Factor Endowments

1 Introduction

Ricardian model insufficient for understanding objections to free trade.

Cannot explain the effect of trade on distribution of income since there is only factor of production.

HOS model attempts to fill this void – introduces a second factor and usage of factors varies across sectors.

Owners of a country's relatively scarce factor lose from trade.

New source of comparative advantage in the HOS model: differences in factor endowments (rather than technologies) across countries.

2 Simple HOS Model

2.1 Assumptions

Two goods: cloth (C) and wheat (W).

Two countries: Home and Foreign *.

Two factors in fixed supply: capital (K) and labor (L).

Factors perfectly mobile between sectors but immobile across countries.

Perfect competition in all markets.

Countries share a common production technology and identical tastes.

2.2 Production Costs

For any given wage (w) and rent (r), firms choose the technique of production (the mix of the two factors) that minimizes the cost of producing each unit of output.

Definition 1 *A unit isoquant curve represents different combinations of capital and labor that yield one unit of output.*

Definition 2 *The budget line represents combinations of capital and labor that cost the same amount at the existing factor prices.*

Definition 3 *The unit cost function records the minimum cost of producing one unit of the good for various factor prices.*

$$C_i(w,r) = wa_{Li}(w,r) + ra_{Ki}(w,r) \tag{1}$$

The first order condition for cost minimization is

$$wda_{Li} + rda_{Ki} = 0$$

which implies optimal input mix occurs at tangency between the budget line and isoquant as in Figure A.8

$$\frac{da_{Ki}}{da_{Li}} = -\frac{w}{r}$$

The derivatives of the cost function (1) for each good with respect to factor prices give (Shephard's Lemma).

$$\frac{\partial C_i}{\partial w} = a_{Li} + \underbrace{\left[w\frac{da_{Li}}{dw} + r\frac{da_{Ki}}{dw}\right]}_{0} = a_{Li} \qquad (2)$$

$$\frac{\partial C_i}{\partial r} = a_{Ki} + \underbrace{\left[w\frac{da_{Li}}{dr} + r\frac{da_{Ki}}{dr}\right]}_{0} = a_{Ki} \qquad (3)$$

2.3 Isocost Curves

Definition 4 *An isocost curve plots combinations of w and r for which minimum cost of production is constant.*

Due to perfect competition in output markets, profits are driven to zero so the price of a good cannot exceed the cost of producing it in equilibrium (costs can exceed price if the good is not produced).

Thus, an isocost curve represents $P_i = C_i(w, r)$ for each industry as in Figure A.9.

The slope of the isocost curve denotes the capital to labor ratio employed in an industry at the prevailing factor prices.

$$\frac{dw}{dr} = -\frac{a_{Ki}}{a_{Li}}$$

The wage-to-rent ratio $\Omega \equiv w/r$ is indicated by the slope of a ray from the origin.

For any wage-to-rent ratio Ω (such as the ray through point A), cloth is relatively capital intensive iff the cloth isocost curve is steeper than the wheat isocost curve

$$\frac{a_{KC}}{a_{LC}} > \frac{a_{KW}}{a_{LW}}$$

since the slope of each isocost curve reflects the capital-to-labor ratio employed in that sector.

Definition 5 *If the identity of which good is relatively intensive in which factor depends on wage-to-rent ratio Ω, then there exists a factor intensity reversal (FIR) at the point where the capital-to-labor ratios are identical.*

In Figure 9b, cloth is relatively capital intensive at point A but relatively labor intensive at point A'.

Definition 6 *Let θ_{ij} denote the share of revenue that goes to factor i in producing good j.*

By definition and then dividing the numerator and denominator by ra_{Li}

$$\theta_{Li} \equiv \frac{wa_{Li}}{wa_{Li} + ra_{Ki}} = \frac{\Omega}{\Omega + \frac{a_{Ki}}{a_{Li}}} \tag{4}$$

The revenue shares for a good must sum to one

$$\theta_{Ki} + \theta_{Li} = \frac{ra_{Ki}}{P_i} + \frac{wa_{Li}}{P_i} = 1 \tag{5}$$

and by the definition are each positive and strictly less than one $0 < \theta_{ji} < 1$.

Lemma 7 *Iff cloth is relatively capital intensive* $a_{KC}/a_{LC} > a_{KW}/a_{LW}$*, then the share of revenue that goes to labor is higher in wheat than cloth* $\theta_{LW} > \theta_{LC}$*, and similarly the share of revenue that goes to capital is higher in cloth than wheat* $\theta_{KC} > \theta_{KW}$*.*

Proof. The labor result follows directly from the definition of revenue shares (4)

$$\frac{a_{KC}}{a_{LC}} > \frac{a_{KW}}{a_{LW}} \rightarrow \frac{\Omega}{\Omega + \frac{a_{KW}}{a_{LW}}} > \frac{\Omega}{\Omega + \frac{a_{KC}}{a_{LC}}} \rightarrow \theta_{LW} > \theta_{LC}$$

and the capital result follows from the property that the revenue shares must sum to one (5)

$$\theta_{LW} > \theta_{LC} \rightarrow 1 - \theta_{LW} < 1 - \theta_{LC} \rightarrow \theta_{KC} > \theta_{KW}$$

■

2.4 Production Equilibrium

The point of intersection of the two isocost curves represents an equilibrium in which both goods are produced. The intersection then determines the two factor prices based on the prices of the two goods. This relationship will be important for the Stolper-Samuelson result to follow.

Definition 8 *The factor-price frontier is the outer frontier traced by the two isocost curves and denotes the possible equilibria.*

At the kink (point A), both goods are produced whereas for any other point, only one of the two goods is produced since for the other good, price is below marginal cost. Factor intensity reversals allow the possibility of more than one kink.

