2 COMPARATIVE ADVANTAGE AND GAINS FROM TRADE

Introduction

Traditional international trade theory has concerned itself with these central questions:

- What determines the pattern of trade? Who trades what with whom and at what prices?
- What are the sources of gains from trade? How are the gains distributed across countries?
- How does trade alter the structure of production and returns to factors within each country?

The first set of questions leads to the notion that the pattern of trade is based on comparative advantage. The second set of questions is addressed by the result that there are always gains from trade, and both countries will gain from trade provided the relative price under free trade differs from both country's relative prices under autarky. The last question is concerned about redistributive consequences. Trade policies may be motivated by shifting the distribution of income within a country (or countries). Owners of a country's relatively scarce factor (associated with the import competing sector) will lose as a result of trade, even though the country as a whole gains.

We start with the Ricardian model, which nicely illustrates comparative advantage and gains from trade - where trade occurs due to technology differences across countries. We will explore distribution implications in the next chapter on factor endowment models of international trade.

Ricardian Model

Assumptions

- Two goods: cloth C and wheat W.
- Two countries: home and foreign *
- One factor: labor L, in fixed supply and immobile across countries
- CRS technology: a_{Lj} units of labor produce one unit of $j \in \{C, W\}$

Comparative Advantage (CA)

Home has comparative advantage over Foreign in wheat relative to cloth$\Leftrightarrow$ Foreign has comparative advantage over Home in cloth relative to wheat

$$\frac{a_{LW}}{a_{LC}} < \frac{a^*_{LW}}{a^*_{LC}}$$

If a country has comparative advantage in a good, that country has a lower opportunity cost of producing that good than the other country.

Production Possibilities Frontier (PPF):

A country's PPF describes the maximal bundles of cloth and wheat that can be produced given factor supply and technology. The labor constraint restricts the sum of labor demands, labor used making cloth $a_{LC}S_C$ and labor used making wheat $a_{LW}S_W$, in each country to not exceed total labor supply L.

$$a_{LC}S_C + a_{LW}S_W = L \Longleftrightarrow S_C = \frac{L}{a_{LC}} - \frac{a_{LW}}{a_{LC}}S_W$$

The opportunity cost of wheat in terms of cloth appears as (absolute value of) the slope of the PPF.

World PPF

The world PPF describes the maximal bundles the world can produce. The world PPF is found by pasting

together the PPFs for the two countries, with a kink in the middle (due to difference in opportunity costs). We need to paste home and foreign PPFs together so that the world PPF kinks outward. The kink occurs where each country is specialized in its comparative advantage good. The objective is to find the maximum production; a kinked in version would have less produced (each country would be specialized in non-comparative advantage good at the inward kink). Correctly constructed, the world PPF has home produce wheat first, then foreign, in accordance with each country's comparative advantage. The country with comparative advantage in wheat should start producing what first because producing an additional unit of wheat there requires less sacrifice of cloth than in the other country.

Example

Given the technology

	Cloth	Wheat
Home	$a_{LC} = 1$	$a_{LW} = 1$
Foreign	$a^*_{LC} = 2$	$a^*_{LW} = 6$

we establish that Home has comparative advantage in wheat (and Foreign in cloth)

$$\frac{a_{LW}}{a_{LC}} = 1 < 3 = \frac{6}{2} = \frac{a^*_{LW}}{a^*_{LC}}$$

Given labor endowments $L = 500$ and $L^* = 600$, we construct each country's PPF as in **Figure 1.1**.

$$S_C + S_W = 500 \Longleftrightarrow S_C = 500 - S_W$$

$$2S^*_C + 6S^*_W = 600 \Longleftrightarrow S^*_C = 300 - 3S^*_W$$

The world PPF is drawn in **Figure 1.2**. Initially, foreign specializes in cloth ($S^*_C = 300$, $S^*_W = 0$) and any increases in wheat occur according to home's PPF ($S_C = 500 - S_W$). One home becomes specialized in wheat ($S_C = 0$, $S_W = 500$), any further increases in wheat must occur according to foreign's PPF ($S^*_C = 300 - 3S^*_W$). The numbers in the above example are not special: as long as the opportunity costs differ across countries, there exists a basis for trade. Absolute advantage is not at all important; a country could have an absolute advantage (or disadvantage) in both goods and still gain from trade.

