Random innovation subsidies

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Abstract

This paper constructs a model where the identity of subsidized industries changes over time. An increase in the R&D subsidy raises innovation in industries that are currently subsidized, lowers innovation in the others, and raises aggregate innovation.

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1 Introduction

In existing R&D models such as Grossman and Helpman (1991) and Segerstrom (1991), any subsidies to innovation are applied to all industries. Here, only some industries receive innovation subsidies. Which industries are subsidized changes over time so that R&D subsidies are temporary. Both innovation that is subsidized and that is not can coexist. An increase in the level of the subsidy increases innovation in subsidized industries but decreases innovation in the others. One might expect that aggregate innovation could fall if sufficiently few industries are subsidized. However, aggregate innovation is shown to necessarily rise. An increase in the fraction of industries that are subsidized decreases innovation in subsidized industries as well as the others. One might expect that aggregate innovation must therefore fall. However, aggregate innovation can still increase due to a composition effect: more industries have the higher innovation level due to becoming subsidized.

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2 Consumer behavior

Consumers choose from a continuum of products $j \in [0, 1]$, available in a discrete number of quality levels m. A consumer has additively separable intertemporal preferences given by lifetime utility

$$U = \int_0^\infty e^{-\rho t} logu(t)dt, \tag{1}$$

where ρ is the common subjective discount factor. Instantaneous utility is

$$\log u(t) = \int_0^1 log \sum_m (\lambda)^m x_m(j,t)dj, \tag{2}$$

where λ^m is the assessment by consumers of quality level m, $x_m(j,t)$ is consumption by consumers of quality level m of product j at time t, and $\lambda > 1$ due to higher quality levels being preferred.

A consumer of maximizes lifetime utility subject to an intertemporal budget constraint. Since preferences are homothetic, aggregate demand is found by maximizing lifetime utility subject to the aggregate intertemporal budget constraint

$$\int_0^\infty e^{-R(t)} E(t)dt \leq A(0) + \int_0^\infty e^{-R(t)} Y(t)dt, \tag{3}$$

where $R(t) = \int_0^t r(s)ds$ is the cumulative interest rate up to time t and $A(0)$ is the aggregate value of initial asset holdings by consumers. Aggregate labor income of consumers is $Y(t) = Lw(t)$ where $w(t)$ is the wage at time t and L is the labor supply. Normalize the wage to one: $w(t) = 1$. Aggregate spending of consumers is

$$E(t) = \int_0^1 \left[\sum_m p_m(j,t)x_m(j,t) \right] dj, \tag{4}$$

where $p_m(j,t)$ is the price of quality level m of product j at time t.

A consumer's maximization problem can be broken into three stages: the allocation of lifetime wealth across time, the allocation of expenditure at each instant across products, and the allocation of expenditure at each instant for each product across available quality levels. In the final stage, consumers allocate spending for each product at each instant to the quality level $\widetilde{m}(j,t)$ offering the lowest quality-adjusted price, $p_m(j,t)/\lambda^m$. Settle indifference in favor of the higher quality level so the quality level selected is unique.

In the second stage, consumers then evenly spread spending across the unit measure of all products, $E(j,t) = E(t)$, as the elasticity of substitution is constant at unity. Consumers of demand $x_{\widetilde{m}}(j,t) = E(t)/p_{\widetilde{m}}(j,t)$ units of quality level $\widetilde{m}(j,t)$ of product j and no units of other quality levels of that product. Thus instantaneous utility (2) becomes

$$\log u(t) = \log E(t) + \int_0^1 \left[\widetilde{m}(j,t)\log\lambda - \log p_{\widetilde{m}}(j,t)\right] dj, \tag{5}$$

and lifetime utility (1) becomes

$$U = \int_0^\infty e^{-\rho t}\left[\log E(t) + \int_0^1 \left[\widetilde{m}(j,t)\log\lambda - \log p_{\widetilde{m}}(j,t)\right] dj\right] dt \tag{6}$$

In the first stage, consumers evenly spread lifetime spending across time, $E(t) = E$, as the utility function is time separable and the aggregate price level does not vary across time $\log p_{\widetilde{m}}(j,t) = \log p_{\widetilde{m}}(j)$. Since aggregate spending is constant across time, the interest rate at each point in time reflects the discount rate $r(t) = \rho$, so $R(t) = \rho t$ in the intertemporal budget constraint. The steady-state solution to the consumer's problem follows Grossman and Helpman (1991).