Definition 9 *Define* $\lambda_{Li} \equiv \frac{L_i}{L}$ *as the fraction of the labor supply used in sector* i *and similarly* $\lambda_{Ki} \equiv \frac{K_i}{K}$ *as the fraction of the capital stock used in sector* i*.*

Summing the fraction of the labor supply used across the two sectors must sum to one as the total labor supply must be fully employed by the labor constraint $L_C + L_W = L$,

$$\lambda_{LC} + \lambda_{LW} = \frac{L_C}{L} + \frac{L_W}{L} = \frac{L}{L} = 1 \tag{6}$$

and similarly for capital $\lambda_{KC} + \lambda_{KW} = 1$. By definition, the factor shares are each positive and strictly less than one $0 < \lambda_{ji} < 1$.

Lemma 10 *If cloth is relatively capital intensive* $a_{KC}/a_{LC} > a_{KW}/a_{LW}$*, then the fraction of the capital stock used in cloth production is greater than the fraction of the labor supply used in cloth production* $\lambda_{KC} > \lambda_{LC}$ *and similarly the fraction of the labor supply used in wheat production is greater than the fraction of the capital stock used in wheat production* $\lambda_{LW} > \lambda_{KW}$*.*

Proof. Cloth being relatively capital intensive implies that the capital-to-labor ratio in the cloth sector must exceed that in the factor endowments, which yields the result for cloth.

$$\begin{aligned} \frac{a_{KC}}{a_{LC}} &> \frac{a_{KW}}{a_{LW}} \rightarrow \frac{a_{KC}}{a_{LC}} > \frac{K}{L} \rightarrow \frac{K_C}{L_C} > \frac{K}{L} \\ &\rightarrow \frac{K_C}{K} > \frac{L_C}{L} \rightarrow \lambda_{KC} > \lambda_{LC} \end{aligned}$$

Using that the factor shares must sum to one $\lambda_{LW} = 1 - \lambda_{LC}$ and $\lambda_{KW} = 1 - \lambda_{KC}$ from (6) gives the result for wheat

$$\lambda_{KC} > \lambda_{LC} \rightarrow 1 - \lambda_{KC} < 1 - \lambda_{LC} \rightarrow \lambda_{LW} > \lambda_{KW}$$

$\blacksquare$

Lemma 11 *An equilibrium where both goods are produced is possible if the capital-to-labor ratios used in the two sectors in such an equilibrium span the capital to labor ratio in the economy as a whole.*

$$\frac{a_{KC}}{a_{LC}} > \frac{K}{L} > \frac{a_{KW}}{a_{LW}}$$

Proof. Dividing the capital constraint $K = K_C + K_W$ by L,

$$\frac{K}{L} = \frac{K_C}{L} + \frac{K_W}{L}$$

and multiplying each term on the right-hand-side (RHS) by $\frac{L_i}{L_i}$,

$$\begin{aligned} \frac{K}{L} &= \frac{L_C}{L_C}\frac{K_C}{L} + \frac{L_W}{L_W}\frac{K_W}{L} \\ &\rightarrow \frac{K}{L} = \frac{L_C}{L}\frac{K_C}{L_C} + \frac{L_W}{L}\frac{K_W}{L_W} \end{aligned}$$

using the definition of $\lambda_{Li} = \frac{L_i}{L}$

$$\frac{K}{L} = \lambda_{LC}\frac{K_C}{L_C} + \lambda_{LW}\frac{K_W}{L_W}$$

and finally substituting $\frac{K_C}{L_C} = \frac{a_{Ki}}{a_{Li}}$

$$\frac{K}{L} = \lambda_{LC}\frac{a_{KC}}{a_{LC}} + \lambda_{LW}\frac{a_{KW}}{a_{LW}}$$

have that the capital-to-labor ratio in the economy as a whole (the economy's factor endowment ratio) is a weighted sum of the capital-to-labor ratios in the two sectors when both goods are produced.

Thus, we can envision a graph of capital versus labor with two rays from the origin (Figure A.15). The first ray has slope $\frac{a_{KW}}{a_{LW}}$ representing the capital-to-labor ratio in the wheat sector at the equilibrium (slope of C_C at point A in Figure A.9). The second ray has slope $\frac{a_{KC}}{a_{LC}}$ representing the capital-to-labor ratio in the cloth sector at the equilibrium. The ray for the cloth sector will lie everywhere above the ray for the wheat sector if cloth is relatively capital intensive $\frac{a_{KC}}{a_{LC}} > \frac{a_{KW}}{a_{LW}}$. ■

Definition 12 *These two rays reflecting the capital-to-labor ratio in the two sectors form a factor endowment cone.*

The factor endowment is represented by a third ray from the origin with slope $\frac{K}{L}$. Iff the factor endowment lies in the cone $\frac{a_{KC}}{a_{LC}} > \frac{K}{L} > \frac{a_{KW}}{a_{LW}}$, then both goods are produced. If the endowment lies outside the cone, only one of the two goods will be produced (cloth if $\frac{K}{L} > \frac{a_{KC}}{a_{LC}}$ and wheat if $\frac{a_{KW}}{a_{LW}} > \frac{K}{L}$). This concept of a factor endowment cone will be important for the factor price equalization result to follow.

