The distribution of the number of other workers hearing about the vacancy is no longer binomial. More complex networks lead to different probabilities of seeing a job for each worker. In the next section, I use simulations to get a sense for the distribution of individual job-finding probabilities.

Heterogeneity in the probability of finding a job due to social referrals can lead to duration dependence. To see how, consider a simple example with two types of workers. One type, *H*, has a high probability of finding employment, $\Pr_H$. The other type, *L*, has a lower probability of finding employment,[8] $\Pr_L < \Pr_H$. The number of newly unemployed workers of each of the two types is given

[8] In these case with a network of pairs these types would be unemployed workers with an employed partner (*H*) and unemployed workers with an unemployed partner (*L*).

by D_0^L and D_0^H . One period later type H workers are more likely reemployed and the ratio of high and low type workers who have been unemployed for one period is given by:

$$\frac{D_1^H}{D_1^L} = \frac{D_0^H \left(1 - \Pr_H\right)}{D_0^L \left(1 - \Pr_L\right)} < \frac{D_0^H}{D_0^L} .$$

A higher share of type *L* workers decreases the average probability of finding employment. The average job finding probability of workers who are newly unemployed is higher than that of workers who have been unemployed for one (or more) periods:

$$\frac{\Pr_H D_1^H + \Pr_L D_1^L}{D_1^H + D_1^L} = \frac{\Pr_H D_0^H \left(1 - \Pr_H\right) + \Pr_L D_0^L \left(1 - \Pr_L\right)}{D_0^H \left(1 - \Pr_H\right) + D_0^L \left(1 - \Pr_L\right)} < \frac{\Pr_H D_0^H + \Pr_L D_0^L}{D_0^H + D_0^L} .$$

Consequently, we observe duration dependence of unemployment. Note that, more conventional explanations of duration dependence are based on the same logic, but assume unobserved worker characteristics as the reason for differences in the probability of finding employment.

5. Quantitative Results

It is not possible to solve the matching model with referrals though social networks analytically. Therefore, I simulate the model.[9] I consider 4 different network structures. The characteristics of the networks are displayed are in Table 1. Table 2 summarizes the parameters used for the simulations. I use the parameter values of Blanchard and Diamond (1994) where possible. The unit period is one week. The ratio of total jobs, *K*, to the number of workers, *N*, is 1.05. The probability that a productive job remains productive in the next period is .982. The probability of that an unproductive job becomes productive is .22. Like Blanchard and Diamond, I also consider a different set of values to describe the economy during a recession, π_{11}=.981 and π_{01}=.129.

[9] I simulate an economy with N=2000 for 5000 periods. The first 100 periods are discarded and the remaining 4900 are used to calculate the steady state results. I also simulate the random and the clustered social networks.

I calibrate the probability that an individual sees a job, *a*, to generate the unemployment rate of 5% obtained by Blanchard and Diamond (1994). The value depends on the subsequent information transmission through social networks.

For the random network and the clustered network, I select the network density, i.e. the average number of friends, to produce the stylized fact that 50% of all jobs are obtained through referrals. The required network density depends on the structure of the network.

5.1 Aggregate results

Table 3 describes the aggregate variables of the economy for the 4 different network structures considered. Rows 1 through 4 present the results for "normal" economic conditions. The unemployment rate is displayed in row one. All models are calibrated to generate a 5% unemployment rate. Row 2 shows the probability that a vacancy is filled. Row 3 displays the probability of moving from unemployment to employment. It is slightly lower in the presence of referrals through complex networks. The reason is that well connected workers find reemployment in the period of their job loss and never appear as unemployed. Row 4 displays the ratio of workers who find their jobs through referrals to workers who find their job directly without referrals. The random and clustered networks are calibrated to a ratio of one. A network of pairs leads to a slightly lower share of workers who found their jobs through referrals.

First, I investigate the importance of referrals for these aggregate variables. I do this by simulating the 4 models with the same set parameters but suppress referrals. I "turn off" information transmission through social networks. Unemployed workers are less likely to see a vacancy and leave unemployment (Row 7 vs. Row 3). The probability that a vacancy is filled drops from .45 to less than .34 (Row 6 vs. Row 2). This leads to an increase of the unemployment rate to about 6.5% (Row 4 vs. Row 1). In other words, information transmission through social networks reduces the unemployment rate from 6.5% to 5%. The network structure has almost no effect for any of these comparisons.