Relative Prices

The relative price under free trade is found at the intersection of world relative supply (step function with steps at each country's opportunity cost of wheat in terms of cloth)

$$RS = \frac{S_W + S_W^*}{S_C + S_C^*} = \begin{array}{c} 0 \ldots \frac{L/a_{LW}}{L^*/a_{LC}^*},\ \frac{P_W}{P_C} = \frac{a_{LW}}{a_{LC}} \\ \frac{L/a_{LW}}{L^*/a_{LC}^*} \ldots \infty,\ \frac{P_W}{P_C} = \frac{a_{LW}^*}{a_{LC}^*} \end{array}$$

and relative demand (such as $D_W/D_C = P_C/P_W$). The free trade relative price lies weakly between the two autarkic relative prices.

$$\frac{a_{LW}}{a_{LC}} \leq \frac{P_W}{P_C} \leq \frac{a_{LW}^*}{a_{LC}^*}$$

Neither country would produce wheat if the relative price of wheat were lower than both opportunity costs of wheat, and likewise neither would produce cloth if the relative price of wheat exceeded both opportunity costs (because then the relative price of sloth would be below both opportunity costs of cloth).

Production Pattern

Efficiency in the world economy requires that at least one country specialize in its comparative advantage good. Three different cases can occur depending on relative demand for the two goods. If relative demand is particularly strong for one of the goods, the country that does not have comparative advantage in that good will not gain from trade (but does not lose from trade either). Such a country will have to remain incompletely specialized to help produce its non-comparative advantage good, and thus, free trade relative price will not differ from its autarkic relative price (opportunity cost). The failure of the free trade relative price to differ from the autarky relative price for that country eliminates the opportunity to obtain the non-comparative advantage good through trade at a lower expense than producing it directly. In cases of moderate relative demand is most common, especially when comparative advantage is strong. Strong comparative advantage (large differences in opportunity costs across countries) make the case where both counties specialize in their comparative advantage good more likely to occur (occurs for more levels of relative demand).

Trade Possibilities Frontier (TPF)

A country's TPF describes the maximal consumption bundles under free trade. The TPF is a budget constraint based on the total value of a county's production bundle at the free trade relative price. The slope of the TPF is the free trade relative price.

Trade Pattern

The Ricardian model predicts the direction of trade: each country exports its comparative advantage good. Regardless of the pattern of production (a country may produce both goods or just its comparative advantage good), the pattern of trade is clear. Home exports cloth and imports wheat; foreign exports wheat and imports cloth. We need knowledge of relative demand to pin down the exact volume of trade.

Gains From Trade

Provided (and to the extent that) the free trade relative price differs from autarkic relative price, a country (as a whole) gains from trade. In the Ricardian model, the condition for gains from trade is equivalent to saying a country gains whenever it becomes completely specialized in its comparative advantage good.

Higher Dimensions

When countries can produce more than two goods, comparison of bilateral labor coefficient ratios is not enough to determine comparative advantage since only considers direct, not indirect, rearrangements. Problems arise in predicting pattern of trade due to potential complementarity in production. Proper criterion is maximizing the product of all the labor coefficients. Welfare implications do not depend on dimensionality, as the positive statements do. To make a statement of comparative advantage for a general technology, we need to introduce some basic concepts first.

National Income

Transformation Curve

Definition 1 *Factor supplies and technology determine the PPF (or transformation curve)*

$$S_C = T(S_W)$$

where S_C denotes the supply of cloth and S_W denotes the supply of wheat.

In **Figure A.1**, C_o denotes maximal production of cloth and W_o denotes maximal production of wheat. The derivative of the PPF is negative, $T'(S_W) \equiv \frac{\partial S_C}{\partial S_W} < 0$, since producing more wheat requires producing less cloth.