2.5 Resource Constraints

Lemma 13 *If cloth is relatively capital intensive compared to wheat production* $a_{KC}/a_{LC} > a_{KW}/a_{LW}$*, then the capital resource constraint is flatter than the labor resource constraint.*

Proof. The resource constraints dictate that the demand for each factor must equal its supply

$$a_{LC}(w,r)S_C + a_{LW}(w,r)S_W = L$$

$$a_{KC}(w,r)S_C + a_{KW}(w,r)S_W = K$$

If factor prices remain constant, then the least cost production technique remains unchanged. Differentiating

$$a_{LC}dS_C + a_{LW}dS_W = dL$$

$$a_{KC}dS_C + a_{KW}dS_W = dK$$

and thus the slope of each resource constraint is

$$\frac{dS_C}{dS_W} = -\frac{a_{KW}}{a_{KC}}, \; \frac{dS_C}{dS_W} = -\frac{a_{LW}}{a_{LC}}$$

So indeed, the capital constraint is flatter than the labor constraint when cloth is relatively capital intensive compared to wheat production

$$\frac{a_{KW}}{a_{KC}} < \frac{a_{LW}}{a_{LC}} \longleftrightarrow \frac{a_{KC}}{a_{LC}} > \frac{a_{KW}}{a_{LW}}$$

Thus, relative factor intensity can be determined by the slope of the resource constraints as in Figure A.11. ■

The intersection of the two resource constraints determines the output of the two goods based on the supply of the two factors. This intersection will be important for the Rybczynski result to follow.

3 Main Results for Simple HOS Model

3.1 Stolper-Samuelson

How do changes in the price of goods affect the prices of factors?

Theorem 14 *(Simple Stolper-Samuelson) A small increase in the relative price of a good will increase, in terms of the price of both goods, the price of the factor used intensively in producing the good whose relative price has risen and will decrease, in terms of the price of both goods, the price of the other factor, provided both goods are initially produced.*

Proof. Due to perfect competition and the assumption that cloth is initially produced, the price of cloth equals the cost of producing cloth

$$P_C = wa_{LC}(w,r) + ra_{KC}(w,r)$$

Suppose the price of cloth increases; differentiating

$$dP_C = a_{KC}dr + a_{LC}dw + \underbrace{[rda_{KC} + wda_{LC}]}_{0}$$

implies (dividing through by P_C)

$$\frac{dP_C}{P_C} = \frac{a_{KC}}{P_C}dr + \frac{a_{LC}}{P_C}dw$$

and implies (multiplying by $\frac{r}{r} = \frac{w}{w} = 1$)

$$\frac{dP_C}{P_C} = \frac{a_{KC}r}{P_C}\frac{dr}{r} + \frac{a_{LC}w}{P_C}\frac{dw}{w}$$

which can be written (letting $\widehat{z} \equiv \frac{dz}{z}$ represent the proportional change in any variable z)

$$\widehat{P}_C = \theta_{KC}\widehat{r} + \theta_{LC}\widehat{w} \tag{7}$$

where $\theta_{KC} \equiv \frac{ra_{KC}}{P_C}$ is the proportion of cloth revenue that goes to capital and similarly $\theta_{LC} \equiv \frac{wa_{LC}}{P_C}$ is the proportion of cloth revenue that goes to labor. Therefore, the proportional change in the price of cloth equals a weighted sum of the proportional change in both factor prices. Similarly, the proportional change in the price of wheat equals a weighted sum of the proportional change in both factor prices

$$\widehat{P}_W = \theta_{KW}\widehat{r} + \theta_{LW}\widehat{w} \tag{8}$$

Since all $0 < \theta_{ij} < 1$, must be that $\widehat{P}_C$ and $\widehat{P}_W$ lie in between $\widehat{r}$ and $\widehat{w}$. Since $\theta_{KC} > \theta_{KW}$ and $\theta_{LW} > \theta_{LC}$, $\widehat{P}_C > \widehat{P}_W$ implies $\widehat{r} > \widehat{w}$. Thus, the change in the relative factor price is magnified relative to the change in the relative price of the two goods

$$\widehat{r} > \widehat{P}_C > \widehat{P}_W > \widehat{w}$$

■

In Figure A.10, start from an equilibrium at point A where the cloth isocost $C_C = P_C$ intersects the wheat isocost $C_W = P_W$. Then, the cloth isocost shifts out to $C_C = P'_C$, reflecting the new higher price of cloth. The point of intersection moves down along the wheat isocost to point A'. At the new equilibrium, the rent is higher and the wage is lower due to the increase in the price of cloth.

The Stolper-Samuelson theorem determines the impact of international trade on the domestic distribution of income. The free trade relative price lies in between the two autarkic relative prices; therefore, each country experiences an increase in the relative price of its comparative advantage good. The real income of the scarce factor suffers whereas the real income of the abundant factor increases. In a labor abundant country, capital should oppose the movement from autarky to free trade (if no transfers compensate for losses) and in a capital abundant country, labor should oppose. The essence of the Stolper-Samuelson theorem is that international trade creates a conflict among domestic factors.

3.2 Rybczynski

How do changes in factor endowments affect the production mix of goods?