3 Producer behavior

To produce a quality level of a product, a firm must first design it. Firms are willing to endure the costs of developing higher quality levels of existing products because they earn profits in the product market if successful. The potential for quality improvement is unbounded. Each quality level invented of each product is better than the previous quality level of that product.

When a firm succeeds at innovation, it finds itself with a one quality level lead over the previous innovator. The top (highest quality) firm in each industry charges price $p = \lambda$ and makes sales $x = E/\lambda$, yielding instantaneous profits

$$\pi = E\left(1 - \delta\right), \tag{7}$$

where $\delta \equiv 1/\lambda$. The firm engages in limit pricing behavior against the previous innovator for that product one quality level below. The instantaneous profits earned in any industry do not depend on whether innovation in that industry is currently being subsidized.

However, the value of a firm, the present discounted value of those profits, does depend on whether innovation is currently being subsidized. In the next instant following the revelation of whether any innovation was successful, the government randomly reallocates the innovation subsidies across industries. With probability β, an industry has its innovation subsidized and with probability $1 - \beta$ it is not.

Let v_S denote the value of a successful innovator in an industry that is currently subsidized. The firm earns instantaneous profits π, loses its value v_S completely with probability ι_S (the chance of a rival innovation), and experiences a capital loss of $\Delta v \equiv v_S - v_N$ with probability $1 - \beta$ (the chance that subsidy status is lost).

$$\rho v_S = \pi - \beta\left(v_S - v_N\right) - \iota_S v_S \rightarrow v_S = \frac{\pi + \left(1 - \beta\right) v_N}{\rho + \iota_S + 1 - \beta} \tag{8}$$

Similarly, let v_N denote the value of a successful innovator in an industry that is not currently subsidized. The firm earns instantaneous profits π, loses its value v_N completely with probability ι_N (the chance of a rival innovation), and experiences a capital gain of $\Delta v \equiv v_S - v_N$ with probability β (the chance that subsidy status is gained).

$$\rho v_N = \pi + \beta\left(v_S - v_N\right) - \iota_N v_N \rightarrow v_N = \frac{\pi + \beta v_S}{\rho + \iota_N + \beta} \tag{9}$$

For $\beta = 0$, the valuation condition (9) reduces to that for the Grossman and Helpman (1991) model.

Modeling innovation success as a continuous Poisson process follows the assumptions made by Grossman and Helpman (1991): generating a probability ιdt of innovation success requires $a\iota dt$ units of labor. In a steady-state equilibrium with positive rates of innovation (both subsidized and not), two valuation conditions ensure the cost of innovation matches the expected gains. For industries that are currently subsidized, the innovator only bears a share of innovation costs

$$v_S = a\,(1 - s)\,, \tag{10}$$

while the innovator bears the full cost in other industries

$$v_N = a. \tag{11}$$

For both conditions to hold, the value in a subsidized industry must fall short of the value in other industries $v_S < v_N$.

4 Steady-state equilibrium

In equilibrium, consumers maximize their lifetime utility subject to the intertemporal budget constraint, firms maximize their value given prices and innovation intensities chosen by other firms and labor markets clear. Provided the economy has enough resources to support innovation, in equilibrium, producing firms earn profits sufficient in expectation to compensate for their innovation expenses. This section first finds innovation and expenditure in a steady-state equilibrium and them performs comparative statices.