National Income Function

At world prices denoted by P_j, $j \in \{C, W\}$, with vector of prices $\mathbf{p} \equiv (P_W, P_C)$, national income is the sum of income from producing cloth and wheat.

$$y = P_C S_C + P_W S_W$$

Budget line shows combinations of (S_C, S_W) that yield a given level of income y given prices $\mathbf{p}$. Maximizing y subject to the constraint $S_C = T(S_W)$ yields the first order condition

$$P_C dS_C + P_W dS_W = 0$$

Assuming an interior solution, income is maximized by producing where the slope of the budget line equals the slope of the PPF.

$$T'(S_W) \equiv \frac{dS_C}{dS_W} = -\frac{P_W}{P_C} \tag{2.1}$$

Otherwise, produce only cloth or only wheat. Denote by w_o the slope of the PPF at maximal wheat production W_o. Similarly, denote by c_o the slope of the PPF at maximal cloth production C_o. If $\frac{P_W}{P_C} > w_o$ then the budget line is steeper than the PPF everywhere and the country specializes in wheat (produces no cloth). Similarly, if $\frac{P_W}{P_C} < c_o$

then the budget line is flatter than the PPF everywhere and the country specializes in cloth (produces no wheat). Finally, if $c_o < \frac{P_W}{P_C} < w_o$ the point of production is the tangency between the PPF and the budget line and the country makes both goods. Let $S \equiv (S_W, S_C)$ denote the production vector.

Definition 2 *The* **National Income Function** *(NIF) records the highest income attainable for a country under different circumstances*

$$y(\mathbf{p}; \ldots) = \sum_i P_i S_i = \mathbf{p} \cdot \mathbf{S}$$

where outputs are optimally chosen by competitive producers, given factor prices and goods prices.

For the two good case, tangency between the budget line and the transforation curve determines the optimal production point.

NIF Properties

Consider fixing the price of cloth P_C and varying the price of wheat P_W. How national income changes depends upon the pattern of production. If $P_W < c_o P_C$, the country produces no wheat so national income $y(\mathbf{p}; \ldots) = P_C C_o$ is unaffected.

$$\frac{\partial y(\mathbf{p}; \ldots)}{\partial P_W} = 0, \frac{\partial^2 y(\mathbf{p}; \ldots)}{\partial^2 P_W} = 0; P_W < c_o P_C$$

If $P_W > w_o P_C$, the country produces only wheat so national income $y(\mathbf{p}; \ldots) = P_W W_o$ increases with price of wheat.

$$\frac{\partial y(\mathbf{p}; \ldots)}{\partial P_W} = W_o, \frac{\partial^2 y(\mathbf{p}; \ldots)}{\partial^2 P_W} = 0; P_W > w_o P_C$$

If $c_o P_C > P_W > w_o P_C$, the country produces both goods and national income increases with price of wheat at an increasing rate. The derivative of the NIF with respect to the price of wheat P_W equals the supply of wheat. Differentiating the NIF with respect to the price of wheat and using the property that optimization in production requires that the slope of the PPF equal ratio of prices (2.1), an Envelope theorem,

$$\frac{\partial y(\mathbf{p};\ldots)}{\partial P_W} = S_W + \underbrace{\left[P_C \frac{dS_C}{dP_W} + P_W \frac{dS_W}{dP_W}\right]}_{0} = S_W > 0$$

so national income increases in the price of wheat. Since production of wheat increases with the price of wheat, national income increases at an *increasing* rate.

$$\frac{\partial^2 y(\mathbf{p};\ldots)}{\partial^2 P_W} = \frac{\partial S_W}{\partial P_W} > 0$$

In **Figure A.2**, the slope of y shows production of wheat S_W since national income is plotted against the price of wheat. In autarky, what is produced is the same as what is consumed $S_W = D_W$, so the point of production where national income is maximized for at the autarky prices is also the point of consumption.