Next, I see whether the different network structures affect the employment response to business cycle fluctuations. I simulate the models again now using, π_{11}=.981 and π_{01}=.129. The new steady state variables are described in rows 9 to 12. The reactions to changes in the economic environment are very similar for all the network structures considered.[10] The unemployment increases to about 9.3% for all 4 models (Row 9 vs. Row 1). The probability of transitioning from unemployment increases and the probability that a vacancy is filled decreases by similar amounts. Moreover, the vacancy to unemployment ratio changes to by similar degree for all models. This suggests that there is little need to mull over social networks when designing labor market matching models to study the steady state of aggregate variables. The only difference between the cases with and without social referrals is that a higher unemployment rate leads to a slight decrease in fraction of workers who find their job through referrals (Row 12 vs. Row 4). The reason is that unemployed workers do not referrer their friends.

5.2 Individual Level Results

The simulations results in Table 4 describe the effect of referrals through social networks on individual workers. Row 1 displays the standard deviation of the average time unemployed. More complex networks introduce heterogeneity among workers; some workers are better connected than others.[11] This leads to variation in the average time unemployed. The variation in the model with a random network is smaller than with the clustered and skewed network. This is due to the fact that in the latter case the variation in the number of friends is higher.

Row 2 shows that the persistence of the employment status as measured by the first order autocorrelation of the employment status. The information transmission through social networks leads to a slight increase in this persistence. Workers with 'bad' networks stay unemployed for longer periods of time. Workers with 'good' networks are likely to find reemployment soon after the loss of a job.

[10] I also considered more extreme changes to the economic environment the differences induced by different networks are still minor.
[11] I discuss the influence of the connections in more detail below.

Row 3 displays the duration dependence of exiting unemployment, as measured by the ratio of the probability of finding a job in the first week of unemployment divided by the probability of finding a job after two weeks of unemployment. Without the information transmission through social networks all unemployed workers are identical and there is no duration dependence. In the presence of social referrals, workers with good connections are more likely to find employment quickly. Workers with bad connections are unemployed for longer periods of time and less likely to find a job in a given period. The clustered and skewed network generates the highest duration dependence with a one vs. two week ratio of 1.135. Lynch (1989) investigates duration dependence using NLSY data, the equivalent ratio reported by her is 1.75.[12] While social referrals can explain part of the observed duration dependence, other sources of heterogeneity seem to be more important.

In Rows 4, 5, and 6, I report the results for simulations for an economy in a recession. The standard deviation of the average unemployment probability increases, as unemployment becomes more likely. Rows 7, 8, and 9 display the ratio between the values under normal economic conditions and during a recession. The relative increase of this variation is very similar for all four models (Row 7). Consistent with the above observation that referrals play a lesser role when more workers are unemployed the social network induced duration dependence decreases during recessions. This is not consistent with Shimer (2008) who finds – using CPS data – that the effect of unemployment duration on the job finding probability is not affected by the business cycle.

Workers with a high number of friends have a high probability of receiving a referral and are less likely unemployed. Table 5 shows the results of regressions of the probability of being employed on the number of friends and the number of friends of friends. For both the random network and the clustered network, additional friends increase the probability of employment at a decreasing rate. The number of friends of friend has very little effect after controlling for the number of friends.

[12] Based on CPS data Shimer (2008) reports a similar level of duration dependence.

6. Wage Determination through Bargaining

Mortensen (2003 p.1 writes "Observable worker characteristics that are supposed to account for productivity differences typically explain no more than 30 percent in the variation in compensation." In this section, I quantify the wage dispersion that can be generated by introducing social referrals into a standard wage bargaining model. Social contacts determine the job finding probabilities of workers and therefore the outside options during the wage bargaining process between workers and firms. Consequently, information transmission through social networks can lead to wage differences among otherwise homogeneous workers.

I assume that there are no binding long-term contracts and wages are renegotiated each period. Both firms and workers have perfect information about the expected referrals through the social network of workers.

When a worker and a firm with a vacancy meet they can gain from forming a connection and filling the vacancy. Let β be the discount rate. Let $V_E(i)$ and $V_U(i)$ express how valuable the states of employment and unemployment are to worker *i*. These values depend on the individual specific wage rate, w_i, the monetized utility of unemployment, *z*, and the job finding probability, p_i.

