

Product Differentiation

1 Introduction

We have seen earlier how pure external IRS can lead to intra-industry trade.

Now we see how product differentiation can provide a basis for trade due to consumers valuing variety.

When trade occurs due to product differentiation, even identical countries will trade by exchanging different varieties of the same good.

The value consumers place on variety generates another source of gains from trade.

IRS due to a fixed cost of producing each variety limits the number of varieties produced by a country.

Two key versions of modeling preferences for the differentiated products are the love of variety approach and the ideal variety (bliss point or spatial) approach.

Both provide a subutility function that increases in the number of varieties available, but the love of variety approach is easier to employ.

2 Product Differentiation (Helpman and Krugman 1985)

The CES utility function has proved very useful in models of product differentiation.

The typical form of modeling preferences is to assume an upper-tiered utility function

$$u(x_0, V) = U(x_0, V(x_1, \dots, x_n))$$

where x_0 is consumption of some homogeneous numeraire good, x_1, \dots, x_n are consumptions of n differentiated goods, and V is a sub-utility function for a set of differentiated products.

Utility is separable between the set of differentiated goods and the numeraire.

Apply a two stage budgeting procedure to allocate spending across differentiated products and then between the set of differentiated products and the numeraire.

We assume that preferences are homothetic between the numeraire good x_0 and the set of differentiated goods $x_1 \dots x_n$ so consumers spend a fixed share of their income on the two categories of goods.

Suppose the upper tier utility function is Cobb-Douglas in the numeraire good and the set of differentiated goods

$$u(x_0, V) = x_0^\alpha V^{1-\alpha}$$

so the elasticity of substitution between the differentiated goods and the numeraire good equals one.

Normalizing the price of the numeraire good to one $p_0 = 1$, the consumer's budget constraint sets expenditure equal to income

$$x_0 + \sum_{i=1}^n p_i x_i = I$$

where p_i is the price of good i and I is income in terms of the numeraire good.

When preferences are homothetic, the consumer spends a fixed proportion of income I on the two set of goods: $x_0 = \alpha I$ on the numeraire good and $\sum_{i=1}^n p_i x_i = (1 - \alpha)I$ on all the differentiated goods.

Let

$$E \equiv I - x_0 = (1 - \alpha)I$$

be expenditure on the set of differentiated products.

Budget constraint for spending on differentiated goods is

$$\sum_{i=1}^n p_i x_i = E$$

2.1 Love of Variety

Suppose the sub-utility function is a symmetrical CES function

$$V = \left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}}, \quad \rho < 1$$

This subutility function has several nice properties:

- Every pair of varieties is equally substitutable:

$$\sigma = \frac{1}{1 - \rho} > 1 \iff \rho = 1 - \frac{1}{\sigma}$$

- Degree of substitution does not depend upon the level of consumption of the goods.

- Variety has value. Suppose n varieties are available at the same price p . Then the consumer buys equal amounts of all goods. Subutility can be written as

$$V = \left(\sum_i x_i^\rho \right)^{\frac{1}{\rho}} = (nx^\rho)^{\frac{1}{\rho}}$$

$$= \left[n \left(\frac{E}{np} \right)^\rho \right]^{\frac{1}{\rho}} = n^{\frac{1}{\rho}} \frac{E}{np} = n^{\frac{1}{\rho}-1} \frac{E}{p}$$

and increases as the number of varieties n increases.

$$\frac{\partial V}{\partial n} = \left(\frac{1}{\rho} - 1 \right) \frac{V}{n} > 0$$

If the number of differentiated goods is large, the set of differentiated goods may be represented by a continuum, so the sum is then replaced by an integral in the subutility function,

$$V = \left[\int_0^n x_i^\rho di \right]^{\frac{1}{\rho}}$$

The goal is to maximize subutility V subject to the budget constraint

$$\int_0^n p_i x_i di = E$$

Given an expenditure E on the differentiated goods (from the first stage), the consumer's problem becomes

$$\max \left[\int_0^n x_i^\rho di \right]^{\frac{1}{\rho}} + \lambda \left[E - \int_0^n p_i x_i di \right]$$

The first order condition for good i is

$$\frac{1}{\rho} \left[\int_0^n x_i^\rho di \right]^{\frac{1}{\rho}-1} \rho x_i^{\rho-1} - \lambda p_i = 0$$

and similarly for another good j

$$\frac{1}{\rho} \left[\int_0^n x_i^\rho di \right]^{\frac{1}{\rho}-1} \rho x_j^{\rho-1} - \lambda p_j = 0$$

The above two first order conditions imply

$$\frac{x_i}{x_j} = \left[\frac{p_j}{p_i} \right]^{\frac{1}{1-\rho}} = \left[\frac{p_j}{p_i} \right]^\sigma$$

If goods are equally priced, then they will be equally demanded

$$p_i = p_j \rightarrow \frac{p_j}{p_i} = 1 \rightarrow \frac{x_i}{x_j} = 1 \rightarrow x_i = x_j$$

For a CES utility function, the elasticity of substitution between two different varieties is $\sigma = \frac{1}{1-\rho}$.

