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Substitution in R&D across countries

Amy Jocelyn Glass*

Department of Economics, Texas A&M University, 3035 Bush Academic Building West, 4228 TAMU, College Station, TX 77845-4228, USA

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Abstract

Suppose imitation becomes easier in developing countries due to international technology diffusion or other reasons. If imitation also occurs in the North, Northern imitators (as well as innovators) would become less secure from low-cost Southern imitations. Does the resulting rise in Southern imitation lead to a decrease in Northern imitation (and perhaps even innovation)? If so, does the overall rate of imitation rise or fall? This paper builds a product cycle model with imitation in both the North and the South (along with innovation in the North) to address questions related to the level and composition of imitation across countries.

JEL classification: F1; F4; O3

Keywords: R&D; Innovation; Imitation; Product cycles

1. Introduction

The rapid advancement in high-technology products contributes substantially to economic growth. Innovations generally occur in developed countries, while imitations occur in all but the poorest of countries. A literature on product cycles has emerged to explain the implications of imitation for the speed of innovations. In these models, innovation generally occurs only in the North and imitation only in the South. This paper makes several contributions to this product cycle literature.

Compared to two-country models of Northern innovation and Southern imitation such as Grossman and Helpman (1991), we add Northern imitation, with imitation easier

* Tel.: +1-979-845-8507; fax: +1-979-847-8757.
E-mail address: aglass@econ.tamu.edu (A.J. Glass).

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than innovation. Similar to Segerstrom (1991), Northern imitation can be supported by Northern imitators colluding with the innovators whose products they targeted. Southern imitators have a cost advantage in production compared to Northern imitators. So Southern firms may have an incentive to imitate even if a Northern firm has already imitated. The Southern firm can charge a price equal to (or a hair below) the Northern imitator's cost and thus capture the whole market while still earning a profit.

This paper addresses how changes in parameters—such as resource supplies, R&D efficiency, and R&D subsidies—alter the rates of innovation, Northern imitation, and Southern imitation. The changes in these rates determine changes in the overall rate of imitation (the sum of Northern and Southern imitation) and in the mix of imitation between the North and the South and the mix of Northern R&D between innovation and imitation. Previous studies could not examine these compositions as there was no Northern imitation along with Southern imitation.

Previous work could not address the effect of a subsidy to all Northern R&D (both innovation and imitation) on Southern imitation. If imitation occurs in the North together with innovation, the government may not be able to effectively target subsidies to only innovation. Hence, we examine the effects of an overall R&D subsidy, applied to all Northern R&D (both innovation and Northern imitation). Does such an overall R&D subsidy increase both Northern imitation and innovation? What about Southern imitation? The incentives for Southern imitation could be reduced due to a shorter expected duration of profits when there is more innovation. But if aggregate expenditure rises or the Northern-to-Southern relative wage rises, the level of profits might rise through a larger volume of sales or a larger price–cost margin. Thus, a subsidy to Northern R&D might increase or decrease Southern imitation.

Also, we are able to address the impact of a R&D subsidy to Southern imitation in the presence of Northern imitation. The expansion in Southern imitation might crowd out Northern imitation by decreasing the expected duration of a Northern imitator's profit stream. However, other effects such as increased sales prior to Southern imitation or a lower Northern-to-Southern relative wage might preserve the incentive for Northern firms to imitate. Similarly, this model is needed to address the impact of increased Southern imitation due to an increase in the labor supply in countries such as China and India on Northern imitation and innovation.

Even though Southern firms engage in imitation, Northern firms may be more efficient at imitation due to proximity to innovators and a more advanced state of knowledge. Enhanced knowledge flows across countries could cause a diffusion of imitation technology across borders that could reduce the difficulty of Southern imitation. Here too Southern imitation should rise but the effects on Northern imitation and innovation are not clear without a model. Or in the reverse direction, negotiated agreements could force the South to strengthen its intellectual property rights (IPR) protection, which could make imitation more difficult. The goal of such agreements is to increase innovation, but any resulting reduction in Southern imitation might lead to additional incentives for imitation in the North, which might cause innovation to fall. The effects of changing R&D efficiency on the level and composition of R&D help shed light on some of these issues.