Combine profits (7) and valuation conditions (8) - (11) to obtain the R&D condition if subsidized

$$\frac{E\left(1-\delta\right)\left(\rho+1+\iota_N\right)}{\rho\left(\rho+1+\iota_S+\iota_N\right)+\iota_N\left(1-\beta\right)+\iota_S\left(\iota_N+\beta\right)}=a\left(1-s\right) \tag{12}$$

and if not subsidized

$$\frac{E\left(1-\delta\right)\left(\rho+1+\iota_S\right)}{\rho\left(\rho+1+\iota_S+\iota_N\right)+\iota_N\left(1-\beta\right)+\iota_S\left(\iota_N+\beta\right)}=a \tag{13}$$

These two conditions imply that subsidized industries innovate more as $s>0$.

$$\frac{\rho+1+\iota_N}{\rho+1+\iota_S}=1-s\rightarrow\iota_S>\iota_N \tag{14}$$

Impose the resource constraint that labor demand for innovation and production must equal labor supply

$$a\left[\beta\iota_S+\left(1-\beta\right)\iota_N\right]+E\delta=L, \tag{15}$$

where aggregate innovation is $\iota=\beta\iota_S+\left(1-\beta\right)\iota_N$. The equilibrium is the expenditure E and innovations ι_S and ι_N that solve the valuation conditions (12) and (13) and the resource constraint (15).

How does innovation respond to an increase in the magnitude of the subsidy or an increase in the fraction of industries subsidized? An increase in the magnitude of the subsidy increases innovation in subsidized industries

$$\frac{\partial\iota_S}{\partial s}=\frac{\left(\frac{L}{a}+\rho\right)\left(\lambda-1\right)\left[\left(\lambda-1\right)\left(1-\beta\right)+1\right]+\lambda^2\left(1-\beta\right)}{D}>0, \tag{16}$$

while decreasing innovation in industries where R&D is not currently subsidized

$$\frac{\partial\iota_N}{\partial s}=-\frac{\beta\left[\left(\frac{L}{a}+\rho\right)\left(\lambda-1\right)^2+\left(\lambda-s\right)^2+\left(1-\beta\right)\left(\lambda-1\right)s^2\right]}{D}<0, \tag{17}$$

where the denominator is positive $D \equiv [\lambda(1-s) + s\beta(\lambda-1)]^2 > 0$. Aggregate innovation increases.

$$\frac{\partial \iota}{\partial s} = \frac{\beta\left[\left(\frac{L}{a}+\rho\right)(\lambda-1) + s(1-\beta)(s\beta(\lambda-1)+\lambda(2-s))\right]}{D} > 0 \tag{18}$$

The effect of a subsidy on innovation increases as the fraction of industries subsidized increases. Therefore, $\partial\iota/\partial s$ here is smaller than in the Grossman and Helpman model (where all industries are subsidized).

An increase in the fraction of industries subsidized decreases innovation in both subsidized industries

$$\frac{\partial \iota_S}{\partial \beta} = -\frac{s\left[\left(\frac{L}{a}+\rho\right)(\lambda-1)^2 + \lambda(\lambda-s)\right]}{D} < 0 \tag{19}$$

and in other industries.

$$\frac{\partial \iota_N}{\partial \beta} = (1-s)\frac{\partial \iota_S}{\partial \beta} < 0 \tag{20}$$

Aggregate innovation may nonetheless increase

$$\frac{\partial \iota}{\partial \beta} = \frac{s\left[\left(\frac{L}{a}+\rho\right)(\lambda-1)(1-s) + s\lambda(1-s)(1-\beta)^2 - s\beta^2(\lambda-s)\right]}{D} \tag{21}$$

as $\partial\iota/\partial\beta > 0$ when evaluated at $\beta = 0$. Aggregate innovation can increase due to the composition effect: more industries are being subsidized and subsidized industries do more innovation $\iota_S > \iota_N$.

5 Conclusion

This paper has modified the Grossman and Helpman (1991) model to have innovation subsidized in only some industries. Which industries are subsidized changes over time following a random process. An increase in the subsidy increases innovation in subsidized industries, decreases innovation in industries that are not subsidized, and increases aggregate innovation. An increase in the fraction of industries that have their innovation subsidized decreases innovation in all industries. Nonetheless, aggregate innovation may rise due to a composition effect, since a greater share of all industries now have the higher level of innovation.

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