Theorem 15 *(Simple Rybczynski): At constant prices, a small increase in the relative endowment of one factor causes the output of the good making relatively intensive use of the factor that has become relatively more abundant to rise relative to both factors and the output of the other good to fall relative to both factors, provided the economy remains diversified.*

Proof. Because the economy remains diversified, and by the HOS assumption of full employment, the capital and labor constraints bind

$$a_{KC}(w,r)S_C + a_{KW}(w,r)S_W = K$$

Differentiate (with a's fixed as relative factor prices constant so technology choice constant)

$$dK = a_{KC}dS_C + a_{KW}dS_W$$

Divide by the capital supply

$$\frac{dK}{K} = \frac{a_{KC}}{K}dS_C + \frac{a_{KW}}{K}dS_W$$

and multiply by $\frac{S_C}{S_C} = \frac{S_W}{S_W} = 1$ to achieve

$$\frac{dK}{K} = \frac{a_{KC}S_C}{K}\frac{dS_C}{S_C} + \frac{a_{KW}S_W}{K}\frac{dS_W}{S_W}$$

or equivalently (using the fraction of the economy's capital stock employed in good i as weights) the proportional change in the capital stock is a weighted average of the proportional changes in production of the two goods

$$\widehat{K} = \lambda_{KC}\widehat{S}_C + \lambda_{KW}\widehat{S}_W \qquad (9)$$

Similarly, the proportional change in the labor supply is also a weighted average of the proportional changes in production of the two goods

$$\widehat{L} = \lambda_{LC}\widehat{S}_C + \lambda_{LW}\widehat{S}_W \qquad (10)$$

Since all $0 < \lambda_{ji} < 1$, must be that $\widehat{K}$ and $\widehat{L}$ lie in between $\widehat{S}_C$ and $\widehat{S}_W$. Since $\lambda_{KC} > \lambda_{LC}$ and $\lambda_{LW} >$

λ_{KW}, $\widehat{K} > \hat{L}$ implies $\widehat{S}_C > \widehat{S}_W$. Thus the change in output is magnified relative to the change in the factor endowment

$$\widehat{S}_C > \widehat{K} > \hat{L} > \widehat{S}_W$$

■

In Figure A.11, start from equilibrium at point A where capital constraint K intersects the labor constraint L. Then the capital constraint shifts out to K', reflecting the new larger endowment of capital. The point of intersection moves up along the labor constraint to point C. At the new equilibrium, the supply of cloth is higher and the supply of wheat is lower due to the increase in the endowment of capital.

Note that the theorem holds only for constant commodity prices even though outputs of the two goods change, so it is (strictly speaking) applicable only to a small open economy that does not affect world prices.

3.3 Factor Price Equalization

Theorem 16 *(Simple Factor Price Equalization) If two countries have similar factor endowments (or produce both goods and no factor intensity reversals), free trade equalizes factor prices in the two countries.*

Proof. If the production technology is the same in the two countries and trade equalizes the prices of goods, then it must equalize prices of factors as well for costs of production to be the same in the two countries.

- With free trade, countries face identical prices of each good.

- With identical technologies, countries have the same isocost curve for each good.

- Due to perfect competition, the price of each good must equal the marginal cost of production if both goods are produced.

- Each country produces both goods if the two countries have sufficiently similar factor endowments. To produce both goods, a country's capital to labor ratio must lie between the capital-to-labor ratio in the production of each good (see Figure A.15), as indicated by the slope of the two isocost curves at the point of intersection (see Figure A.9). The capital-to-labor ratio must lie in between the ratio in the two sectors, as under diversification the country's capital to labor ratio is a weighted sum of the ratios in the two sectors.

- If no factor intensity reversal, then the intersection of the two isocost curves is unique. If one or more factor intensity reversals exist, sufficiently similar factor endowments do still achieve FPE, but diversification is not sufficient since one country could be at one intersection and the other country at another (see Figure 9b).

Thus factor prices are determined by the unique intersection of the same two isocost curves and achieve factor price equalization. Countries have sufficiently similar factor endowments if each country's factor endowment ratio lies in the cone in the capital-labor plane bounded by the capital-to-labor ratios in the two sectors. ■

In Figure A.9, if countries remain diversified and no FIR, unique equilibrium is at point A. There is a cone in the capital-labor plane as in Figure A.15 such that all countries with endowments in the cone will remain diversified. If a FIR exists, then there would be multiple cones, one cone ensuring reach equilibrium A and one ensuring reach equilibrium A' (in Figure A.9b). Again sufficiently similar countries will achieve FPE, but in some sense it is harder (they must be more similar) when factor intensity reversals exist.

An interesting implication of the FPE theorem is that trade in goods substitutes for factor mobility. In the HOS model, in effect, the countries are trading bundles of factors of production when they trade goods. Trade in goods takes the place of trade in factors because most goods are easier to trade than most factors.

3.4 Heckscher Ohlin

A country is relatively capital abundant in the quantity sense if its capital to labor ratio exceeds that of the rest of the world $\frac{K}{L} > \frac{K^*}{L^*}$. A country is relatively capital abundant in the price sense if in autarky its wage rental ratio exceeds that of the rest of the world $\left(\frac{w}{r}\right)^A > \left(\frac{w^*}{r^*}\right)^A$. The first definition is easier to apply since we never observe countries in autarky so we have no way of knowing what their factor prices would be in isolation. If the ratio of capital to labor is higher in one country, by Rybczynski theorem, its relative output of cloth must be higher; given identical preferences and world market clearing, the capital abundant country must export cloth.

