6 OLIGOPOLY

Traditional trade theory has been developed in models of perfect competition in which firms lack market power and do not act strategically. Models of monopolistic competition allow firms to have a limited degree of market power but rule out strategic interaction by assumption – each firm perceives itself too small to effect the prices of other firms. Oligopoly theory adds a second layer of complication by allowing firms to act strategically. Oligopolistic firms recognize that their decision affects the decisions of their rivals and they recognize that their rivals recognize this and they recognize that their rivals recognize that they recognize this and so on (common knowledge). Our goal is to examine trade under these conditions, but first we develop some basic oligopoly models.

Cournot Model (background)

Consider an industry comprised of n firms, each of which produces a homogenous good. Demand function facing the industry is given by $p(Q)$ where $Q = \sum_i q_i$ is total industry output. Each firms chooses its own output q_i, *taking the output of all its rivals* Q_{-i} ($n-1$ vector) as given, to maximize its profits

$$\pi_i = p(Q)q_i - c(q_i)$$

where $c(\cdot)$ is the cost function for firm i. Firms recognize that they should account for the output decisions of their rivals, yet when making their own decision, they view their rivals' outputs as fixed. Each firm views itself as a monopolist on the *residual demand curve* – the demand left over after subtracting the output of its rivals. The output vector $(q_1 ... q_n)$ is a Cournot Nash equilibrium iff (given Q_{-i})

$$\pi_i(q_i, Q_{-i}) \geq \pi_i(q_i', Q_{-i})$$

for all i. In a Cournot-Nash equilibrium, firms do not have an incentive to unilaterally deviate by altering their output levels: the chosen quantity maximizes the profits of each firm given the quantities chosen by the other firms.

The first order condition (FOC) for firm i is given by

$$\frac{\partial \pi_i}{\partial q_i} = p'(Q)q_i + p(Q) - c_i'(q_i) = 0 \tag{6.1}$$

The Cournot Nash equilibrium is found by simultaneously solving the first order conditions for all n firms. Let $s_i \equiv q_i/Q$ denote firm i's share of industry output in equilibrium. Rewrite the FOC as

$$p(Q)\left[1 + \frac{dp/p}{dQ/Q}\frac{q_i}{Q}\right] = c_i'(q_i)$$

and rewrite again as

$$p(Q)\left[1 - \frac{s_i}{\varepsilon(Q)}\right] = c_i'(Q) \tag{6.2}$$

Suppose marginal cost is constant. The above condition allows some interesting comparisons. For a monopolist $s_i = 1$. For the monopolist, the bracketed term is smaller than for a Cournot firm, and thus price is higher and output is lower than the Cournot equilibrium total output. For a perfectly competitive firm $\varepsilon(Q)$ is infinite. For a perfectly competitive firm, the bracketed term equals exactly 1, so price equals marginal cost, and price is lower and output is higher than in the Cournot equilibrium. Thus, Cournot industry output must fall in between monopoly and perfectly competitive output. Adding together the FOCs for all firms gives

$$np(Q) + p'(Q)Q = \sum_i c_i$$

Thus, Q depends only upon the sum of the marginal costs of production and not upon their distribution. In a symmetric equilibrium, firms have equal market shares $s_i = 1/n$ so that

$$p(Q)\left[1 - \frac{1}{n\varepsilon(Q)}\right] = c$$

To keep exposition simple, we continue with constant marginal cost and two firms.

The FOC for firm 1 is

$$\pi_1^1 \equiv \frac{\partial \pi^1}{\partial q_1} = p'(Q)q_1 + p(Q) - c = 0$$

where $Q \equiv q_1 + q_2$. The above FOC defines firm 1's *reaction function* (R_1): the set of best responses for different output levels of its rivals. We can trace out R_1 simply by varying q_2. Let $q_1(q_2)$ denote firm 1's reaction function. According to the FOC

$$\pi_1^1(q_1(q_2), q_2) \equiv 0$$

Total differentiation gives

$$\pi_{11}^1 \frac{dq_1}{dq_2} + \pi_{12}^1 = 0$$

Solving gives the slope of firm 1's reaction function

$$\frac{dq_1}{dq_2} = -\frac{\pi_{12}^1}{\pi_{11}^1}$$

For the second order condition to hold, must have

$$\pi_{11}^1 = 2p'(Q) + p''(Q)\, q_1 \leq 0$$

Thus, the sign of the slope is the same as the sign of π_{12}^1. If $p'' \leq 0$, the second order condition is automatically satisfied.

When this cross partial is negative $\pi_{12}^1 \leq 0$, we say that q_1 and q_2 are *strategic substitutes* – an increase in the output of firm 2 lowers the marginal profitability of firm 1. Clearly, when the outputs of two firms are strategic substitutes, the reaction functions will be downward sloping. The Cournot assumption by itself does not mean that the reaction functions are downward sloping. From the FOC

$$\pi_{12}^1 = p' + p''q_1 > 2p' + p''q_1 = \pi_{11}^1$$

Thus, the assumption of strategic substitutes $\pi_{12}^1 \leq 0$ implies the second order condition. When $\pi_{12}^1 \geq 0$, then q_1 and q_2 are strategic complements.

The intersection of the two reaction functions gives the equilibrium. The comparative statics properties of the Cournot equilibrium depend upon the sign of a particular condition (the stability condition). Suppose some parameter θ effects the profit functions of both firms. The FOC can be written as a function of this parameter.

$$\pi_1^1(q_1(\theta), q_2(\theta); \theta) \equiv 0$$

Total differentiation of both FOCs yields

$$\pi^1_{11}\frac{dq_1}{d\theta}+\pi^1_{12}\frac{dq_2}{d\theta}+\pi^1_{1\theta}=0$$

where

$$\pi^1_{1\theta}\equiv\frac{\partial^2\pi_1}{\partial q_1\partial\theta}$$

and similarly

$$\pi^2_{21}\frac{dq_1}{d\theta}+\pi^2_{22}\frac{dq_2}{d\theta}+\pi^2_{2\theta}=0$$

The above equations can be written as follows

$$\underbrace{\begin{bmatrix}\pi^1_{11} & \pi^1_{12}\\ \pi^2_{21} & \pi^2_{22}\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\frac{dq_1}{\partial\theta}\\ \frac{dq_2}{\partial\theta}\end{bmatrix}}_{x}=\underbrace{\begin{bmatrix}-\pi^1_{1\theta}\\ -\pi^2_{2\theta}\end{bmatrix}}_{b}$$

We can use Cramer's rule to solve for the elements of x.[1]

$$\frac{\partial q_1}{\partial\theta}=\frac{\begin{vmatrix}-\pi^1_{1\theta} & \pi^1_{12}\\ -\pi^2_{2\theta} & \pi^2_{22}\end{vmatrix}}{|A|}$$

where $|A|=\pi^1_{11}\pi^2_{22}-\pi^1_{12}\pi^2_{21}>0$ by assumption (*stability condition*).[2] Thus the sign of $\frac{dq_1}{d\theta}$ is the same as the sign of $\pi^2_{22}\pi^1_{1\theta}-\pi^1_{12}\pi^2_{2\theta}$. In the special case where θ only directly effects the profit function of firm 1, $\pi^2_{2\theta}=0$ so that only the sign of $\pi^1_{1\theta}$ will matter. Thus, in this case, we can differentiate only the FOC of firm 1 to determine the sign of $\frac{dq_1}{d\theta}$ (the envelope theorem). Recall that $\pi^2_{22}\leq 0$ by the SOC for firm 2.