(9) $$V_E(i) = w_i + \beta\{\pi_{11}V_E(i) + (1-\pi_{11})V_U(i)\}$$

(10) $$V_U(i) = z + \beta\{p_i V_E(i) + (1-p_i)V_U(i)\}$$

The gain of accepting a job with a wage rate w_i is:

$$G_A(w_i, p_i) = V_E(i) - V_U(i) = \frac{w_i - z}{1-\beta(\pi_{11}-p_i)}.$$

$V_F(i)$ is the value of filled job for a firm, V_V the value of a vacancy, and V_I the value of an idle job. Note that V_V and V_I do not depend on the specific worker that is interacting with the firm. Let $\overline{V}_F$ represent the expected value of $V_F(i)$, then:

(11) $$V_F(i) = y - w_i + \beta\{\pi_{11}V_F(i) + (1-\pi_{11})V_I\}$$

(12) $$V_V = \beta\left\{\pi_{11}\theta_S\overline{V}_F + \pi_{11}(1-\theta_S)V_V + (1-\pi_{11})V_I\right\}$$

$$V_I = \beta\left\{\pi_{01}\theta_S\overline{V}_F + \pi_{01}(1-\theta_S)V_V + (1-\pi_{01})V_I\right\}$$

For the firm the gain of filling the vacancy is given by:

$$G_B(w_i) = V_F(w_i) - V_V .$$

The total surplus of filling the vacancy is split between the worker and the firm. The share each side receives depends on *α,* the bargaining power of workers:

(13) $$(1-\alpha)G_A(i) = \alpha G_B(i),$$

or using equations (9) through (12),

(13') $$\underbrace{(1-\alpha)\frac{w_i - z}{1-\beta\pi_{11}+\beta p_i}}_{(1-\alpha)G_A(i)} = \underbrace{\alpha\frac{y-w_i}{(1-\beta\pi_{11})} + \alpha Q}_{\alpha G_B(i)},$$

with $$Q = \left[\frac{(\beta-\beta\pi_{11})V_I}{(1-\beta\pi_{11})} - V_V\right].$$

The term Q captures the value of an idle job and the outside option of the firm. It does not vary across workers.

Without social referrals all workers have the same probability of finding a job and thus the same wage.[13] With social referrals, the connections of a worker affect the probability of finding a job, p_i. A higher probability implies a higher outside option for the worker and decreases the gain from a contract for a given wage:

$$\left.\frac{\partial G_A}{\partial p_i}\right|_{w_i} < 0.$$

The job finding probability of a specific worker does not affect the outside option of the firms. Consequently equation (13) implies that a high value of p_i will lead to a high value of w_i. Some of the surplus of the firm is transferred to the worker.

[13] See Appendix for the closed form expression of the wage.

Equation (13) can be solved to obtain an expression for the wage as a function of p_i, the exogenously given parameters (π_{11} and π_{01}) and an expression capturing the expected values of a vacancy and an idle job:

$$(14) \qquad w_i = \frac{(1-\beta\pi_{11}+\beta p_i)\alpha y + (1-\beta\pi_{11})(1-\alpha)z}{(1-\beta\pi_{11}+\alpha\beta p_i)} + \frac{(1-\beta\pi_{11}+\beta p_i)(1-\beta\pi_{11})}{(1-\beta\pi_{11}+\alpha\beta p_i)}\alpha Q .$$

To get a sense for the magnitude of the variation in wages due to social referrals I parameterize the values in equation (14). I pick the period discount rate $\beta = .999062$. This corresponds to a yearly interest rate of 5%. I normalize $y = 1$ and pick $z = .15$. The values $\pi_{11} = .982$ and $\theta = .45$ are obtained from Table 2. I approximate Q by calculating its value for the case of homogeneous workers with a job finding probability that equal is to the average job finding probability for heterogeneous workers.[14]

Table 6 presents the resulting wage distribution for the different network types and different values for α.[15] Without social referrals (or with a network of pairs) all workers earn the same wage. The two other network structures introduce very little heterogeneity in wages. With the bargaining process assumed here, social referrals contribute very little to the wage variation of otherwise homogeneous workers.