Theorem 17 *(Simple Heckscher-Ohlin, Quantity Version): Suppose two countries with identical homothetic demands, identical technologies of production and not separated by a FIR engage in free trade. Each country exports the good that makes relatively intensive use of its relatively abundant factor (in the quantity sense).**

Proof. With identical relative prices due to free trade and identical homothetic preferences by assumption, the two goods are consumed in the same proportion in the two countries (and the world).

$$\frac{D_C}{D_W} = \frac{D_C + D_C^*}{D_W + D_W^*} = \frac{D_C^*}{D_W^*}$$

Suppose the home country is relatively capital abundant (in the quantity sense) $\frac{K}{L} > \frac{K^*}{L^*}$. By the Rybczynski Theorem, the home country will produce a larger relative supply of the capital intensive good

$$\frac{S_C}{S_W} > \frac{S_C^*}{S_W^*}$$

* *In this case, demand for wheat relative to cloth depends only the relative prices and is independent of income, thus allowing us to determine world relative demand in a simple fashion.*

In equilibrium, world relative demand must equal world relative supply

$$\frac{D_C + D_C^*}{D_W + D_W^*} = \frac{S_C + S_C^*}{S_W + S_W^*}$$

Putting these inequalities together yields

$$\frac{S_C}{S_W} > \frac{D_C}{D_W} = \frac{S_C + S_C^*}{S_W + S_W^*} = \frac{D_C^*}{D_W^*} > \frac{S_C^*}{S_W^*}$$

which indicates that home exports cloth and imports wheat (first inequality) and foreign exports wheat and imports cloth (last inequality). ■

Theorem 18 *(Simple Heckscher-Ohlin, Price Version): Suppose two countries with identical technologies of production and no factor intensity reversals engage in free trade. Each country exports the good that makes relatively intensive use of its relatively abundant factor (in the price sense).*

The wage-to-rent ratio increases in the capital-to-labor ratio. Also, the wage to rent ratio decreases in the relative demand for cloth to wheat. A country then has comparative advantage in the good that intensively uses the abundant factor (abundant in the price sense). Note that a country could be capital abundant in the quantity sense but labor abundant in the price sense if it has a stronger relative demand for the capital intensive good (cloth) than the other country. The autarky factor prices incorporate demand information, so the price version has less stringent assumptions on demand. However, the down side of the price version is that we do not usually observe autarky prices.

4 Confrontation with Reality

Let us see how the major implications of the traditional trade theory stack up against the empirical evidence.

1. Since trade is based on differences in factor endowments across countries, it should be greatest between countries with greatest differences in economic structure, that is, between developed and less developed countries. DC-DC trade is the largest proportion of world trade.

2. The gains from trade should be greatest between countries with greatest differences in economic structure. LDCs have traditionally opposed free trade.

3. Trade should cause more specialization in production and cause countries to export goods distinctly different from their imports. Simultaneous expansion of

all sectors in all countries; relatively little specialization. Much trade is intraindustry, with import and export goods having essentially the same factor content.

4. The HO Theorem states that countries should export goods that make relatively intensive use of their relatively abundant factor. Since the United States is relatively capital abundant (compared to the rest of the world), the United States should export capital-intensive goods. Leontief paradox: HO Theorem empirically rejected for the United States - the United States imports capital-intensive goods! The Leontief Paradox has generated a voluminous literature: it robustly rejects such an intuitively plausible result. There are three main responses to the Leontief paradox:

   - The first explanation (demand reversal) relaxes the assumption of identical tastes across countries. A country could have a higher autarkic price for a good despite having relative abundance in the factor used relatively intensively in producing the good if local consumers have a substantial bias in favor of the local good.

- The second explanation acknowledges the possibility of a factor intensity reversal. Suppose the technology of production for cloth is flexible – easy to substitute capital for labor and the appropriate factor mix varies with factor prices whereas the technology for wheat allows no substitution and requires the usage of capital and labor in fixed proportions (Leontief technology). cloth could be capital intensive in the capital abundant country but labor intensive in the labor abundant country. The HO theorem would require both countries to export cloth, so the HO theorem must be violated for one of the countries. In other words, if a FIR separates the United States from the rest of the world due to its extreme capital abundance combined with inflexible technology, the United States may export wheat instead of cloth, a violation of the HO theorem.

- The third explanation allows for a large number of factors of production. For example, many capital intensive goods maybe intensive in natural resources, land or human capital. A country that seems to be importing the capital intensive good may actually be importing goods intensive in some other complementary factor.

5. FPE survives relatively well. Similarly developed countries have roughly equal factor prices. Strong indication that international trade causes factor prices to converge at least partially.

6. Opposition to free trade should depend upon the identity of the factor providing income rather than on the identity of the industry of employment. Lobbying, unions and labor strikes are organized on the basis of industry identity. The misalignment of protectionism – a particular industry lobbies rather than labor from all industries - can be fixed by considering the Specific Factors model, a sort of short-run version of the HOS model.

7. International investment should be stimulated by differences in factor endowments. The incentive to undertake international investment arises iff FPE does not hold, which happens when factor endowments differ substantially across countries. Most of foreign direct investment takes place between developed countries.

8. International trade brings about FPE, while international investment arises to exploit differences in factor prices across countries. Thus, international trade and international investment should be negatively correlated. Both international investment and international trade take place between developed countries, where factor prices are essentially equal. High volumes of international trade appear to occur together with high levels of international investment.

The overall lesson is that the traditional factor endowments based theory is excellent for explaining trade in a world comprised of countries that differ radically in their factor endowments, but not for a world that involves exchange of similar products between similar (industrialized) countries. The $2 \times 2 \times 2$ structure's insights survive a general setting, although in a considerably weaker sense.

However, traditional trade theory leaves unexplained a number of significant topics:

- intraindustry trade (importing and exporting essentially the same products, at least in terms of factor content)

- foreign direct investment, especially two-way FDI

- impact of international trade and FDI on economic growth

Intraindustry trade and FDI are substantial between similarly developed countries. Without an explanation of these phenomenon, international trade theory was substantially incomplete. The desire to fill these holes in our knowledge gave rise to the blossoming of new trade theory. The new trade theory differs from the old in allowing for product differentiation, IRS, market power, etc., that give rise to intraindustry trade (or FDI) even between ex ante identical countries. But before we turn to the new, we finish the old.