The Cournot equilibrium is not Pareto optimal from the viewpoint of the two firms, as seen from the isoprofit contours of the two firms. Isoprofit contours trace out combinations of q_1 and q_2 that yield the same level of profits for a firm.

$$\pi_1(q_1(q_2),q_2)\equiv\pi$$

Totally differentiating the above identity

$$\pi^1_1\frac{dq_1}{dq_2}+\pi^1_2=0$$

[1] *Cramer's rule*: replace the ith column of A by vector b to form the matric A_i; the solution to $Ax=b$ is then $x_i=\frac{|A_i|}{|A|}$.

[2] This condition ensures that the reaction functions intersect in the right way – if the equilibrium were slightly perturbed, outputs would converge back to the Cournot equilibrium.

determines the shape of the isoprofit contours for each firm

$$\frac{dq_1}{dq_2} = -\frac{\pi_2^1}{\pi_1^1}$$

The denominator is negative since the products are substitutes for one another $\pi_1^1 < 0$ (we are not saying that variables are strategic substitutes). Thus, when profits increase with q_1, the contour is positively sloped, and its slope reaches a maximum at $\pi_1^1 = 0$ (it equals infinity) and has negative slope for higher values of q_2.

$$\frac{d^2q_1}{d^2q_2} = -\frac{\pi_{12}^1\pi_1^1 - \pi_{12}^1\pi_1^1}{(\pi_1^1)^2} < 0 \text{ iff } \pi_1^1 < 0$$

Similarly, we can derive the shape of the isoprofit contours for firm 2.

The reaction function for each firm goes through the peak points of all the different isoprofit contours. This fact immediately implies that there exists a Pareto improving region where the two firms can increase total profits. Cournot output exceeds monopoly output and thus fails to maximize joint profits. The locust of all industry profit maximizing output levels is called the *contract curve* – all along this curve industry output equals monopoly output. The goal of collusion is to achieve an equilibrium on this contract curve. However, the Nash Equilibrium does not lie on this curve. Firms are in a prisoner's dilemma: they could both gain by restricting output but each firm unilaterally has an incentive to increase its output.[3]

Stackelberg Model (optional)

The Stackelberg model is a sequential game, whereas the Cournot model is a simultaneous game. One of the firms, say firm 1, is assigned the role of the first mover. Firm 2 chooses output, knowing firm 1's output and behaves like a Cournot firm who reacts to a given level of sales on its rival's part. Thus the first order condition for firm 2 is

$$p'(q_1 + q_2)q_2 + p(q_1 + q_2) - c = 0$$

[3] Collusion can be sustained in repeated games if firms are patient enough.

This FOC defines R2. Knowing how firm 2 responds, firm 1 chooses its own output. Firm 1 solves the problem of picking its output to maximize its profits

$$\pi_1 = [p(q_1 + q_2(q_1)) - c]q_1$$

The key difference from the Cournot case is the fact that firm 1 does not treat firm 2's output as fixed but rather takes into account how firm 2 responds to its output choice: firm 1 views firm 2's output as a function of its own output. The FOC for firm 1 is

$$p(Q) + p'(Q)\left[1 + \frac{dq_2}{dq_1}\right] - c = 0$$

The above first order condition is often generalized into the *conjectural variation model*. Let v_{12} denote firm 1's conjectures about how firm 2 will respond to a change in output on its own part. In the Stackelberg model, $v_{12} = \frac{dq_2}{dq_1}$, in the competitive model, $v_{12} = -1$ whereas in the Cournot model $v_{12} = 0$. Firm 1 earns higher profits in this model than it does as a Cournot competitor whereas firm 2 earns lower profits. Essentially, firm 1 picks the optimal point for itself on firm 2's reaction function.

Bertrand Model (optional)

The Bertrand model assumes that firms set prices, so that marginal cost pricing is the Nash equilibrium. Assume differentiated goods so can have positive profits. Suppose

$$p_i = a - q_i - sq_j$$

where s measures the extent of differentiation. Solving for the Nash equilibrium yields upward sloping reaction functions. Hence in this model, prices will be strategic complements. Comparative statics analysis can be done as in the case of Cournot.

Oligopoly as a Basis for Trade

The fundamental idea of oligopoly as a basis for trade (and gains from trade) is that international trade increases competition. This argument for trade is logically distinct from comparative advantage, IRS, and product differentiation. Opening up to international trade forces domestic firms to compete with foreign firms. In one model, we assume trade results in a single world market, while in the other model we assume that markets are segmented: each firm decides how much to sell in each market so that prices in the two markets can differ.

Helpman and Krugman 1985

The assumptions for a general model of trade under oligopoly are:

- Two countries: Home and Foreign (denote foreign variables by an asterisk)
- Two goods: Y and X
- Good Y is competitively produced, with price normalized to one $P_Y = 1$
- Good X oligopoly: n Cournot firms at home and n^* abroad.
- One factor of production: labor
- m consumers in the home country and m^* in the foreign country
- $D(p; w)$ is the individual consumer's demand curve: given a price p a consumer demands quantity $D(p; w)$ of good X

Autarky

Producers of good Y are perfectly competitive, which implies zero profit so wage must equal one

$$(w - 1)Y = 0 \rightarrow w = 1$$

If a producer of good X makes x units of output, then total output its total cost is given by $c(w, x)$. Profit maximization occurs at the output such that marginal cost equals marginal revenue

$$c_x(w,x) = p\left(1 + \frac{x}{p}\frac{dp}{dx}\right) \tag{6.3}$$

The total amount of X produced equals $X = nx \leftrightarrow x = \frac{X}{n}$ by symmetry, so we have

$$c_x(w,x) = p\left(1 + \frac{1}{n\frac{p}{X}\frac{dx}{dp}}\right) \tag{6.4}$$

The elasticity of demand is $\varepsilon(p,w) \equiv \frac{p}{X}\frac{dx}{dp}$, so we have

$$c_x(w,x) = p\left(1 - \frac{1}{n\varepsilon(p,w)}\right) \tag{6.5}$$

Assume marginal cost is constant and equals $c_x(w,x) = c$ in each country and the elasticity of demand is constant at $\varepsilon(p,w) = \varepsilon$. Therefore, our first order condition for X producers becomes

$$p\left(1 - \frac{1}{n\varepsilon}\right) = c = p^*\left(1 - \frac{1}{n^*\varepsilon}\right) \tag{6.6}$$

This relationship allows us to compare prices under autarky.

$$p^A = \frac{c}{1 - \frac{1}{n\varepsilon}},\ p^{A*} = \frac{c}{1 - \frac{1}{n^*\varepsilon}} \tag{6.7}$$

Proposition 1 *If the domestic country has more firms $n > n^*$, then the domestic country must have the lower autarkic price $p^A < p^{A*}$.*

Does this imply that the domestic country has comparative advantage in good X? No! We cannot apply the comparative advantage theorem here since its defined under perfect competition. What prediction can we obtain regarding the pattern of trade?

Free Trade

Assume free trade generates a single world market. Under trade the total number of firms in the market becomes $n + n^*$. Consequently,

$$p^T\left[1 - \frac{1}{(n+n^*)\varepsilon}\right] = c$$

Free trade will equalize prices in both countries so that

$$p^T = \frac{c}{1 - \frac{1}{(n+n^*)\varepsilon}} \tag{6.8}$$

Clearly, competition reduces the price of good X.