2. Model with Northern and Southern imitation

Consumers live in one of two countries, North and South, and choose from a continuum of products available in discrete quality levels. Consumer preferences are as specified in Grossman and Helpman (1991), so the consumer's problem shares the same solution.

2.1. Consumers

Consumers choose from a continuum of products $j \in [0, 1]$. Quality level m of product j provides quality $q_m(j) \equiv \lambda^m$. By the definition of quality improvement, new generations are better than the old: $q_m(j) > q_{m-1}(j) \rightarrow \lambda^m > \lambda^{m-1} \rightarrow \lambda > 1$. All products start at time $t = 0$ at quality level $m = 0$, so the base quality is $q_0(j) = \lambda^0 = 1$.

A consumer from country $i \in \{\mathrm{N}, \mathrm{S}\}$ has additively separable intertemporal preferences given by lifetime utility

$$U_i = \int_0^\infty \mathrm{e}^{-\rho t} \log u_i(t)\, \mathrm{d}t, \tag{1}$$

where ρ is the common subjective discount factor. Instantaneous utility is

$$\log u_i(t) = \int_0^1 \log \sum_m (\lambda)^m x_{im}(j, t)\, \mathrm{d}j, \tag{2}$$

where $x_{im}(j, t)$ is consumption by consumers from country i of quality level m of product j at time t.

Consumers maximize lifetime utility subject to an intertemporal budget constraint. Since preferences are homothetic, aggregate demand is found by maximizing lifetime utility subject to the aggregate intertemporal budget constraint

$$\int_0^\infty \mathrm{e}^{-R(t)} E_i(t)\, \mathrm{d}t \leq A_i(0) + \int_0^\infty \mathrm{e}^{-R(t)} Y_i(t)\, \mathrm{d}t, \tag{3}$$

where $R(t) = \int_0^t r(s)\, \mathrm{d}s$ is the cumulative interest rate up to time t and $A_i(0)$ is the aggregate value of initial asset holdings by consumers from country i. Individuals hold assets in the form of ownership in firms, but with a diversified portfolio, any capital losses appear as capital gains elsewhere so only initial asset holdings remain. Aggregate labor income of all consumers from country i is $Y_i(t) = L_i w_i(t)$, where $w_i(t)$ is the wage in country i at time t and L_i is the labor supply there, so $L_i w_i(t)$ is total labor income in country i at time t. Aggregate expenditure of all consumers in country i is

$$E_i(t) = \int_0^1 \left[\sum_m p_m(j, t) x_{im}(j, t) \right] \mathrm{d}j, \tag{4}$$

where $p_m(j, t)$ is the price of quality level m of product j at time t, and $E_i(t)$ is aggregate expenditure of consumers in country i, where overall aggregate expenditure is $E(t) = E_\mathrm{N}(t) + E_\mathrm{S}(t)$. Due to assumed free trade, price levels do not vary across countries.

A consumer's maximization problem can be broken into three stages: the allocation of lifetime wealth across time, the allocation of expenditure at each instant across products, and the allocation of expenditure at each instant for each product across available quality levels. In the final stage, consumers allocate expenditure for each product at each instant to the quality level $\tilde{m}(j,t)$ offering the lowest quality-adjusted price, $p_m(j,t)/\lambda^m$. Consumers are indifferent between quality level m and quality level $m-1$ if the relative price equals the quality difference $p_m(j,t)/p_{m-1}(j,t) = \lambda$. Settle indifference in favor of the higher quality level so the quality level selected is unique (or view the high-quality firm as offering a quality-adjusted price slightly below that of the low-quality firm). Only the highest quality level available will sell in equilibrium. Glass (1997) has dealt with the case where more than one quality level of each product sells (but without Northern imitation).