5 Specific Factors Model

Retain the basic structure of the HOS model but introduce the following assumptions:

- There are two kinds of capital, K_W and K_C (in addition to labor), and each is specific to a particular industry. Let returns to capital be denoted by r_W and r_C respectively. Since each type of capital is specific to one industry, these returns to capital can differ across industries.

- Let $k_i \equiv \frac{K_i}{L_i}$ denote the ratio of capital to labor employed in sector i. The conditions for equilibrium in the two capital markets are

  $$VMPK_i = P_i f^{'}(k_i) = r_i,\ i \in \{C, W\}$$

  The condition for equilibrium in the labor market is

  $$VMPL_i = P_i(f(k_i) - k_i f^{'}(k_i)) = w$$

Since labor is mobile across sectors, there is just one wage w, not different wages in each sector. Figure 6.1 illustrates the equilibrium wage and division of labor between the two sectors, as required to equalize the value of the marginal product of labor across the sectors.

5.1 Results in the Specific Factor Model

Here we examine whether the Stolper-Samuelson, FPE, Rybczynski and HO-type results hold if factors are specific.

1. Stolper-Samuelson: Suppose price of wheat increases and price of cloth is constant. As seen in Figure 6.3, the higher price of wheat shifts up the $VMPL_W$, the labor demand in the wheat sector. The wage goes up but not by as much as the increase in price

of wheat: $\widehat{P}_W > \widehat{w} > 0$. If workers spend enough of their income on wheat, the purchasing power of their earnings could fall, which is not consistent with the Stolper-Samuelson result. As for the two kinds of capital, one loses and one gains (capital specific to the sector which has had a price increase gains). How do we know that return to specific factor increases relative to the price of the good itself? As in the HOS model, because the price of each good is a weighted average of the wage and the rental rate to the specific factor. Thus if the wage has increased by less than the price, the return to the specific factor must have increased by more: $\widehat{r}_W > \widehat{P}_W$. Similarly, the return to the other capital must have fallen if price of cloth has not changed but wage has gone up: $0 < \widehat{r}_C$. An increase in the relative price of a good increases the real income of the factor specific to that industry, reduces the real income of the other specific factor, and has an ambiguous effect on the mobile factor.

2. Factor Price Equalization: There are three factor prices and only two goods. We need international trade in at least as many goods as there are factors (immobile across borders) to get factor price equalization. Since there are more factors than goods here, Factor Price Equalization not guaranteed here and in general will not hold.

3. Rybczynski: Suppose the endowment of cloth-specific capital K_C increases. As seen in Figure 6.2, the additional capital increases the marginal product of labor in cloth production. Labor moves from the wheat to the cloth sector. The wage is higher in the new equilibrium. The output of cloth increases while the output of wheat falls. This effect is consistent with the Rybczynski effect for the HOS model (as would be an increase in wheat-specific capital). Suppose instead the endowment of labor L increases. The output of both goods increases – not consistent with the Rybczynski result in the HOS model. An increase in

the endowment of a specific factor increases the output of the good that uses it and decreases the output of the other good, but an increase in the nonspecific factor increases the output of both goods.

4. Heckscher-Ohlin: Assuming identical stocks of the mobile factor labor, each country exports the good that is relatively intensive in its relatively abundant specific factor. If stocks of the mobile factor differ, we cannot make an unambiguous claim about the pattern of trade without additional assumptions. Having the trade pattern reflect specific factor endowments requires symmetric endowments of the mobile factor.

Essentially, were it not for the mobile factor, the HOS-type results would go through. The key gain is that the group that is harmed by free trade, the owners of the relatively scarce specific factor, are indeed employed within an industry. If it takes long enough for factors to become mobile across sectors, then the specific factors model provides a better description of who within an economy will object to free trade.

6 Higher Dimensional HOS Model

- Consider a world of n goods and m factors.

- Retain all other assumptions of the HOS model.

- Two approaches the literature to generalize the results for the $2 \times 2 \times 2$ version of the HOS model.

- The first is to hold results (nearly) constant and determine assumptions required to recover basic HOS results.

- The second is to hold assumptions (nearly) constant and determine what results emerge with no (or few) additional assumptions.

- The first approach led to discovery of some technical conditions with little or no chance of being satisfied (requires the economy to essentially have a 2×2 structure).

- The second approach proved to be more insightful and not too damaging to the spirit of the HOS theory. We will concentrate on the second approach.

6.1 FPE Theorem

- Start with the FPE – will free trade in goods equalize factor prices when there are many goods and many factors?

Theorem 19 *(General Factor Price Equalization) Suppose several countries are alike in every way except factor endowments and engage in free trade. Let K_W be the cone of factor endowments spanned by the columns of $A(W)$. All countries with factor endowments in the cone K_W will have factor price equalization.*

Proof. Suppose that the number of goods n and number of factors m are equal: $m = n$ (often called the even case). Perfect competition requires that price equal cost

$$p = WA(W)$$

where p is the (n dimensional) vector of prices, W is the (m dimensional) vector of factor prices, and $A(W)$ is the matrix of optimal production techniques

$$A(W) \equiv \begin{bmatrix} a_{11}(W) & \dots & a_{1n}(W) \\ a_{m1}(W) & \dots & a_{mn}(W) \end{bmatrix}$$

given factor prices W. If all goods are produced in each country, we must have full employment. The resource constraint equates factor endowments with factor demands

$$K = A(W)S$$

where K is the (m dimensional) vector of factor endowments and S is the (n dimensional) vector of commodity outputs. The above equation gives us the vector of endowments required to produce the outputs in S. Let K_W denote the collection of all vectors that solve the preceding equation for a given S with non-negative components (to rule out a negative endowment of any factor). Clearly, if factor endowments lie in K_W, both countries can produce all goods at factor prices W, so FPE is possible. We still must show that FPE must indeed hold. Suppose not: that there is a second set of factor prices W' as well such that

$$p = W'A(W')$$

and

$$K = A(W')S'$$

$A(W)$ is the least cost technique at factor prices W but $A(W)$ is not the least cost technique at the other factor prices W'. Thus, using the technique $A(W)$ at factor prices W must be less costly than using technique $A(W)$ at factor prices W'.