Using demand functions,

$$E = \int_0^n x_j p_i^{1-\sigma} p_j^\sigma di = x_j p_j^\sigma \int_0^n p_i^{1-\sigma} di$$

implies

$$x_j = \frac{E p_j^{-\sigma}}{\int_0^n p_i^{1-\sigma} di} = \frac{E p_j^{\frac{1}{\rho}-1}}{\int_0^n p_i^{-\frac{\rho}{1-\rho}} di}$$

An individual firm views $\int_0^n p_i^{1-\sigma} di$ as fixed and thus faces a constant elasticity demand curve

$$x_j = k p_j^{-\sigma}$$

where

$$k \equiv \frac{E}{\int_0^n p_i^{1-\sigma} di}$$

with demand elasticity equal to the elasticity of substitution σ between a pair of the differentiated goods.

Each firm chooses the price of its variety to maximize its profits, taking as given the price charged by other firms.

Assume that every variety is produced with the same production function.

Focus on a representative firm (producing a unique variety), whose problem is to pick its price to maximize its profits

$$\pi = px - C(x)$$

Suppose the cost function takes the forms of a fixed cost plus a constant marginal cost

$$C(x) = b + cx$$

Then profits are

$$\pi = (p - c)x - b = (p - c)kp^{-\sigma} - b$$

The first order condition for profit maximization is

$$p \left(1 - \frac{1}{\sigma} \right) = c$$

which implies that all varieties are priced equally at

$$p = \frac{c}{1 - \frac{1}{\sigma}}$$

and in the limit as elasticity becomes infinite, price equals cost and $\lim_{\sigma \rightarrow \infty} p = c$.

With all varieties priced equally, the budget constraint $nxp = E$ implies that consumers evenly spread consumption over all available varieties

$$x = \frac{E}{np}$$

A zero profit condition $\pi = 0$ then pins down the measure of varieties available.

$$n = \frac{E}{b\sigma}$$

The measure of varieties available decreases in the elasticity σ and the fixed cost of variety b

$$\frac{\partial n}{\partial \sigma} = -\frac{E}{b\sigma^2}, \quad \frac{\partial n}{\partial b} = -\frac{E}{b^2\sigma}$$

as intuitively should happen based on cost and benefit.

3 Krugman AER 1980

- Explanation for trade between countries with similar (even identical) factor endowments (same technology and tastes too).
- Having a large domestic market operates as a source of comparative advantage (export goods for which have a larger domestic market than other goods relative to ROW).
- Model derived from the famous Dixit and Stiglitz (1977) model of monopolistic competition (horizontal product differentiation).

3.1 Consumers

- A large number of potential goods enter symmetrically into utility according to the utility function

$$U = \sum_{i=1}^n c_i^\theta, \quad 0 < \theta < 1 \quad (1)$$

where c_i is consumption of good i .

- Utility exhibits love of variety.
- Preferences exhibit a constant elasticity of substitution between any two goods.
- Consumers choose their consumptions c_i to maximize their utility (1) subject to the budget constraint

$$\sum_{i=1}^n p_i c_i = I$$

where p_i is the price of good i and I is income.

- Consumers solve

$$\max_{c_i} \sum_{i=1}^n c_i^\theta + \lambda \left(I - \sum_{i=1}^n p_i c_i \right)$$

where λ is the shadow price on the budget constraint (the marginal utility of income).

- The first order conditions (FOC) are

$$\theta c_i^{\theta-1} - \lambda p_i = 0, \quad i = 1, \dots, n$$

which can be rearranged to give individual consumer's demand

$$p_i = \frac{\theta}{\lambda} c_i^{\theta-1}$$

- Since everyone is identical, consumption of each good equals production x_i of each good per consumer

$$c_i = \frac{x_i}{L}$$

where L is the labor supply (measure of consumers as each consumer has one unit of labor supply).