In the second stage, consumers then evenly spread expenditure across the unit measure of all products, $E_i(j,t) = E_i(t)$, as the elasticity of substitution between any two products is constant at unity. Consumers demand $x_{i\tilde{m}}(j,t) = E_i(t)/p_{\tilde{m}}(j,t)$ units of quality level $\tilde{m}(j,t)$ of product j and no units of other quality levels of that product. In the first stage, consumers evenly spread lifetime expenditure across time, $E_i(t) = E_i$, as the utility function for each consumer is time separable and the aggregate price level will be shown to not vary across time $\log p_{\tilde{m}}(j,t) = \log p_{\tilde{m}}(j)$. Since aggregate expenditure is constant across time, the interest rate at each point in time reflects the discount rate $r(t) = \rho$, so $R(t) = \rho t$ in the intertemporal budget constraint.

2.2. Production

All innovation occurs in the North, while imitation occurs in both the North and the South. We normalize prices by the Southern wage $w_S = 1$. The Northern wage relative to the Southern wage thus equals the Northern wage $w \equiv w_N/w_S = w_N$. Normalizing the labor supplies in each country so that one unit of labor is needed to produce one unit of output (of any quality) in either country, Northern firms have production cost w, while Southern firms have production cost 1. Successful Northern imitators collude with innovators since they lack any cost advantage, whereas Southern imitators undercut the price of innovators (and any Northern imitators) to capture the full market due to their production cost advantage (details will be discussed in Section 2.4).

In the production stage, each firm chooses its price given the prices (and R&D intensities) of other firms. A Northern firm with a one quality level lead over its closest rival charges a price reflecting the willingness to pay for quality improvements $p_N = \lambda$ and thus earns instantaneous profits

$$\pi_N = E\left(1 - \frac{w}{\lambda}\right), \tag{5}$$

where E is aggregate expenditure. Northern innovators are assumed to face Southern firms as their rivals one quality level below—regardless of whether they innovated over a Northern or a Southern firm due to an assumption of free access to discarded technology or equivalently costless licensing of obsolete technology. The technology for producing any quality level of a good that is below the highest available (the state-of-the-art) has no value as it can generate no profits in equilibrium. Thus, as in Glass (1997), any firm that previously innovated or imitated a now obsolete technology would have no reason to protect its product design.

If another Northern firm then imitates that quality level, the two Northern firms have the same costs of production and collude by maintaining the limit price $p_M = \lambda$ and splitting the market evenly, each earning instantaneous profits

$$\pi_M = \frac{E}{2}\left(1 - \frac{w}{\lambda}\right) = \frac{\pi_N}{2}. \quad (6)$$

The profits for each firm after Northern imitation are exactly half of the profits for an innovator prior to imitation, so total industry profits remain unchanged. This property follows from the ability of the innovator and Northern imitator to perfectly collude using a tit-for-tat strategy (see Section 2.4 for details). The model can be modified to allow the profits after imitation to be any fraction less than one-half of monopoly profits, if post-imitation collusion is imperfect.

If a Southern firm imitates that quality level (either before or after a Northern firm has imitated it), the Southern firm can capture the whole market due to its lower costs by charging a price equal to a Northern firm's cost $p_S = w$ and earning instantaneous profits

$$\pi_S = E\left(1 - \frac{1}{w}\right). \quad (7)$$

Southern firms capture the whole market by undercutting the price of Northern firms, rather than colluding to split the market.

The instantaneous profits are integrated over time to determine the value of an innovation or imitation. The probability that a stream of profits will be terminated depends on the R&D targeting the market. We assume all Northern firms are equally able to conduct innovation (regardless of whether they made the previous innovation), so innovation targets all markets, including products that have not yet been imitated. New introductions are exposed to both Northern imitation and Southern imitation. Following Northern imitation, the Northern innovator and imitator collude to share the market, but are still exposed to Southern imitation due to their higher costs than a Southern firm.

A successful innovator earns profits (5) until subsequent innovation or imitation occurs. If innovation or Southern imitation occurs, the profit stream ends and the firm looses all value. Thus, the reward to innovation (the value gained by a successful innovator) is the present discounted value

$$v_N = \frac{\pi_N + \iota_M(v_M - v_N)}{\rho + \iota_N + \iota_S} \rightarrow v_N = \frac{\pi_N + \iota_M v_M}{\rho + \iota_N + \iota_M + \iota_S}, \quad (8)$$

where ι_N is the intensity of innovation, ι_M the intensity of Northern imitation, and ι_S the intensity of Southern imitation. However, if Northern imitation occurs, the profit stream is merely reduced, and the original innovator as well as the new imitator each have the value

$$v_M = \frac{\pi_M}{\rho + \iota_N + \iota_S}, \quad (9)$$

until innovation or Southern imitation occurs. A second Northern imitation will not occur in equilibrium, provided second imitations are as costly as first imitations (or not much less costly), because a second Northern imitator would earn a one-third share of profits, which would be dominated by a one-half share from achieving a first imitation. Finally, a