$$WA(W) < W'A(W) \tag{11}$$

Similarly, using the technique $A(W')$ at factor prices W' must be less costly than using technique $A(W')$ at factor prices W.

$$W'A(W') < WA(W') \tag{12}$$

Post-multiplying both sides of the inequality (11) by S yields

$$WK < W'K$$

while doing the same (using S') to the second inequality (12) yields

$$WK > W'K$$

a contradiction. If factor endowments lie in K_W, factor prices must equal W. Hence sufficient similarity in factor endowments still gives FPE for the general case of $n = m > 2$. ■

When more goods than factors $n > m$ we do not run into any problem with FPE but when more factors than goods $n < m$, FPE is quite unlikely (would be a fluke). In general (where some goods are nontraded and some factors are traded), we need at least as many traded goods and traded factors as nontraded factors for FPE to hold.

6.2 Stolper Samuelson Theorem

- We could prove that the price of goods is correlated with the price of factors

$$(\Delta W)\overline{A}(\Delta P) > 0$$

  so that changes in factor prices are positively correlated with changes in goods prices. On average, the Stolper-Samuelson result holds, but it may fail for a given factor price.

- We can also prove that the $2 \times 2 \times 2$ type results still hold for 2 factors, but now there may be many more than 2 factors, so many factors that we are not saying anything about (like adding water to wine – eventually you lose the flavor).

- Thus, the essence of the Stolper Samuelson theorem remains: changes in factor prices cause conflict – the real income of some factor increases while the real income of some other factor decreases.

- Each good is friend of some factor and enemy of another.

Theorem 20 *(General Stolper-Samuelson) A rise in a single commodity price will cause the price of some factor to rise in terms of the price of all goods, and will cause the price of some other factor to fall, provided only that the good whose price has risen is initially produced and that every factor the good employs is also subsequently employed elsewhere in the economy.*

Proof. Consider an increase in the price of a good (call it good 1): $\widehat{P}_1 > 0$. As good 1 was initially produced (by assumption), its price can have increased by no more than its costs (whether or not good 1 remains produced)[†]

$$0 < \widehat{P}_1 \leq \widehat{C}_1 = \theta_1 \widehat{W}$$

[†]As good 1 was initially produced, $P_1 = C_1$. If good 1 remains produced, then $P_1' = C_1'$ and price increased the same as cost increased $\widehat{P}_1 = \widehat{C}_1$. If good 1 is no longer produced, then $P_1' < C_1'$ and price increased by less than costs increased $\widehat{P}_1 < \widehat{C}_1$.

where $\theta_1 \equiv [\theta_{11}, \ldots, \theta_{m1}]$ is vector of the share of revenues going to each factor and thus $\sum_{j=1}^{m} \theta_{j1} = 1$. The increase in cost is a weighted average of the changes in factor prices. Consequently, at least one factor (call it factor 1) used to produce good 1(so $\theta_{11} > 0$) must have had its price increase by at least as much as the increase in the price of good 1 to absorb the price increase.

$$\widehat{W}_1 > \widehat{P}_1 > 0$$

An increase in the price of good 1 causes the price of some factor to rise in at least equal proportion. Next consider some other good (call it good 2) whose price has not changed $\widehat{P}_2 = 0$. By assumption, there must be some other good such as good 2 that is subsequently produced and also uses factor 1 (so $\theta_{12} > 0$), the factor whose price rises. As good 2 is subsequently produced, its price must have increase by at least as much as its costs (whether or not good 2 was initially produced)[‡]

$$0 = \widehat{P}_2 \geq \widehat{C}_2 = \theta_2 \widehat{W}$$

[‡]As good 2 remains produced, $P_2' = C_2'$. If good 2 was also initially produced, then $P_2 = C_2$ and price increased the same as costs increased $\widehat{P}_2 = \widehat{C}_2$. If good 2 was not initally produced, then $P_2 < C_2$ and price increased by more than costs increased $\widehat{P}_2 > \widehat{C}_2$.

Since the price of factor 1 has increased, the price of some factor 2 also used to produce good 2 (so $\theta_{22} > 0$) must fall for price of good 2 to remain constant:

$$\widehat{W}_2 < 0 < \widehat{P}_1 < \widehat{W}_1$$

So an increase in the price of an initially produced good using a non-specific assortment of at least two factors causes some factor price to rise in greater proportion and some factor price to fall. ■

6.3 Rybczynski Theorem

- Similarly, we could prove that factor endowment changes are correlated with production changes

$$(\Delta K)A(\Delta S) > 0$$

so that changes in factor endowments are positively correlated with changes in outputs. On average, the Rybczynski result holds, but it may fail for a given output of a good.

- We can also prove that the $2 \times 2 \times 2$ type results still hold for 2 goods, but now there may be many more than 2 goods, so many goods that we are not saying anything about (again like adding water to wine, eventually you lose the flavor).