- Substituting for c_i , we get the market demand function for good i

$$p_i = \frac{\theta}{\lambda} \left(\frac{x_i}{L} \right)^{\theta-1} \quad (2)$$

- Firms face a CES demand function with elasticity $\frac{1}{1-\theta}$.
- No strategic interdependence among firms as the number of available varieties is assumed to be large.
- Each good i will be produced by only one firm: all firms differentiate their product from all products offered by other firms.

3.2 Producers

- Suppose there is only one factor, labor. Let the cost function for each good be given by

$$l_i = \alpha + \beta x_i, \quad \alpha > 0, \quad \beta > 0 \quad (3)$$

where l_i is the labor needed to produce x_i units of good i .

- α gives the fixed cost and β the marginal cost (in terms of labor).
- IRS internal to the firm: average cost declines with output (at a decreasing rate).

- Each producer chooses its output x_i to maximize its profits

$$\pi_i = p_i x_i - w l_i$$

where w is the wage paid to labor. Substituting the demand function (2) for p_i and the cost function (3) for l_i , the firm picks its output x_i to maximize its profits

$$\pi_i = \frac{\theta x_i^\theta}{\lambda} \left(\frac{1}{L}\right)^{\theta-1} - w(\alpha + \beta x_i).$$

- The first order conditions are

$$\frac{\partial \pi_i}{\partial x_i} = \left(\frac{\theta^2}{\lambda}\right) x_i^{\theta-1} \left(\frac{1}{L}\right)^{\theta-1} - \beta w = 0, \quad i = 1, \dots, n$$

which can be rearranged to read

$$\frac{\theta}{\lambda} \left(\frac{x_i}{L}\right)^{\theta-1} = \frac{\beta w}{\theta}$$

- The left-hand side is recognized as p_i from the demand function, so the FOC implies

$$p = p_i = \frac{\beta w}{\theta}$$

where $\frac{1}{\theta} > 1$ is the markup over marginal cost βw : price is a fixed mark up over marginal cost reflecting the substitutability of varieties.

- Clearly, the price increases with the marginal cost parameter β and decreases with θ

$$\frac{\partial p}{\partial \beta} = \frac{w}{\theta} > 0, \quad \frac{\partial p}{\partial \theta} = -\frac{\beta w}{\theta^2} < 0$$

- Profits can be expressed as

$$\pi_i = (p - \beta w) x_i - w\alpha = \left[\left(\frac{1}{\theta} - 1 \right) \beta x_i - \alpha \right] w$$

- Free entry drives equilibrium profits to zero. Solving $\pi_i = 0$ for x_i gives

$$x = x_i = \frac{\alpha}{\frac{p}{w} - \beta} = \frac{\alpha\theta}{\beta(1 - \theta)}$$

- Equilibrium output increases in the fixed cost of variety α and θ but decreases in the marginal cost parameter β

$$\frac{\partial x}{\partial \alpha} = \frac{\theta}{\beta(1 - \theta)} > 0, \quad \frac{\partial x}{\partial \theta} = \frac{\alpha}{\beta(1 - \theta)^2} > 0$$

$$\frac{\partial x}{\partial \beta} = -\frac{\alpha\theta}{\beta^2(1 - \theta)} < 0$$

- Equilibrium output is constant across goods and independent of the resources in the economy.
- An increase in resources L increases only the number of varieties n that are produced (see below), not the output of each variety x .

3.3 Full Employment

- Full employment requires that labor demand nl_i equal labor supply L

$$L = n(\alpha + \beta x) = \frac{n\alpha}{1 - \theta}$$

where n is the number of varieties actually produced.

- Solving the full employment condition for n gives

$$n = \frac{L}{\alpha + \beta x} = \frac{L(1 - \theta)}{\alpha}$$

The above equations provide a complete description of the autarkic equilibrium in an economy.

- As resources L increase, the number of available varieties n increases.

$$\frac{dn}{dL} = \frac{1 - \theta}{\alpha} > 0$$

- An increase in the substitutability of goods θ decreases n .

$$\frac{dn}{d\theta} = -\frac{L}{\alpha} < 0$$

- An increase in the fixed cost of variety α decreases n .

$$\frac{dn}{d\alpha} = -\frac{L(1-\theta)}{\alpha^2} < 0$$

- The gains from trade are related to expanded resources L and imperfect substitutability of goods $\theta < 1$.

3.4 International Trade

- Assume two countries (alike in every way except size) engage in free trade (with zero transportation costs).
- What does trade do? Fixed costs in the production of each good create an incentive to concentrate production within a firm.
- Increasing returns to scale imply that concentration is efficient so that two identical countries will specialize in the production of different sets of goods and intraindustry trade will occur in similar but differentiated products.
- Gains from trade stem from the increased diversity of goods available under free trade due to market expansion.