Proposition 2 *Free trade results in a lower price than the autarky price of either country:* $p^T < \min\{p^A, p^{A*}\}$.

Total world output is given by $(n + n^*)x$. Consumers have identical tastes so domestic quantity demanded as a percentage of world quantity demanded of X is given by

$$\frac{mD(p^T)}{mD(p^T) + m^*D(p^T)} = \frac{m}{m + m^*}$$

Domestic production of X as a percentage of world production is given by

$$\frac{nx}{nx + n^*x} = \frac{n}{n + n^*}$$

Therefore if home's share of firms exceeds home's share of resources

$$\frac{n}{n + n^*} > \frac{m}{m + m^*}$$

then the home country exports good X.

Proposition 3 *If the number of firms per capita at home exceed the number of firms per capita abroad* $n/m > n^*/m^*$*, then the home country exports good* X*.*

While autarkic prices in the country depend upon the absolute size of the X industry, direction of trade is determined by the per capita number of firms. Depending upon the relative sizes of the two countries, the country with the lower relative price in autarky may still be importing under free trade (if it is large enough). *Comparative advantage does not predict the pattern of trade in this model.* If the two countries are identical ($n = n^*$ and $m = m^*$), no trade (intraindustry or otherwise) arises! Is there no basis for trade between identical countries then? Not quite. *While goods are not traded, competition does have beneficial effects.* There are gains from the potential to trade.

Segmented Markets (optional)

Under segmented markets, prices in the two countries need not equal each other under free trade and each firm makes a separate decision on how much to sell in each market. In other words, taking sales of foreign firms in its own market as a given, home firms decide on how much to sell at home. And then a similar decision is made about the foreign market. Let

$$h = \frac{nx}{X} \text{ and } f = \frac{nx^*}{X^*}$$

denote home firm's share of the home and foreign market respectively. Foreign shares are denoted by $1-h$ and $1-f$. Note that total output is

$$X = nx + n^*y$$

and

$$X^* = nx^* + n^*y^*$$

A typical home firm's equilibrium in the home market is given by

$$p\left[1 + \frac{x}{p}\frac{dp}{dx}\right] = c$$

which can be rewritten as

$$p\left[1 + \frac{x/X}{\frac{p}{X}\frac{dx}{dp}}\right] = p\left[1 - \frac{h}{n\varepsilon}\right] = c \tag{6.9}$$

and a typical foreign firm's equilibrium is given by

$$p\left[1 - \frac{1-h}{n^*\varepsilon}\right] = c + t$$

where per unit transportation cost is t. We can plot the above equilibrium conditions (called HFE and FFE) in the (h, p) space. The slope of the HFE curve is found by total differentiation and from (6.9) we have, along the HFE,

$$dp - \frac{h}{n\varepsilon}dp - dh\frac{p}{n\varepsilon} = 0$$

This implies

$$\frac{dp}{dh} = \frac{\frac{p}{n\varepsilon}}{1 - \frac{h}{n\varepsilon}} > 0$$

We can show that the HFE curve has an intercept of c at $h = 0$ and at $h = 1$, it must have

$$p = \frac{c}{1 - \frac{1}{n^*\varepsilon}} > c$$

Similarly, the FFE is downward sloping,

$$\frac{dp}{dh} = -\frac{\frac{p}{n^*\varepsilon}}{1 - \frac{1-h}{n^*\varepsilon}} < 0$$

Furthermore, at $h = 0$, the FFE condition gives

$$p = \frac{c+t}{1 - \frac{1}{n^*\varepsilon}} > c$$

and at $h = 1$, we have $p = c + t$. Equilibrium in the home market is given by the intersection of the two curves. An analogous picture gives the equilibrium in the foreign market with the role of the firms reversed. As t increases, the foreign curve shifts up and home firm's market share expands.

To evaluate welfare, consider the following utility function that gives rise to a constant elasticity demand curve of the kind we have employed above

$$u = \frac{\varepsilon}{\varepsilon - 1} X^{\frac{\varepsilon-1}{\varepsilon}} + Y$$

Write the Lagrangean as:

$$L = \frac{\varepsilon}{\varepsilon - 1} X^{\frac{\varepsilon-1}{\varepsilon}} + Y + \lambda(I - Y - pX)$$

Consumer's FOCs are:

$$\frac{\partial L}{\partial X} = X^{\frac{-1}{\varepsilon}} - \lambda p = 0$$

and

$$\frac{\partial L}{\partial Y} = 1 - \lambda = 0$$

These FOCs imply

$$X = p^{-\varepsilon}$$

Therefore, we can employ the above utility function to provide a micro foundation for our demand function.

National income equals national expenditure (this is an identity) so that

$$n(\pi + \pi^*) + L = Y + pX$$

This implies the amount of Y that is produced is given as

$$Y = L - pX + n(\pi + \pi^*)$$

Therefore we can substitute out Y and X to write the following value function for the domestic economy

$$u = L + n(\pi + \pi^*) - \frac{1}{1 - \varepsilon} p^{1-\varepsilon}$$

Two forces affect domestic welfare: Allocative efficiency – $p > c$ implies that too little X is produced and this negative effect of higher prices is captured in the second term - and profits from producing X matter as well.

Suppose to simplify matters we set $\pi^* = 0$ (a high t could do this – we wish to focus on domestic market equilibrium). This implies that domestic welfare can be written as

$$u = L + (p - c)\overbrace{hp^{-\varepsilon}}^{sales} - \frac{1}{1-\varepsilon}p^{1-\varepsilon}$$

This can be used to draw isoutility contours in the (p, h) space. Slope of the contours for the above value function are given by

$$u' = \frac{dp}{dh} = \frac{p}{\left(\frac{p}{p-c}\right)(1-h) + h\varepsilon} > 0$$

Free trade moves us from point A to a point like C. At point A,

$$p = \frac{c}{1 - \frac{1}{n\varepsilon}}$$

As domestic firms' share of the home market falls, price moves toward marginal cost. Therefore, trade reduces the allocative inefficiency. However, as price moves toward marginal cost some of the lost producer surplus is exported to the foreign country. This generates a countervailing force for the home country.

At A, the slope of the IU curve equals

$$u'(h = 1) = \frac{p}{\varepsilon}$$

whereas that of HFE curve equals

$$\frac{p}{n\varepsilon - 1}$$

Therefore, the isoutility curve is steeper than the HFE iff

$$(n - 1)\varepsilon > 1$$

If the above holds there *could be losses from trade.* The above says that if the number of firms in the home market is large or if demand elasticity is high, increased competition from trade does not bring gains that outweigh losses resulting from lower profits (both a high n and a high ε imply low autarkic mark ups over marginal cost). If $(n - 1)\varepsilon < 1$, *there is no possibility of losses from trade.* In this case the IU curve intersects the HFE curve from above and utility increases in the downward direction (IU curve is flatter than HFE at A).