Southern imitator's value is eroded only by innovation, since neither Northern nor further Southern imitation would be profitable. Thus, a Southern imitator gains the value

$$v_S = \frac{\pi_S}{\rho + \iota_N}. \tag{10}$$

Again, assuming second Southern imitations would be as costly as first Southern imitations, a second Southern imitator, even if it colluded with the first Southern imitator, would earn only half the profit stream, and thus be dominated by making a first imitation instead.

2.3. Research and development

In the R&D stage, each firm chooses its intensity of R&D to maximize its expected value, given the R&D intensities of other firms. R&D races occur simultaneously for all products. Following Grossman and Helpman (1991), a firm undertaking R&D intensity ι_k for a time interval $\mathrm{d}t$ requires $a_k \iota_k\,\mathrm{d}t$ units of labor and leads to success with probability $\iota_k\,\mathrm{d}t$, $k = \mathrm{N, M, S}$. For simplicity, Northern firms have the same innovation costs regardless of whether they innovated the previous generation and the quality increment λ is fixed. Absence of Southern innovation can be supported by a sufficiently large resource requirement in Southern innovation.

Focusing on a steady-state equilibrium with positive intensities of innovation $\iota_N > 0$, Northern imitation $\iota_M > 0$, and Southern imitation $\iota_S > 0$, the expected value generated by each R&D success must match the cost of achieving the R&D success. The expected reward cannot exceed the cost due to free entry (R&D would become infinite), and the cost cannot exceed the reward if R&D is to occur. Therefore, the R&D equilibrium conditions require $v_N = wa_N$ for innovation, $v_M = wa_M$ for Northern imitation and $v_S = a_S$ for Southern imitation.

Defining the inverse of the quality improvement $\delta \equiv 1/\lambda$, these R&D valuation conditions are combined with the expressions for firm value and profits to generate

$$E(w - 1) = wa_S(\rho + \iota_N), \tag{11}$$

$$E(1 - w\delta) = 2wa_M(\rho + \iota_N + \iota_S), \tag{12}$$

$$E(1 - w\delta) = wa_N\left[\rho + \iota_N + \iota_S + \iota_M\left(1 - \frac{a_M}{a_N}\right)\right]. \tag{13}$$

These expressions complete the description of firm behavior.

2.4. Implicit collusion

Consider markets where a Northern firm has successfully imitated another firm's invention. A collusive equilibrium can be supported as a subgame perfect equilibrium by the firms playing trigger strategies in the repeated game: both firms pick the price $p_M = \lambda$ unless either firm defects, in which case the other firm would pick $p_D = w$ during the permanent punishment phase. Given this strategy, neither firm ever (for finite discount rate and sufficiently short lag in retaliation) deviates from $p_M = \lambda$, as higher instantaneous profits would require sacrifice of all future profits following retaliation.

Following Segerstrom (1991), suppose retaliation can occur only after some lag, $0 < \ell < \infty$. For either firm to cooperate, its value v_M from cooperation must exceed the value v_D stemming from a deviation. Let $\pi_D = \pi_N$ represent instantaneous profits a firm earns when deviating before the other retaliates (the deviator charges an ε below λ, but profits are arbitrarily close to π_N due to ε being arbitrarily close to zero). A firm that defects earns instantaneous defecting profits until either the other firm retaliates or rival innovation or imitation occurs and terminates the profit stream

$$v_D = \frac{\pi_N}{\rho + \iota_N + \iota_M + \iota_S}[1 - e^{-(\rho+\iota_N+\iota_M+\iota_S)\ell}]. \tag{14}$$

At each instant, the defecting firm earns instantaneous profits $\pi_D = \pi_N$ if rival innovation or imitation has not yet occurred and the lag until retaliation has not yet expired. Once the lag has passed, neither firm earns any further profits.