Ricardian Model Revisited

What does the NIF look like in the Ricardian model? Recall that the PPF is linear in the Ricardian model. Maximal production of cloth is $C_o = \frac{L}{a_{LC}}$, maximal production of wheat is $W_o = \frac{L}{a_{LW}}$ and the (absolute value of the) slope of the PPF equals $\frac{a_{LW}}{a_{LC}} = c_o = w_o$. The prices of wheat $c_o P_C$ and $w_o P_C$ that were distinct points above and below the autarky price of wheat collapse to the autarky price of wheat. As in **Figure A.3**, the NIF is a horizontal line until the price of wheat exceeds $P_W^A = \frac{a_{LW}}{a_{LC}} P_C$ when the country switches from specializing in cloth to specializing in wheat. For prices of wheat above the autarky price $P_W > P_W^A$, the NIF increases at a constant rate with the price of wheat – all resources are allocated to wheat production so there can be no further increase in wheat production as the price of wheat rises. The kink means that at the critical price, any pattern of production yields the same income so that the slope of the NIF is undefined.

National Expenditure

National Expenditure Function

We assume the existence of community indifference curves $u(D_C, D_W)$.[1] Total expenditure is sum of expenditures on cloth and wheat

$$E = P_C D_C + P_W D_W$$

where D_C is consumption of cloth and D_W is consumption of wheat. Minimizing E subject to the constraint of providing at least a chosen level of utility (such as autarky), $u(D_C, D_W) \geq u^A$, or equivalently, maximizing utility for a chosen level of expenditure, yields first order condition

$$P_C dD_C + P_W dD_W = 0$$

so expenditure is minimized by consuming where slope of budget line equals slope of indifference curve as in **Figure A.4**.

$$\frac{dD_C}{dD_W} = -\frac{P_W}{P_C} \tag{2.2}$$

Definition 3 *The* **National Expenditure Function** *(NEF) records the minimum expenditure required to achieve a chosen level of utility for various prices*

$$E(\mathbf{p}; u) = \sum_i P_i D_i = \mathbf{p} \cdot \mathbf{D}$$

where consumption D_i are optimally chosen by consumers.

NEF Properties

The partial derivative with respect to prices gives quantity consumed (envelope theorem) due to (2.2)

$$\frac{\partial E(\mathbf{p}; u)}{\partial P_W} = D_W + \left[P_C \frac{dD_C}{dP_W} + P_W \frac{dD_W}{dP_W} \right] = D_W > 0$$

[1] Under homothetic preferences, these exist and are well defined. Intuition is clear: if all consumers spend a certain proportion θ of their income on one good and $1 - \theta$ on the other, the economy as a whole must have the same pattern of consumption. Preferences are said to be homothetic when consumers spend a fixed proportion of their income on each good. More technically, preferences are homothetic if given that you prefer a bundle (x_1, y_1) to (x_2, y_2), you also prefer the bundle (tx_1, ty_1) to (tx_2, ty_2): as long as the ratio of x to y is fixed, your ranking of the two bundles does not change.

Furthermore, the expenditure function is concave in price

$$\frac{\partial^2 E(\mathbf{p}; u)}{\partial^2 P_W} = \frac{\partial D_W}{\partial P_W} < 0$$

because consumers substitute toward the cheaper good as the price of a good rises. Expenditure increases as price increases but at a decreasing rate as in **Figure A.5**.

Autarkic Equilibrium

The autarky equilibrium in the economy is given by the tangency of the NIF and the NEF. Consider an autarkic price of wheat P_W^A yielding an autarkic level of utility u^A. The national expenditure needed to achieve the autarkic level of utility is described by $E(\mathbf{p}; u^A)$. Meanwhile, national income is described by $y(\mathbf{p}; \ldots)$. National income equals national expenditure at autarky prices

$$y(\mathbf{p}^A; \ldots) = E(\mathbf{p}^A; u^A)$$

and the quantity supplied equals the quantity demanded $S^A = D^A$.