Proposition 4 *Suppose there exist m sectors at least some of which are imperfectly competitive and the others are perfectly competitive. Then there exist gains from trade if $\sum_i p_i(x_i - x_i^A) \geq 0$.*

Note that

$$\sum_i p_i(x_i - x_i^A) = \sum_i MR_i(x_i - x_i^A) + \sum_i (p_i - MR_i)(x_i - x_i^A)$$

Therefore a sufficient condition for gains from trade is that on average, $x_i - x_i^A$ is largest when $p_i - MR_i$ is largest. In other words, output should expand in markets that are most imperfectly competitive (see Markusen *JIE* 1981). The key to understanding this point is that the presence of imperfect competition creates a distortion in an economy (price exceeds marginal cost): if trade reduces the magnitude of this distortion, it improves welfare. This point can be seen quite clearly for the case of a small open economy. Suppose $p^A = p^*$: autarkic price ratio equals free trade price ratio so that there is no basis for trade. Imagine a policy intervention (tax/subsidy). To be concrete say there is a consumption tax on good Y causing its consumption price q_Y to differ from the price received by the producers p_Y, $q_Y = (1+t)p_Y$. This will imply that while producers face world prices, consumers do not. Since consumers see too high a price of good Y, they will consume too little of it. *The tax distortion will induce trade and this trade will be welfare reducing for the small country.* However, the rest of the world gains from this trade – it gets to trade at a price different from its autarkic price.

The argument applies to a large country as well. Suppose two identical countries under perfectly competitive markets. Autarkic equilibria are identical (p^*) – no basis for trade. Suppose H imposes a consumption tax on Y. Its clear that p^* cannot be an equilibrium anymore – H consumers will want to import good X whereas F country does not wish to trade. Hence, p^* must rise (its relative price of good X). At the new free trade price p', H country will import good X and it will be worse off whereas the foreign country will be better off.

What if the economy had the tax in place and then opened to trade? In other words, what if *the economy opens to trade under a pre-existing distortion*? Here the general result is that trade need not increase welfare. This is theory of the second best: *in the presence of a distortion, a second distortion can actually improve welfare by lowering the first distortion.* In general, we cannot rank the autarkic and free trade equilibria.

Let p^T denote the free trade price vector and X^T denote the production bundle. Similarly, define p^A and X^A. We know

$$p^T X^T > p^T X^A$$

Trade is balanced when

$$p^T D^T = p^T X^T$$

and autarkic equilibrium requires

$$X^A = D^A$$

Thus we must have

$$p^T D^T > p^T D^A \Leftrightarrow p_X^T D_X^T + p_Y^T D_Y^T > p_X^T D_X^A + p_Y^T D_Y^A$$

Suppose that there is a consumption tax on good Y only so that

$$q_Y^T = (1+t)p_Y^T \text{ and } q_X^T = p_X^T$$

Thus we have

$$\begin{aligned} q_X^T D_X^T + (1+t)p_Y^T D_Y^T &> p_X^T D_X^A + p_Y^T D_Y^A + t p_Y^T D_Y^T \\ &= q_X^T D_X^A + q_Y^T D_Y^A + t p_Y^T (D_Y^T - D_Y^A) \end{aligned}$$

which gives

$$q_X^T D_X^T + q_Y^T D_Y^T > q_X^T D_X^A + q_Y^T D_Y^A + t p_Y^T (D_Y^T - D_Y^A)$$

Clearly, if the second term on the RHS is positive i.e. when free trade consumption of the taxed good is more than the autarkic consumption, the value of the free trade consumption (the LHS) necessarily exceeds the value of autarkic consumption at free trade prices and the country must benefit from free trade. Thus, $D_Y^T - D_Y^A > 0$ is a sufficient condition for gains from trade under the presence of a distortion. Markusen (1981) has made this point explicitly in a model of imperfect competition.

Markusen *JIE* 1981 (optional)

Two countries: S and L (later to be read as small and large when the asymmetric case is considered). Two goods: X and

Y. X is produced by a monopolist and Y under perfect competition. Let p denote the relative price of X. Factor markets are perfect. Let the transformation curve in country i be

$$Y^i = F^i(X)$$

Slope of the PPF gives MRT.

Assume identical factor endowments to eliminate the role of factor endowments as a basis for trade. Identical countries implies that MRT depends only upon the ratio of the good being produced

$$\frac{X^L}{Y^L} = \frac{X^S}{Y^S} \Leftrightarrow F^{L'} = F^{S'}$$

Also assume homothetic utility and monopolist maximizes its profits not utility. Since X is produced by a monopolist, we must have too little X is produced – the distortion in the economy.

$$p\left(1 - \frac{1}{\eta_X}\right) = MRT < p = MRS$$

Assume a CES utility function

$$U = \left[aX^{-\beta} + bY^{-\beta}\right]^{\frac{1}{\beta}}$$

Define

$$\gamma = \frac{1}{1+\beta}$$

Demand for good X

$$X = \frac{I}{p(1 + \alpha^{\gamma} p^{-\beta\gamma})}$$

where $\alpha = b/a$ and $I = pX + Y$.

Monopolist maximizes

$$p(X)X - c(X) \text{ s.t. } X = \frac{I}{p(1 + \alpha^{\gamma} p^{-\beta\gamma})}$$

Under homothetic utility, one can think of the monopolist treating I as a fixed constant I'. The above problem can be solved easily – we get

$$\eta_X = \frac{1 + \alpha^{\gamma} p^{-\beta\gamma}}{1 + \alpha^{\gamma}(1 - \beta\gamma)p^{-\beta\gamma})} < 1 \;\Leftrightarrow \beta < 0 \;\Leftrightarrow \gamma > 1$$

Thus the countries have an identical autarkic equilibria. What happens if they open up to trade? The key point is that the monopolist in each country now has a competitor. Assume Cournot-Nash competition. Under free trade, the L monopolist solves

$$p(X^L + X^S)X^L - c(X^L) \text{ s.t. } X^L + X^S = \frac{I^L + I^S}{p(1 + \alpha^\gamma p^{-\beta\gamma})}$$

Note that $c'(X^L) = F'$ or the slope of the PPF. The first order condition for L firm under free trade becomes

$$p\left(1 - \frac{\sigma^L}{\eta_X}\right) = -F^{L'}(X^L)$$

Thus trade increases the elasticity of demand faced by each producer and total

$$\sigma^L > \sigma^S \Leftrightarrow -F^{L'}(X^L) < -F^{S'}(X^S) \Leftrightarrow \frac{X^L}{Y^L} < \frac{X^S}{Y^S}$$

Conclusions:

1. No actual trade
2. When countries are asymmetric, large country imports the good.
3. Small country always gains while large may lose if its monopoly produces less under free trade. Sufficient condition for gains is that monopolist expand it output.
4. Trade may make relative factor prices more unequal relative to autarky.

Brander and Krugman (*JIE* 1983)

The assumptions made to generate reciprocal dumping under segmented markets are:

- Two identical countries: Home and Foreign (starred variables denote the foreign country)
- One homogeneous good with domestic demand $p(Z)$ and foreign demand $p^*(Z^*)$ implying *segmented markets*, and some other numeraire good.

- Duopoly (under trade): One firm in each country (no entry) producing one homogeneous good. In the home market, $Z \equiv x + y$, where x denotes the home firm's sales in the home market and y the foreign firm's sales in the home market; in the foreign market $Z^* \equiv x^* + y^*$, where x^* denotes home firm's sales abroad and y^* denotes foreign firm's sales abroad.

- Cournot behavior: Firms choose quantities for each market, given quantities chosen by the other firm

- Constant marginal costs c and c^* with fixed costs F and F^*. Assume home and foreign firms have the same marginal costs $c = c^*$.

- Iceberg type transportation costs: for every one unit exported, only g arrives, $0 \leq g \leq 1$. Thus, the marginal cost of export c/g is inflated by the extent that not all of the exports arrive safely abroad.