- Full employment determines the varieties produced in each country under free trade

$$n = \frac{L}{\alpha + \beta x} = \frac{L(1 - \theta)}{\alpha} \quad (4)$$

$$n^* = \frac{L^*}{\alpha + \beta x} = \frac{L^*(1 - \theta)}{\alpha}$$

- Each country produces the same number of varieties under free trade as under autarky, but under free trade, consumers can consume all $n + n^*$ varieties.
- Since consumers love variety, they gain from trade.
- The simplifying assumptions of this model imply that free trade does not alter the prices charged by firms or their output levels.
- Under free trade, consumers solve the same problem but now distribute their expenditures over $n + n^*$ goods so that they enjoy greater variety.

3.5 Results

1. Both countries gain from trade because of increased diversity of goods. To see the gains from trade, compare (domestic) utility under autarky

$$U_A = n \left(\frac{x}{L} \right)^\theta = n \left(\frac{w}{np} \right)^\theta = n^{1-\theta} \left(\frac{\theta}{\beta} \right)^\theta \quad (5)$$

to utility under free trade,

$$U_T = (n + n^*)^{1-\theta} \left(\frac{\theta}{\beta} \right)^\theta \quad (6)$$

Utility under free trade is higher

$$U_R \equiv \frac{U_T}{U_A} = \left(\frac{n + n^*}{n} \right)^{1-\theta} = \left(1 + \frac{n^*}{n} \right)^{1-\theta} > 1 \quad (7)$$

due to the greater availability of variety $n + n^* > n$ as $n^* > 0$ due to $L^* > 0$ and $\theta < 1$. Furthermore, the gains from trade decrease with θ as the goods become closer substitutes so variety is less important

$$\frac{dU_R}{d\theta} = - \left(\frac{n + n^*}{n} \right)^{1-\theta} \ln \left(\frac{n + n^*}{n} \right) < 0 \quad (8)$$

Considering the two extremes is illustrative of the vital role of the goods being differentiated

$$\lim_{\theta \rightarrow 1} U_R = 1, \lim_{\theta \rightarrow 0} U_R = 1 + \frac{n^*}{n} \quad (9)$$

Gains from trade vanish as goods become perfect substitutes (lose love of variety).

2. Direction of trade is indeterminate, but nothing important depends on which country produces (and thus exports) which good as the goods are symmetric. Of the $n + n^*$ goods consumed, n^* are imported by the home country. The value of home country imports measured in wage units equals value of foreign country imports, so trade is balanced. Thus, the volume of trade is determinate.
3. One peculiarity of the model is that the output of each good remains the same even under trade. Krugman (JIE 1979) develops a more general (but more tedious) model where the scale of production of each variety does increase with trade.

4. When there are transportation costs, bigger country has the higher wage. Iceberg type transportation costs are assumed: when send one unit of any good abroad, only $g \leq 1$ arrives as the rest melts on the way. The higher wage in the bigger country gives a production cost advantage in the smaller country to offset the higher total transportation costs paid to reach consumers.

5. Transportation costs lead to the home market effect – countries export those products for which they have bigger markets at home.

4 Ethier AER 1982

- A large proportion of world trade involves exchange of similar products between industrialized countries.
- These products are typically intermediate production goods rather than consumption goods as in Krugman's models.
- Ethier (1982) constructs a model in which two kinds of IRS coexist.
- One is the usual external IRS (as the number of intermediate goods expands, final output increases).
- The second is IRS internal to the firm due to the presence of fixed costs involved in the production of each intermediate good.

- Full general equilibrium model with two goods and two factors of production.
- Allows us to examine the fate of the traditional HOS theory in the presence of international IRS.
- Most of the traditional theorems of the HOS model survive quite well.
- Intra-industry trade is complementary to international factor mobility.
- Lastly, while IRS are essential to obtain intra-industry trade, the degree of IRS is not of much consequence in determining the magnitude of intra-industry trade.

4.1 Model

- Two goods: wheat W and manufactures M
- Two factors: capital K and labor L
- Wheat has CRS technology
- Manufactures has IRS. Define $M = km$ where k is an index of scale economies and m is the size of operation in the M industry.
- Let $W = T(m)$ define the transformation curve between W and m .
- M has two sub-sectors: component production and assembly of finished components.

- Each component's firm produces one type of input x_i using m as an input.
- Let n be the (endogenous) number of components produced

$$m = \sum_{i=1..n} m_i$$

where m_i is the factor use in component i .