Free Trade Equilibrium

Let u^T represent the level of utility under free trade. Two conditions need to be satisfied in a free trade equilibrium: national income equals national expenditure in each country

$$y(\mathbf{p}^T; \ldots) = E(\mathbf{p}^T; u^T)$$

$$y^*(\mathbf{p}^T; \ldots) = E^*(\mathbf{p}^T; u^{T*})$$

and the market for each good clears at the world level (world demand equals world supply).

$$S + S^* = D + D^*$$

Gains From Trade

Revisiting **Figure A.5**, for the autarky price of wheat $P_W = P_W^A$, income equals expenditure $y = E$ since autarkic supply equals autarkic demand $S^A = D^A$. Due to the curvature of the national expenditure and national

income functions, national income exceeds national expenditure $y > E$ for all prices differing from the autarky price $P_W \neq P_W^A$. Whenever free trade price differs from autarky, national income is more than enough to yield autarkic level of utility. The gains are larger the more the price differs from its autarkic level. A country can increase its utility by trading and thus gains from trade at the national level.

Comparative Advantage

By the gains from trade, utility under free trade exceeds utility under autarky. Thus, the minimum expenditure required to reach the free trade level of utility at autarkic prices is higher than the minimum expenditure required to reach the autarkic level of utility at autarkic prices

$$E(\mathbf{p}^A; u^T) \geq E(\mathbf{p}^A; u^A)$$

The expenditure function shifts up as in **Figure A.6**, yielding two intersections of the expenditure and the income functions. At point B to the right of the tangency at point A, the slope of the y curve exceeds the slope of the E curve so the supply of wheat exceeds the demand of wheat and the country exports wheat

$$X_W \equiv S_W - D_W > 0$$

Also, the national income y curve is steeper at point B than at point A so the supply of wheat S_W has risen. At point C to the left of the tangency at point A, the slope of the E curve exceeds the slope of the y curve so the demand of wheat exceeds the supply of wheat and the country imports wheat

$$M_W \equiv D_W - S_W > 0$$

Similarly, the national income y curve is flatter at point C than at point A so the supply of wheat S_W has fallen. Whichever country exports wheat, the other country must be importing wheat (by the world demand equals world supply condition for a free trade equilibrium). Thus, if the home country is at an equilibrium such as point B where $P_W^T > P_W^A$, the foreign country must be at an equilibrium such as point C where $P_W^T < P_W^{A*}$. Since the two countries

share a common free trade price, by transitivity the free trade price must be in between the two autarky prices

$$P_W^{A*} > P_W^T > P_W^A$$

so tangency of NIF and NEF under autarky for the home country lies to the left of the tangency under autarky for the foreign country. Thus, comparing the autarky prices predicts the pattern of trade: the country with the lower autarky price of wheat exports wheat under free trade. Alternatively, the home country could be at a point like C and the foreign country would then be at a point like B (the pattern of comparative advantage is simply reversed). For two countries and two goods, comparative advantage dictates:

- The free trade price lies between the two autarkic prices.
- Each country exports the good for which it has a lower autarkic price than the other country.
- Each country experiences a rise in the price of its comparative advantage good in the move from autarky to free trade.
- Each country expands the production of its comparative advantage good in the move from autarky to free trade.

What can we say about gains from trade and comparative advantage in general?

General Gains from Trade

Let $\mathbf{p}^T$ denote the vector of free trade prices, S^T and S^A denote the free trade and autarkic vectors of output produced and let D^A be the autarkic consumption vector. Then, national income is higher at the free trade price when free trade supply is chosen than were the autarky supply chosen instead

$$\mathbf{p}^T S^T = y(\mathbf{p}^T; \ldots) \geq \mathbf{p}^T S^A$$

Furthermore, the quantities supplied must equal the quantities demanded in autarky (by the definition of an autarkic equilibrium)

$$S^A = D^A$$

Therefore, substituting autarkic demand for autarkic supply, national income is sufficient to purchase the autarkic consumption bundle at free trade prices.

$$\mathbf{p}^T S^T \geq \mathbf{p}^T D^A$$

Thus, the country gains from trade. Does each individual necessarily gain? No. A system of lump-sum taxes and subsidies is required to ensure that each individual can afford the autarkic consumption bundle if individuals are not all alike.