Dumping means charging a lower price abroad than in the home market (or than cost). This model demonstrates that dumping can occur due to oligopolistic behavior. Here, dumping is reciprocal: each firm dumps in the other's market. Reciprocal dumping involves wasteful transportation costs.

Equilibrium

The domestic and foreign profit functions are

$$\pi = xp(Z) + x^* p^*(Z^*) - c\left(x + \frac{x^*}{g}\right) - F$$

$$\pi^* = yp(Z) + y^* p^*(Z^*) - c\left(\frac{y}{g} + y^*\right) - F^*$$

Considering the profits of the domestic firm, the first term reflects sales in the home market, the second term sales revenue from the foreign market, the third term marginal costs, and the last term fixed costs.

The market in each country can be considered separately due to the segmented nature of markets. Consider the domestic economy. The domestic and foreign FOCs for choosing output (x and y) for the domestic market to maximize profits are[4]

[4]SOCs are $\pi_{xx} = xp'' + 2p' < 0$ and $\pi^*_{yy} = yp'' + 2p' < 0$. Also, stability conditions $\pi_{xy} = xp'' + p' < 0$ and $\pi^*_{yx} = yp'' + p' < 0$.

$$\pi_x = xp' + p - c = 0$$

$$\pi_y^* = yp' + p - \frac{c}{g} = 0$$

Let $f \equiv y/Z$ be the foreign firm's share of the home market, $h \equiv x/Z$ be the home firm's share of the home market (so $h + f = 1$), and $\varepsilon = -\frac{p}{Zp'}$ be elasticity of home demand. The home firms' FOC becomes

$$x\left(-\frac{p}{Z\varepsilon}\right) + p = c$$

$$\leftrightarrow p\left(1 - \frac{h}{\varepsilon}\right) = c$$

$$\leftrightarrow p = c\left(\frac{1}{1 - \frac{h}{\varepsilon}}\right)$$

Similarly, the foreign firm's FOC becomes

$$\leftrightarrow p = \frac{c}{g}\left(\frac{1}{1 - \frac{f}{\varepsilon}}\right)$$

Solving for p and f (using $h = 1 - f$), gives the Nash equilibrium price

$$p = \frac{c\varepsilon(1+g)}{g(2\varepsilon - 1)}$$

and foreign market share

$$f = \frac{1 - \varepsilon(1-g)}{1+g}$$

Firms suffer a smaller markup over cost abroad than in their domestic markets (might be found guilty of dumping)

$$\frac{p/(c/g)}{p/c} = g < 1$$

For an interior solution ($0 < f < 1$), we need the autarkic price to exceed the foreign monopoly's marginal cost of selling in the home market

$$\frac{c\varepsilon}{\varepsilon - 1} > \frac{c}{g} \rightarrow g > 1 - \frac{1}{\varepsilon}$$

assuming constant elasticity of demand $p = AZ^{-\frac{1}{\varepsilon}}$ (see Figure 1). Thus, we have a unique stable equilibrium with two-way (intraindustry) trade.

Welfare Analysis

Assume quasilinear preferences

$$U = u(Z) + K$$

where K is consumption of a numeraire good. Overall welfare measures total surplus

$$W = 2(u(Z) - cZ - ty) - F - F^*$$

where $t \equiv c\left(\frac{1}{g} - 1\right)$. The 2 arises due to having two symmetric countries (except for fixed costs), using $ty = tx^*$. Considering the effect of a small increase in transportation costs, we have

$$\frac{dW}{dt} = 2\left[u'\frac{dZ}{dt} - c\frac{dZ}{dt} - t\frac{dy}{dt} - y\right]$$

which can be rewritten as

$$\frac{dW}{dt} = 2\left[(p-c)\frac{dZ}{dt} - t\frac{dy}{dt} - y\right]$$

by collecting terms, since the relative price of Z equals the marginal utility u'. Focus on a special case where transportation costs are prohibitively high. At a prohibitive transportation cost (per unit) t, imports are zero $y = 0$.[5] Furthermore, $\frac{dZ}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$. Using these properties, we have $(t = p - c)$

$$\frac{dW}{dt} = 2(p-c)\frac{dx}{dt} = 2t\frac{dx}{dt} > 0$$

Therefore, when t is large, a small lowering of t lowers welfare. But for small t, there are clear gains because of increased competition. For negligible transportation costs $t = 0$,

$$\frac{dW}{dt} = 2(p-c)\frac{dZ}{dt} < 0$$

In general, three effects determine welfare. First is the *increased consumption* of the good that comes from increased competition, second is the *reduced profits* of domestic firms since increased consumption comes from increased imports and third are the *wasteful transportation costs* incurred. When t is large, this last effect dominates.

[5] The prohibitive transport cost t makes $p = c + t$.

The existence of positive profits in oligopoly models alters the implications for trade policy. Since profits are split between domestic and foreign firms, the domestic government wants to implement policies that capture a greater share of world profits for its own firms. The traditional argument for intervention is that a large country can improve its terms of trade: by restricting trade (imports or exports), a large country can achieve a higher price of its exports relative to its imports.

The policy literature on oligopoly (called strategic trade theory) is unique in that it furnishes an intuitive argument for promoting exports rather than restricting them, thereby reversing conventional wisdom. This export promotion view has struck a chord with policy makers and business people who see exports as a means of achieving greater profits and thus feel that exports should be encouraged rather than restricted. Government policy in many NICs such as South Korea has sought to expand exports. Even U.S. policy pushing foreign countries such as Japan to give U.S. firms more access to their markets. Why would market access be important if profits were not involved?

Examples (optional)

Two examples from Brander (1995) help illustrate the argument for export subsidies.

$G\backslash F$	(B,B)	(B,NB)	(NB,B)	(NB,NB)
S	$5,15$	$5,15^*$	$-5,5$	$-5,5$
NS	$10,10^*$	$0,0$	$10,10^*$	$0,0$

Example 5 *(Sequential entry game)*[6] *A multinational firm decides whether to build B or not build NB a plant in a potential host country. The government of the potential host country first decides whether to subsidize S (or not subsidize NS) the firm.*

[6] A *strategy* for a player is a complete description of how the player would play the game, and a *strategy set* is a set of all such possible ways of playing the game. An *action* is what the player would do at a particular node. *A strategy tells us what the player would do at each node where action is required.* Thus, in the above game, the strategy set for the government is $\{S, NS\}$ whereas that for the firm is $\{(B,B), (B,NB), (NB,B), (NB,NB)\}$. The firm's strategy set specifies its possible actions in each contingency.

The first number gives the government's payoff while the second gives the firm's payoff. Gross payoff of firm and government is 10 each while 5 is the amount of the subsidy. What are the Nash Equilibrium of this game?

In the first Nash equilibrium, the firm's strategy is to build if subsidized and build if not subsidized and the government gives no subsidy. In the second Nash equilibrium, the firm builds if subsidized and does not build if not subsidized. The second Nash equilibrium is weird - the firm should build the plant no matter what and *the government should realize that the firm will build the plant no matter what.* Thus, the firm's threat to not build the plant in the absence of a subsidy is *not credible.* A Nash equilibrium places no restrictions on the out of equilibrium threats that players can use. *Subgame perfection requires that such threats be credible.* Formally, a subgame perfect Nash equilibrium (SPNE) requires each component of a strategy to be a Nash equilibrium in every subgame (even subgames that are not reached in equilibrium). The unique SPNE, found by backward induction, is for the government to not subsidize and for the firm to build regardless of whether subsidized (NS, (B, B)).