- Cost function

$$m_i = b + ax_i \Rightarrow m = nb + a \sum_i x_i$$

- Component producers are monopolistic competitors and under free entry (so zero profits).

4.2 Final Good

- All components are assembled through a symmetric CES production function

$$M = n^{\alpha - \frac{1}{\beta}} \left(\sum_i x_i^\beta \right)^{\frac{1}{\beta}}, \quad \alpha > 1, 0 < \beta < 1$$

- Every pair of varieties is equally substitutable.
- The degree of substitution does not depend upon the level of usage.

- Variety is valued. Suppose n varieties are available at price q . Then final producer buys equal amounts of all components. If x equals the quantity purchased of each component, then

$$M = n^{\alpha - \frac{1}{\beta}} \left(\sum_i x^\beta \right)^{\frac{1}{\beta}} = n^{\alpha - \frac{1}{\beta}} (nx^\beta)^{\frac{1}{\beta}} = n^\alpha x = n^{\alpha - 1} [nx]$$

Clearly, output increases as n increases. Since $\alpha > 1$, there are IRS in n . These represent the productivity gains that accrue from an increased division of labor. These IRS are external to the final good producer who cannot control n . M displays CRS with respect to x .

- Let q_i be the price of component i in W units.

- Minimize cost of producing each unit

$$\min \sum_i q_i x_i \text{ such that } n^{\alpha - \frac{1}{\beta}} \left(\sum_i x_i^\beta \right)^{\frac{1}{\beta}} = 1$$

First order conditions

$$n^{\alpha - \frac{1}{\beta}} \frac{1}{\beta} \left[\sum_{i=1}^n x_i^\beta \right]^{\frac{1}{\beta} - 1} \beta x_i^{\beta - 1} - \lambda q_i = 0$$

Take another good j and we have

$$n^{\alpha - \frac{1}{\beta}} \frac{1}{\beta} \left[\sum_{i=1}^n x_i^\beta \right]^{\frac{1}{\beta} - 1} \beta x_j^{\beta - 1} - \lambda q_j = 0$$

The above two conditions imply constant elasticity demand curve facing the producer of component i

$$\frac{x_i}{x_j} = \left[\frac{q_j}{q_i} \right]^{\frac{1}{1-\beta}}$$

4.3 Components

- Price of component j is a markup over marginal cost, where the markup equals β .

- Total cost in terms of numeraire good W is given by

$$C(x_i) = -T'(m)(b + ax_i)$$

- The above cost function implies that any given firm is too small to influence $T'(m)$, the opportunity cost of a small amount of m in terms of W .

- Marginal cost for producer i equals

$$c(x_i) = C'(x_i) = -T'(m)a$$

- Profit maximization requires marginal revenue equal marginal cost

$$q = q_j = \frac{-T'(m)a}{\beta}$$

- Since in equilibrium, all components will have the same price $q_i = q$,

$$\pi_i = qx + T'(m)(b + ax)$$

- Free entry implies

$$x = \frac{-T'(m)b}{q + T'(m)a}$$

Substituting for price of components

$$x = \frac{-T'(m)b}{\frac{-T'(m)a}{\beta} + T'(m)a} = \frac{b\beta}{a(1 - \beta)}$$

- Total resources devoted to component production are given by

$$m = n(b + ax)$$

- This implies

$$n = \frac{m(1 - \beta)}{b}$$

- Index of IRS can be rewritten by noting that

$$n^\alpha x = M = km$$

so that

$$k = \left(\frac{(1 - \beta)}{b} \right)^{\alpha-1} \frac{\beta}{a} m^{\alpha-1}$$

- k has the property that it increases in m and decreases in a and b .

4.4 Autarky

- Set $P_W = 1$ and let $P = \frac{P_M}{P_W} = P_M$.

- Zero profits in supply of M requires that

$$P_S M = qn x \longleftrightarrow P_S n^\alpha x = qn x$$

which gives

$$P_S = qn^{1-\alpha}$$

which can be rewritten using values of q and k and n as

$$P_S = \frac{-T'(m)m}{M} = \frac{-T'(m)}{k(m)}$$

Note that due to IRS, the supply function is downward sloping.

- On the demand side, assume homothetic preferences

$$U(M, W) = M^\gamma W^{1-\gamma}$$

- Constant expenditure shares imply

$$P_D M = \gamma(W + P_D M)$$

where γ equals the fraction of income spent on M and $W + P_D M$ equals total expenditure of consumers.

- This implies the demand price equals

$$P_D = \frac{\gamma}{1 - \gamma k(m)m} T(m)$$

Intersection of the two curves gives autarkic equilibrium.