General Comparative Advantage

We again make use of community indifference curves or assume that there are lump sum transfers which make sure that no one is worse off under free trade. Then the vector of commodities consumed under free trade must yield the autarkic level of utility. Therefore,

$$\mathbf{p}^A D^T \geq E(\mathbf{p}^A, u^A) = y(\mathbf{p}^A;) = \mathbf{p}^A S^A \tag{2.3}$$

But the autarkic supplies must yield higher income at autarkic prices than the free trade supplies

$$\mathbf{p}^A S^A \geq \mathbf{p}^A S^T \tag{2.4}$$

From the above two inequalities, we have that producing the free trade supplies would not yield enough income to afford the free trade consumption bundle at autarky prices

$$\mathbf{p}^A D^T \geq \mathbf{p}^A S^T$$

or equivalently

$$\mathbf{p}^A (D^T - S^T) \geq 0$$

But under free trade, value of imports must equal that of exports, so that

$$\mathbf{p}^T (D^T - S^T) = 0$$

Combining the last two expressions implies the following result:

Proposition 4 *A country's imports are positively correlated with lower trade prices relative to autarky:* $(\mathbf{p}^A - \mathbf{p}^T)(D^T - S^T) \geq 0$.

The same logic for the foreign country implies

$$(\mathbf{p}^{A*} - \mathbf{p}^{T})(D^{T*} - S^{T*}) \geq 0$$

Since home country imports equal foreign country exports

$$(D^{T} - S^{T}) = (S^{T*} - D^{T*})$$

we have

$$(\mathbf{p}^{T} - \mathbf{p}^{A*})(D^{T} - S^{T}) \geq 0$$

This inequality together with the above proposition implies the following:

Proposition 5 *Imports are positively correlated with lower foreign autarkic prices relative to domestic:* $(\mathbf{p}^{A} - \mathbf{p}^{A*})(D^{T} - S^{T}) \geq 0$.

Finally, autarky supply is optimal at autarky prices

$$\mathbf{p}^{A}S^{A} = y(\mathbf{p}^{A};) \geq \mathbf{p}^{A}S^{T} \rightarrow \mathbf{p}^{A}\left(S^{A} - S^{T}\right) \geq 0$$

and free trade supply is optimal at free trade prices

$$\mathbf{p}^{T}S^{T} = y(\mathbf{p}^{T};) \geq \mathbf{p}^{T}S^{A} \rightarrow \mathbf{p}^{T}\left(S^{T} - S^{A}\right) \geq 0$$

Thus we have our last main result regarding comparative advantage.

Proposition 6 *Output reductions are positively correlated with lower trade prices relative to autarky:* $(\mathbf{p}^{A} - \mathbf{p}^{T})(S^{A} - S^{T}) \geq 0$.

How is the free trade price determined? The general version of the conditions for a free trade equilibrium dictate that each country's trade must be balanced

$$pX = 0$$

$$pX^{*} = 0$$

and each market must clear, that is one country's exports must be another country's imports

$$X - X^{*} = 0$$

These conditions jointly determine the free trade price vector and the vector of exports for both countries $\{p, X, X^{*}\}$.

General conclusion: *Although comparative advantage may not hold on a commodity by commodity basis, it holds on average in a multi-commodity environment.*

The next question is what determines comparative advantage? The Ricardian model has shown that comparative advantage may stem from differences in technologies across countries. The HOS model gives another source of comparative advantage: differences in factor endowments across countries. Both sources of comparative advantage are likely to be present in any given setting, so the models are not mutually exclusive. However, the HOS model (and the Specific Factors model) help us understand why some groups within an economy may protest free trade movements. While the country as a whole gains from trade, some part of the country may lose from trade if agents derive their income from different factors.