	Intervention			*Non−Intervention*	
$X/Y/G$	y_1	y_2	$X/Y/G$	y_1	y_2
x_1	$2, 0; -1$	$0, 2; -1$	x_1	$1, 1; 0$	$0, 2; 0*$
x_2	$3, 0; 2*$	$1, -1; 0$	x_2	$2, 0; 3$	$-2, 1; 1$

Example 6 *First, the government decides whether or not to intervene and then two firms (one domestic and one foreign) choose their actions (such as low or high output). By intervening the government ensures itself a payoff of 2 by directly altering the strategic interaction between the firms. In the above figure, for a fixed pair of strategies (x_i, y_i), welfare is always lower in the intervention matrix. The gain from intervention stems from the change in the strategic interaction between the two firms. The government is assumed to have the ability to commit. After the firms choose (x_2, y_1), government would like to renege and choose non-intervention.*

This second example is the closest to the Brander and Spencer case for strategic export subsidies. Two Cournot duopolists, one domestic and one foreign, pick their quantities taking the other firm's quantity as given. The domestic government, by giving the domestic firm an export subsidy, induces the domestic firm to take a more aggressive stance (produce more output for any given level of output chosen by the foreign firm). In the new equilibrium, the domestic firm produces more and the foreign firm less; hence, profits are shifted towards the domestic firm.

Brander and Spencer *JIE* 1985

To model profit shifting export subsidies, assume:

- Two homogenous products: one oligopolistic and one numeraire good produced under perfect competition (so that wage equals unity).
- Two Cournot firms: One domestic firm – whose output is denoted by x - and one foreign firm - whose output is denoted by y.
- $\pi_{xy} \leq 0$ and $\pi^*_{yx} \leq 0$ (strategic substitutes), where $\pi(x, y)$ and $\pi^*(x, y)$ denotes the profit functions for the two firms.
- Firms sell output in only a third market (no domestic or foreign consumer surplus) and government in the third country is not a player (not policy active).
- One factor of production, labor L.
- Constant marginal costs c and c^* and fixed costs F and F^*.
- The domestic government chooses a subsidy s (export subsidy is equivalent to production subsidy as no domestic consumption).

The game has two stages: policy and output. The domestic government sets a (specific) subsidy s per unit for the domestic firm and then firms compete in the product market (Cournot competition). The structure of the game implies that the domestic government can commit itself to a specific policy intervention (a level of the export tax or subsidy). The government cannot change its policy if it is no longer optimal once the firms have chosen their output.

We solve for the SPNE of this game by backward induction. In the second stage, the subsidy s is given. The profit functions for the domestic and foreign firms are

$$\pi(x,y) = (p - c + s)\,x - F$$

$$\pi^*(x,y) = (p - c)\,y - F^*$$

The first order conditions for the domestic and foreign firms are

$$\pi_x = xp' + p - c + s = 0$$

$$\pi_y^* = yp' + p - c = 0$$

Totally differentiate the FOCs with respect to the subsidy s and write

$$\begin{bmatrix} \pi_{xx} & \pi_{xy} \\ \pi_{xy}^* & \pi_{yy}^* \end{bmatrix} \begin{bmatrix} \frac{dx}{ds} \\ \frac{dy}{ds} \end{bmatrix} = \begin{bmatrix} -\pi_{xs} \\ -\pi_{ys}^* \end{bmatrix}$$

Using the FOCs, $\pi_{xs} = 1$ and $\pi_{ys}^* = 0$ so,

$$\begin{bmatrix} \pi_{xx} & \pi_{xy} \\ \pi_{xy}^* & \pi_{yy}^* \end{bmatrix} \begin{bmatrix} \frac{dx}{ds} \\ \frac{dy}{ds} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Applying Cramer's rule we find that

$$\frac{dx}{ds} = -\frac{\begin{vmatrix} -1 & \pi_{xy} \\ 0 & \pi_{yy}^* \end{vmatrix}}{D} = -\frac{\pi_{yy}^*}{D} > 0$$

and

$$\frac{dy}{ds} = \frac{\begin{vmatrix} \pi_{xx} & -1 \\ \pi_{xy}^* & 0 \end{vmatrix}}{D} = \frac{\pi_{xy}^*}{D} < 0$$

where $D = \pi_{xx}\pi_{yy}^* - \pi_{xy}^*\pi_{xy} > 0$ is positive by the stability condition.

Proposition 7 *An increase in the domestic export subsidy s causes the output of the domestic firm x to increase and the output of the foreign firm y to decrease.*

Optimal Subsidy

In the first stage, the domestic government chooses its export subsidy s to maximize national income (welfare)

$$W = L + \pi - sx$$

The first term is labor income (labor supply times the wage of one), the second term is domestic profits and the third term is subsidy payments. This specification assumes that the government weights the profits of domestic firms and government revenue equally in evaluating domestic welfare. Thus, as a function of s, the objective function of the government is

$$W(s) = L + \pi(x(s), y(s); s) - sx(s)$$

The FOC for choosing the subsidy to maximize domestic welfare is

$$\frac{dW(s)}{ds} = \underbrace{\pi_x}_{0} \frac{dx}{ds} + \pi_y \frac{dy}{ds} + \underbrace{\pi_s - x}_{0} - s\frac{dx}{ds} = 0 \tag{6.10}$$

But $\pi_x = 0$ (by domestic firm's FOC) and $\pi_s = x$ (by definition of domestic profits) so that the first, third and fourth terms drop out leaving

$$\frac{dW(s)}{ds} = \pi_y \frac{dy}{ds} - s\frac{dx}{ds} = 0$$

Evaluated at an initial point of no intervention $s = 0$, an increase in the subsidy raises domestic welfare

$$\left.\frac{dW(s)}{ds}\right|_{s=0} = \pi_y \frac{dy}{ds} > 0$$

The above expression spells out the intuition for the export subsidy raising domestic welfare: an increase in the subsidy reduces the foreign firm's output $dy/ds < 0$ and an increase in foreign firm output decreases domestic firm profits $\pi_y < 0$, so the subsidy must raise domestic profits and thus domestic welfare. The optimal subsidy is found by solving the FOC (6.10) for

$$s^* = \frac{\pi_y \frac{dy}{ds}}{\frac{dx}{ds}} = \frac{-\pi_y \pi^*_{yx}/D}{\pi^*_{yy}/D} = \frac{-\pi_y \pi^*_{yx}}{\pi^*_{yy}} > 0 \tag{6.11}$$

Since x and y are strategic substitutes ($\pi^*_{yx} < 0$), the optimal subsidy is positive.