7. Conclusion

I augment a labor market matching model by incorporating information transmission through social networks. I simulate the resulting economy. I find that without social referrals the steady state unemployment rate would be 6.5% instead of 5%. The predictions for aggregate outcomes of a matching model with social networks and a standard model without social networks are quantitatively very similar. Social referrals generate heterogeneity in unemployment duration and can explain about 1/5th of the observed duration dependence of unemployment. They cannot explain the variation in wages of otherwise homogeneous workers.

[14] See Appendix.
[15] I obtained similar results when considering an economy in recession.

This paper quantifies the effects of the sharing of information about vacancies among worker. I do not take into account some other mechanisms through which social networks might affect labor market outcomes. For example, social networks might be used to transmit information about worker characteristics to potential employers. Moreover, to isolate the effects of social networks, I assume homogeneous workers that differ only in their position in the network. I do not consider the effects of membership in different social networks due to individual characteristics, such as ethnicity. Effects of social networks on labor market outcomes through any of these channels are not reflected in the results presented here.

Appendix

In this appendix, I derive an expression for the wage for the case of workers with identical job finding probabilities. Then I illustrate how the wage for heterogeneous workers can be approximated.

If all workers have identical employment probabilities the value of all filled jobs to the firm are identical, as well:

$$p_i = \overline{p} \qquad \text{and} \qquad \overline{V}_F = V_F(i) \qquad \text{for all } i$$

The gains for workers and firms are given by:

$$G_A = V_E - V_U = \frac{\overline{w} - z}{1 - \beta(\pi_{11} - \overline{p})}$$

$$G_B = V_F - V_V = \frac{y - \overline{w}}{1 - \beta\pi_{11}(1-\theta)}.$$

The wage, $\overline{w}$, can be derived by using $(1-\alpha)G_A = \alpha G_B$

(A 1) $$(1-\alpha)\frac{\overline{w} - z}{1 - \beta(\pi_{11} - \overline{p})} = \alpha \frac{y - \overline{w}}{1 - \beta\pi_{11}(1-\theta)},$$

(A 2) $$\overline{w} = \frac{\alpha(1 - \beta\pi_{11} + \beta\overline{p})y + (1-\alpha)(1 - \beta\pi_{11} + \beta\pi_{11}\theta)z}{(1 - \beta\pi_{11} + (1-\alpha)\beta\pi_{11}\theta + \alpha\beta\overline{p})}$$

If both parties have equal bargaining power ($\alpha = .5$) the wage is given by:

$$\overline{w} = \frac{(1 - \beta\pi_{11} + \beta\overline{p})y + (1 - \beta\pi_{11} + \beta\pi_{11}\theta)z}{(2 - 2\beta\pi_{11} + \beta\pi_{11}\theta + \beta\overline{p})}.$$

If workers have different job finding probabilities the value of a filled job to a firm depends on the wage negotiated with the respective worker, *i*. Equation (A 1) becomes now:

$$(1-\alpha)G_A(i) = (1-\alpha)\frac{w_i - z}{1 - \beta\pi_{11} - p_i} = \alpha\frac{y - w_i}{(1 - \beta\pi_{11})} + \alpha Q = \alpha G_B(i),$$

with $$Q = \frac{(\beta - \beta\pi_{11})V_I}{(1 - \beta\pi_{11})} - V_V \ .$$

The term Q captures the value of the outside option of the firm and does not vary across workers. I approximate Q using equation (A 2) and using the average job finding probability for heterogeneous workers to obtain $\bar{p}$.

$$w_i = \frac{(1-\beta\pi_{11}+\beta p_i)\alpha y + (1-\beta\pi_{11})(1-\alpha)z}{(1-\beta\pi_{11}+\alpha\beta p_i)} + \frac{(1-\beta\pi_{11}+\beta p_i)(1-\beta\pi_{11})\alpha}{(1-\beta\pi_{11}+\alpha\beta p_i)}Q.$$

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Table 1

Charactersitics of the Simulated Networks

	Pairs	Random	Clustered
Average number friends	1	1.63	2.29
Variance of number of friends	-	1.27	2.32
Skewness of number of friends	-	0.80	1.29
Cluster coefficient	-	0.00	0.36

Note: I simulate the random and clustered networks. The network size is 2000.