Theorem 21 *(General Rybczynski) At constant prices, an increase in any factor endowment will cause the output of some good to rise in terms of all factors and will cause the output of some other good to fall, provided that the factor is subsequently fully employed and that industries using it also use another factor that was initially fully employed.*

Proof. Consider an increase in the endowment of a factor (call it factor 1): $\widehat{K_1} > 0$. We assume that the techniques if production do not respond to changes in factor endowments (that is, we hold factor prices fixed). As factor 1 is subsequently fully employed (by assumption), the supply of the factor can have increased no more than its demand (whether or not factor 1 was initially fully employed).[§]

$$0 < \widehat{K_1} \leq \lambda_1 \hat{S}$$

[§]As factor 1 is subsequently fully employed $K_1' = \lambda_1 S_1'$. If factor 1 was initially fully employed, then $K_1 = \lambda_1 S_1$ and factor endowment increased the same as factor demand increased $\widehat{K}_1 = \lambda_1 \widehat{S}_1$. If factor 1 was initially not fully employed, then $K_1 > \lambda_1 S_1$ and factor endowment increased less than factor demand increased $\widehat{K}_1 < \lambda_1 \widehat{S}_1$.

where $\lambda_1 \equiv [\lambda_{11}, \ldots, \lambda_{1n}]$ is vector of the share of factor supply going to each sector and thus $\sum_{i=1}^{n} \lambda_{1i} = 1$. The increase in factor endowment is a weighted average of the changes in the output of the goods. Consequently, at least one good (call it good 1) that uses factor 1 (so $\lambda_{11} > 0$) must have its output increase by at least as much as the factor endowment has increased to absorb the increased availability of the factor.

$$\widehat{S}_1 > \widehat{K}_1 > 0$$

An increase in the endowment of factor 1 causes the output of some good to rise in at least equal proportion. Next consider a factor (call it factor 2), whose endowment has not changed $\widehat{K}_2 = 0$. By assumption, there must be some nonspecific factor such as factor 2 that is initially fully employed and used by sectors that also use factor 1 (with $\lambda_{21} > 0$), the factor whose endowment will have risen. As factor 2 is initially fully employed, the endowment of factor 2 must have risen by at least as much as the demand for factor 2 (whether or not it remains fully employed)[¶]

$$0 = \widehat{K_2} \geq \lambda_2 \widehat{S}$$

Since the output of good 1 has increased, the output of some good 2 also using factor 1 ($\lambda_{12} > 0$) and some factor 2 (with $\lambda_{22} > 0$) must fall for demand for factor 2 to remain constant (since endowment of factor 2 is unchanged):

$$\widehat{S}_2 < 0 < \widehat{K_1} < \widehat{S}_1$$

So at constant factor prices, an increase in the endowment of a non-specific factor used in at least two sectors that leaves the factor fully employed produces a more than proportional rise in the output of some good and a fall in the output of some other good. ■

[¶] As factor 2 is initially fully employed, $K_2 = \lambda_2 S_2$. If factor 2 remains fully employed, then $K_2' = \lambda_2 S_2'$ and the factor enndowment increased by the same as the factor demand increased $\widehat{K}_2 = \lambda_2 \widehat{S}_2$. If factor 2 is subsequently not fully employed, then $K_2' > \lambda_2 S_2'$ and the factor endowment increased by more than the factor demand increased $\widehat{K}_2 > \lambda_2 \widehat{S}_2$.

6.4 Heckscher Ohlin Theorem

- We have already proved that imports are correlated with high autarkic prices of goods when we discussed the general version of comparative advantage in Chapter 2.

- Here, we are interested in making a link between factor endowments or autarkic factor prices and imports (or exports).

- Both the price version and the quantity version of the HO theorem generalize as correlations.

- For the quantity version (relating imports to factor scarcity), we could prove that on average, an economy tends to import those goods that make relatively intensive use of its relatively scarce factor. Again, we omit the proof for brevity.

- Instead, we focus on the price version of the HO theorem (relates imports to autarkic factor prices).

Theorem 22 *(General Heckscher-Ohlin, Price Version) Autarkic factor prices are positively correlated with the factor content of imports: countries tend to import those factors (through trade in goods) that are relatively expensive in autarky.*

Proof. Define the free trade import vector by $M^T = D^T - S^T$. Let $M_K = \overline{A}M$ denote the factors used to produce M, where the columns of $\overline{A}$ reflect the technique of the country that exports the good. Thus, M_K represents the implied factor flows due to international trade: goods are composed of factors such that trade in goods would be equivalent to as if M_K trade in factors were to occur.[‖] If instead of using international trade, the home country were to produce the goods it imports from factors $K + M_K$ mimicking the foreign techniques, this technique need not be feasible at autarkic prices

$$W^A(K + M_K) \geq p^A D^T$$

[‖]Negative elements of M_K imply net exports of factors, similar to negative elements of M reflecting exports of goods.

Thus, bringing the $W^A K$ term to the right,

$$W^A M_K \geq p^A D^T - W^A K$$

Now, since there are gains from trade, the country must not be able to afford the free trade consumption bundle D^T at autarky prices, $p^A D^T \geq p^A D^A$, so subtracting $W^A K$ from both sides

$$p^A D^T - W^A K \geq p^A D^A - W^A K$$

In autarky, expenditure must match factor income

$$p^A D^A - W^A K = 0$$

Therefore, collecting equations together, by transitivity

$$W^A M_K \geq 0$$

Similarly, for the rest of the world

$$W^{A*} M_K^* \geq 0$$

As foreign imports equal home exports $M_K = -M_K^*$, we can combine the inequalities

$$(W^A - W^{A*}) M_K \geq 0$$

to demonstrate the positive correlation between factor prices in autarky and the factor content of imports. ■