Robustness (optional)

The traditional argument for trade policy calls for trade restrictions rather than subsides to exports because a larger volume of exports causes a deterioration of the country's terms of trade. Denote the utility function by $U(z, m)$. We have

$$dU = u_z dz + u_m dm$$

Define the change in real income (measured in terms of the numeraire good as)

$$dI \equiv \frac{dU}{u_m} = \frac{u_z}{u_m}dz + dm = pdz + dm$$

Denote domestic production by x and m^p. Trade is balanced only when

$$px + m^p = pz + m$$

Totally differentiating the above equation gives

$$xdp + pdx + dm^p = zdp + pdz + dm$$

From $dI = pdz + dm$ we have

$$dI = (x - z)dp + pdx + dm^p$$

The first term captures the terms of trade effect while the second captures the value of output effect. However, $\frac{-dm^p}{dx} = c_x$ or the marginal cost of good x. Thus we have

$$dI = (x - z)dp + (p - c_x)dx$$

Under perfect competition there is only the terms of trade loss since the subsidy causes prices to drop ($dp < 0$ and $p = c_x$). However, under oligopoly the second term is positive and can outweigh the loss from terms of trade. In a Cournot-Nash equilibrium,

$$p - c_x = -xp' - s$$

Thus we have

$$dI = (x - z)dp - (xp' + s)dx$$

The analysis of the optimal export subsidy was derived in a partial equilibrium set up – subsidy only had effect on price of good z. To be able to utilize the comparative static developed before, assume quasilinear preferences

$$U(z, m) = u(z) + m$$

These preferences simplify the analysis by ensuring that the demand for good z only depends upon its price and is independent of consumer income. This simplification is a reasonable assumption if the consumer spends only a small fraction of income on good z – clearly if consumer spends all of his income on good z, demand for this good must with income. Under quasi-linear preferences, consumer surplus provides an exact measure of consumer welfare.[7] Then we have

$$\frac{dI}{ds} = (x-z)\frac{dp}{ds} - (xp' + s)\frac{dx}{ds} = -zp_s + xp'y_s - sx_s$$

At $s = 0, \frac{dI}{ds} > 0$ since $p_s < 0$ and $y_s < 0$. A small subsidy reduces the distortion present due to the under production of good z.

Foreign Policy Retaliation (optional)

Look for a Nash equilibrium in subsidies and show that it involves positive subsidy levels for both countries; however, both countries end up offering too large a subsidy since each ignores the adverse impact of its own subsidy on the welfare of the other. Assume quasilinear preferences and use asterisks for foreign variables. Define the foreign objective function as

$$G^*(s, s^*) = U^*(z^*) - pz^* + \pi^*(x, y, s^*) - s^*y$$

where

$$\pi^*(x, y, s^*) = py - c(y) + s^*y$$

Since $\frac{dU^*}{dz^*} = p$ and $\frac{d\pi^*}{ds^*} = yp'x_x + y$, the first order conditions for the two countries can be written as

$$\frac{dG^*(s, s^*)}{ds^*} = -z^*p_{s^*} + yp'x_{s^*} - s^*y_{s^*} = 0$$

and

$$\frac{dG(s, s^*)}{ds} = -zp_s + yp'x_s - sy_s = 0$$

Solving the above gives the non-cooperative NE:

$$s = \frac{xp'y_s}{x_s} - z\frac{p_s}{x_s} > 0 \text{ and } s^* = \frac{xp'y_{s^*}}{x_{s^*}} - z\frac{p_{s^*}}{x_{s^*}} > 0$$

Criticism

[7] Note that consumer optimization yields $\frac{du}{dz} = p$ where p is the relative price of good z. Now, suppose the consumer buys z_1 units at price p_1. Utility equals $\int_0^{x_1} u'(z)dz$ – sum of marginal utility of each unit. Say price changes from p_1 to p_2, altering consumption from z_1 to z_2. Change in utility $\Delta U = \int_0^{z_2} u'(z)dz - \int_0^{z_1} u'(z)dz = \int_{z_1}^{z_2} u'(z)dz = \int_{z_1}^{z_2} p(z)dz$ which is nothing but the area under the demand curve between quantities z_1 and z_2.

Brander and Spencer made three broad points. First, under oligopoly, profits matter and should count as part of national welfare - unobjectionable. Second, the government can alter the rules of the game and affect the strategic interaction among players - essentially an assumption that may or may not hold. Third, under Cournot competition, export subsidies are the optimal policy - this point has invited most of the criticism since it is highly sensitive to assumptions such as the number of firms in the market (Dixit 1984), the nature of the strategic variable (Eaton and Grossman 1986), general equilibrium concerns (Dixit and Grossman 1986) and more. The literature has grown rapidly and Brander (1995) provides an excellent survey of these and other concerns.

Eaton and Grossman *QJE* 1986

The Eaton and Grossman critique is that if firms play Bertrand instead of Cournot (pick prices rather than quantities), the optimal policy is a tax. Under Bertrand competition, the domestic government wants its firm to be able to commit to a higher price instead of higher output. A tax achieves this objective by raising the firm's marginal cost.

Output

Let small letters denote the home country and capital letters the foreign country. The domestic and foreign revenue functions are $r(x, X)$ and $R(x, X)$. Let t denote the domestic ad valorem tax per unit of output (or subsidy if negative). Again, all consumption occurs in a third country so a tax on all output is a tax on exports. Domestic profits (after tax) and foreign profits are

$$\pi = (1 - t)r(x, X) - c(x)$$

$$\Pi = R(x, X) - C(x)$$

Let γ and Γ denote the home and foreign firm's conjectures about how its rival responds to a change in its own output. Then, the FOCs for choosing x and X to maximize profits are

$$(1 - t)(r_x + \gamma r_X) - c' = 0$$

$$R_X + \Gamma R_x - C' = 0$$

where $r_x = \partial r(x,X)/\partial x$ and $r_X = \partial r(x,X)/\partial X$ are understood to be functions of the quantities x and X (and likewise for R_x and R_X). The solution to the foreign firm's FOC gives its reaction function $X = \Psi(x)$, foreign firm output as a function of domestic firm output. Let the slope of the foreign firm's reaction function be $g \equiv \Psi'(x)$, the actual response of the foreign firm to a change in the output of the domestic firm.

Policy

Let the home welfare function be

$$\begin{aligned} w &= \pi + tr \\ &= (1-t)r(x,X) - c(x) + tr(x,X) \\ &= r(x,X) - c(x) \end{aligned}$$

Differentiating with respect to the tax t gives

$$\frac{dw}{dt} = (r_x - c')\frac{dx}{dt} + r_X \frac{dX}{dt}$$

Substituting from the FOC gives

$$\frac{dw}{dt} = \left(-\gamma r_X + \frac{tc'}{1-t}\right)\frac{dx}{dt} + r_X \frac{dX}{dt} \tag{6.12}$$

Domestic welfare is maximized when

$$\frac{dw}{dt} = \left(-\gamma r_X + \frac{tc'}{1-t}\right)\frac{dx}{dt} + r_X \frac{dX}{dt} = 0$$

or incorporating g as the slope of the foreign firm's reaction function implies the condition

$$\left(-\gamma r_X + g r_X + \frac{tc'}{1-t}\right)\frac{dx}{dt} = 0$$

$$\leftrightarrow (g-\gamma)(-r_X) = \frac{tc'}{1-t}$$

Since $r_X < 0$ and $c' > 0$, $g - \gamma$ on the LHS and $\frac{t}{1-t}$ on the RHS must have the same sign. The term $g - \gamma$ measures the difference between the actual response of X to a change in x and the home firm's conjectural variation. If the actual response in greater than conjectured $g > \gamma$, then a tax is required $t > 0$; if the actual response is smaller than conjectured $g < \gamma$, then a subsidy is required $t < 0$. Government policy allows the domestic firm to achieve the outcome it would as a Stackelberg leader (where picks output before the foreign firm).