Table 2

Parameters for Simulation

Time Periods (The first 100 periods are not used)	T=5000	
Number of workers	N=2000	
Number of Total Jobs	K=2100	(Source: Blanchard & Diamond, 1994)

	Normal		Recession		
	π_{11}	π_{01}	π_{11}	π_{01}	Source
Transition Probabilities	0.982	0.22	0.981	0.129	Blanchard & Diamond (94)

	No-Network	Pairs	Random	Clustered	Source
a (Probability of seeing vacancy)	0.006	0.0032	0.003	0.0031	Calibrated to generate unemployment rate of 5%
Average number of friends	0	1	1.63	2.29	Calibrated to generate referral ratio of one

Table 3
Aggregate Results

Row #			No-Network	Pairs	Random	Clustered
1		Unemployment	0.050	0.050	0.050	0.050
2	Baseline	Prob Vacancy filled, θ	0.453	0.455	0.451	0.461
3		Transition to Employment	0.290	0.291	0.279	0.271
4		Ratio referred / Direct	0.000	0.906	1.002	0.998
5		Unemployment	0.050	0.063	0.065	0.064
6	Turn off referrals	Prob Vacancy filled, θ	0.454	0.333	0.322	0.327
7		Transition to employment	0.290	0.235	0.229	0.232
8		Ratio referred / Direct	0.000	0.000	0.000	0.000
9		Unemployment	0.093	0.094	0.093	0.093
10	Recession	Prob Vacancy filled, θ	0.673	0.667	0.667	0.682
11		Transition to employment	0.170	0.168	0.165	0.162
12		Ratio referred / Direct	0.000	0.829	0.958	0.983

Note: Based on simulations over 5000 periods.
The first 100 Periods are not considered in the calculations.

Table 4

Individual Level Results

Row #			No-Network	Pairs	Random	Clustered
1		Standard deviation of u	36.9	35.8	126.0	155.2
2	Baseline	Persistence of employment	0.69	0.69	0.71	0.71
3		Duration dependence	1.00	1.01	1.07	1.14
4		Standard deviation of u	62.7	64.9	209.6	265.4
5	Recession	Persistence of employment	0.81	0.81	0.82	0.82
6		Duration dependence	1.00	1.01	1.04	1.08
7	Ratio	Standard deviation of u	0.59	0.55	0.60	0.58
8	Baseline / Recession	Persistence of employment	0.85	0.85	0.86	0.87
9		Duration dependence	1.00	1.00	1.03	1.05

Note: Based on simulations over 5000 periods.
The first 100 Periods are not considered in the calculations.

Table 5

The Role of Connections

Linear regression of probability of employment
on number of friends and friends of friends

Dependent Variable: Probability of employment

	Random Network		Clustered Network	
# of friends	0.052	0.050	0.036	0.045
# of friends squared	-0.013	-0.012	-0.005	-0.007
# of friends cubed	0.001	0.001	0.000	0.000
# of friends of friends		0.001		-0.002
# fr. of fr. squared		0.000		0.000
# fr. of fr. Cubed		0.000		0.000
R^2	0.89	0.90	0.91	0.91

Note: The average number of friends is in the Random Network is 1.63.
In the Clustered Network it is 2.29.

Table 6

Effect on Wages

	No-Network	**Pairs**	**Random**	**Clustered**
α=.5				
mean wage	0.48	0.48	0.48	0.48
median wage	0.48	0.48	0.48	0.48
standard deviation wage	-	-	0.01	0.01
10th percentile	0.48	0.48	0.47	0.46
90th precentile	0.48	0.48	0.49	0.49
α=.1				
mean wage	0.21	0.21	0.21	0.21
median wage	0.21	0.21	0.21	0.21
standard deviation wage	-	-	0.01	0.01
10th percentile	0.21	0.21	0.20	0.19
90th precentile	0.21	0.21	0.22	0.22
α=.9				
mean wage	0.87	0.87	0.87	0.87
median wage	0.87	0.87	0.87	0.87
standard deviation wage	-	-	0.00	0.00
10th percentile	0.87	0.87	0.87	0.87
90th precentile	0.87	0.87	0.87	0.88

Note: See Tables 1 and 2 for paremeters used for simulations.
Additional parameters for the bargaining process: $\beta = .999062$