Proposition 8 *An increase in the export tax raises domestic welfare relative to nonintervention if the domestic firm's conjecture is smaller than the foreign firm's actual response.*

Cournot

Under the Cournot case, conjectural variations are zero $\gamma = \Gamma = 0$ (each firm assumes that the other will not respond to changes it output), so the welfare-maximizing condition becomes

$$-r_X g = \frac{tc'}{1-t}$$

Totally differentiating the FOCs allows us to obtain g and write the above condition as

$$\frac{r_X R_{21}}{R_{22} - C''} = \frac{tc'}{1-t}$$

Hence the sign of the optimal policy under Cournot model is determined by the sign of R_{21} (the rest is determined by the SOC for the foreign firm). The usual presumption is that R_{21} is negative (it is negative for linear demand) so that a subsidy is required. Domestic welfare gains at the expense of foreign welfare (sum of profits minus subsidy is lower), but world welfare is higher due to total output higher (and price lower) with the subsidy.

Bertrand

What happens if firms act as Bertrand competitors? Let $d(p, P)$ denote the demand function facing the home firm. The domestic and foreign profits are

$$\pi = (1-t)pd(p, P) - c(d(p.P))$$

$$\Pi = PD(p, P) - c(D(p.P))$$

The domestic and foreign FOCs for profit maximization are

$$\pi_1 = (1-t)(d + pd_1) - c'd_1 = 0$$

$$\Pi_1 = D + PD_1 - c'D_1 = 0$$

We know quantity demanded must equal quantity supplied

$$d(p(P), P) \equiv x$$

$$D(p, P(p)) \equiv X$$

Totally differentiate the demand functions and write

$$\begin{bmatrix} dx \\ dX \end{bmatrix} = \begin{bmatrix} d_1 & d_2 \\ d_1 & D_2 \end{bmatrix} \begin{bmatrix} dp \\ dP \end{bmatrix}$$

Using the first order condition, can write down the actual response of foreign sales to domestic sales

$$g = \frac{dX/dp}{dx/dp} = \frac{D_1 - D_2\frac{\Pi_{21}}{\Pi_{22}}}{d_1 - d_2\frac{\Pi_{21}}{\Pi_{22}}}$$

whereas the Bertrand conjecture is that

$$\gamma = \left.\frac{dX/dp}{dx/dp}\right|_{dP=0} = \frac{D_1}{d_1}$$

We can show using the stability conditions that $g-\gamma$ is positive iff Π_{21} (foreign firm responds to a price cut by cutting its price). Thus, sign t^* is the same as the sign of Π_{21}, which is positive when the products are substitutes and returns to scale are non-increasing, so a tax would be required.

Proposition 9 *When the home firm's conjectures are 1) consistent, free trade is best; 2) Bertrand, export tax is best; 3) Cournot, export subsidy is best.*

Complications

The direction of the optimal policies is robust to foreign policy response. What if the number of firms is greater than two (a multifirm oligopoly)? Assume symmetric countries except for that the number of firms at home equal n and m abroad. Further assume consistent conjectures to isolate the role of the number of firms. The optimal policy is a tax when the number of home firms exceeds one $n > 1$. This result is the usual terms of trade argument – want to increase the price of your exports. By taxing the firms, you lower sales abroad, increase price and transfer foreign consumer surplus to the domestic economy. With entry, profit shifting benefits can be dissipated by increased entry costs or enhanced by foreign exit and domestic entry. With domestic consumption (and consistent conjectures), increasing marginal costs for the foreign firm give rise to export subsidy being optimal.

Dixit and Grossman *JIE* 1986

Dixit and Grossman argue that in reality there are may oligopolistic industries that can potentially be targeted for export subsidies. They show that if these any industries use a common factor that is available in fixed supply, optimal policy cannot be determined in partial equilibrium (one industry at a time). An export subsidy to one industry bids up the factor price and essentially discourages the other industries. Hence the government would need detailed information on which industries would most benefit form receiving export subsidies.

To demonstrate the need for targeted export promotion, assume:

- Two countries: home and foreign
- $n > 1$ symmetric high-tech industries - output $y_i = y$ - and one low-tech industry - output x
- Two factors: workers and scientists (skilled labor), each in fixed supply: l and k (foreign country does not face resource constraint for simplicity)
- CRS technology: one unit of scientists and a units of workers produces one unit of any high-tech good; one unit of workers produces one unit of low-tech good
- each high-tech industry is a Cournot duopoly with one domestic and one foreign firm, low-tech industry has perfect competition
- All consumption of high-tech goods occurs in a third country.

Normalize the price of the low-tech good to one, which then implies that the wage for workers is one (value of the marginal product of workers). The cost of producing one unit of a high-tech good is $c = a + z$ where z is the wage for scientists. Firms take the scientific wage z as given.

Resource Constraints

The demand for workers must equal supply

$$x + any = l \tag{6.13}$$

and the demand for scientists must equal supply

$$ny = k \tag{6.14}$$

The two resource constraints imply that output of the numeraire good is determined by the residual supply of workers after producing the high-tech goods

$$x + ak = l \rightarrow x = l - ak \tag{6.15}$$

(the numeraire good and the workers play no key role in the symmetric model). The second resource constraint implies output of each high tech good equals the number of scientists per industry

$$y = \frac{k}{n} \tag{6.16}$$

High-Tech Output

Consider a symmetric export subsidy for all high-tech industries. Let s be the subsidy per unit of domestic production. Let $r(y, Y)$ be the revenue function for a home firm. Profits for a home firm are thus

$$\pi = r\,(y, Y) - [a + z - s]\,y$$

The first order condition for the domestic firm choosing y to maximize its profits equates marginal revenue to marginal cost (net of the subsidy)

$$r_y\,(y, Y) = a + z - s \tag{6.17}$$

Profits for a foreign firm are

$$\Pi = R\,(y, Y) - CY$$

and the foreign firm's FOC also equates marginal revenue to marginal cost

$$R_Y\,(y, Y) = C$$

The foreign FOC gives the foreign firm's reaction function

$$Y = B(y) \tag{6.18}$$

The equilibrium is the output of domestic and foreign firms, scientific wage, and output of the numeraire good $\{y, Y, z, x\}$ that solve the two resource constraints and the two FOCs. In the symmetric case, the two resource constraints pin down y and x, so the FOCs determine Y and z. Since the output of each domestic firm y is determined exclusively by resource availability, the subsidy fails to shift profits toward domestic firms. All that happens is the scientific wage z rises by the amount of the subsidy. Hence, the domestic firm's costs net of the subsidy $z + a - s$ is unchanged and Y is unchanged. To be effective, any export subsidy must be asymmetric - to target some industries but not others. But to implement an asymmetric subsidy scheme, the government needs information on which industries to favor over others.

Optimal Policy

Define domestic welfare as the sum of factor earnings and profits minus subsidy payments which is equivalent to consumption of the numeraire good plus total revenue from the high-tech sector.

$$w = l + zk + n\pi - nsy$$

$$\to w = l + zk + n(\pi - sy)$$

$$\to w = l + zk + n(r - [a + z - s]y - sy)$$

$$\to w = l + zk + n(r - ay - zy)$$

$$\to w = l - any + zk - nz\frac{k}{n} + nr$$

$$\to w = x + nr$$

Any subsidy level yields the same x, y, Y, and $z - s$, so export subsidies fail to raise domestic welfare as they fail to shift profits towards domestic firms (for symmetric industries). When oligopolistic industries all use a factor available in fixed supply, the export promotion property of export subsidies hinges on the ability to target subsidies to the industries with the greatest profit shifting potential. However, the government is apt to lack the information needed to determine which industries have the greatest profit shifting potential, so export promotion is apt to fail as a method of raising domestic welfare.