Instantaneous profits for a defecting firm are maximized by charging a price infinitesimally below $p_M = \lambda$ and thus capturing the whole market. For cooperation to be supported, the lag must be sufficiently short

$$\ell < \bar{\ell} \equiv \frac{\log(2)}{\rho + \iota_N + \iota_M + \iota_S}. \tag{15}$$

In the limit as the lag becomes infinitesimally small $\ell \to 0$, the firms always cooperate.

Provided the Northern-to-Southern relative wage is sufficiently large $[w > \tilde{w} = 2/(1 + 1/\lambda)]$, Southern firms successful in imitation will prefer not to collude with Northern firms. The condition on the Northern-to-Southern relative wage is less binding as the quality increment λ shrinks $[\lim_{\lambda \to 1} \tilde{w} = 1$ and $\lim_{\lambda \to \infty} \tilde{w} = 2]$, as Southern firms are weighing part of a bigger pie (based on λ) against all of a smaller price (based on w).

2.5. Resource constraints

R&D and production in each country are constrained by the fixed supply of a scarce resources. Let each n_k, $k = \mathrm{N, M, S}$ represent the fraction of all markets where innovation, Northern imitation, or Southern imitation was the most recent R&D success. These market measures sum to one. The fixed supply of Northern resources L_N is allocated to production $(1 - n_S)E\delta$ in both types of Northern markets and to R&D that is composed of both innovation targeting all markets $a_N \iota_N$ and Northern imitation targeting Northern innovator markets $a_M \iota_M n_N$. Thus, the Northern labor constraint requires that

$$a_N \iota_N + a_M \iota_M n_N + (1 - n_S)E\delta = L_N. \tag{16}$$

Similarly, the fixed supply of Southern resources L_S is allocated to production $n_S E/w$ in Southern imitator markets and to imitation targeting all markets that have not yet been imitated by a Southern firm $a_S \iota_S (1 - n_S)$. So the Southern labor constrain requires that

$$a_S \iota_S (1 - n_S) + n_S \frac{E}{w} = L_S. \tag{17}$$

These resource constraint combine with the R&D valuation conditions to form the five key equations of the model.

2.6. Constant market measures

In a steady-state equilibrium, the market measures are linked to the R&D intensities through conditions necessary for the market measures to remain constant. Each market measure must have as many products flowing into as flowing out of that market structure at each point in time. For Northern innovator markets, the flows in are innovation targeting all other markets (innovation targets Northern innovator markets but does not generate any inflows into the market structure), while the flows out are the Northern and Southern imitation targeting these markets. Thus, for the measure of markets served by an innovator to remain constant, we require that

$$\iota_{N}(1 - n_{N}) = (\iota_{M} + \iota_{S})n_{N}. \tag{18}$$

Similarly, for Northern imitator markets, the flows in are Northern imitation (which targets Northern innovator markets), while the flows out are innovation and Southern imitation targeting these markets. So the measure of markets served by a Northern imitator (colluding with the innovator) remains constant if

$$\iota_{M}n_{N} = (\iota_{N} + \iota_{S})n_{M}. \tag{19}$$

Finally, the three market measures must sum to one since they completely describe the possible market structures

$$n_{N} + n_{M} + n_{S} = 1. \tag{20}$$

These equations can be solved to find expressions for each market measure in terms of the R&D intensities.

2.7. Transform system

At this point, we transform the system into the variables of primary interest: the aggregate rate of innovation, Northern imitation relative to innovation, and Southern imitation relative to innovation. The aggregate rate of innovation is the same as the intensity of innovation (since innovation targets all markets): $\iota \equiv \iota_{N}$. The rate of Northern imitation is the intensity of Northern imitation times the measure of Northern innovator markets targeted: $\psi = \iota_{M}n_{N}$. Similarly, the rate of Southern imitation is the intensity of Southern imitation times the measure of Northern innovator and imitator markets targeted: $\mu \equiv \iota_{S}(1 - n_{S})$. The overall rate of imitation is the sum of Northern and Southern imitation $\psi + \mu$. Northern imitation relative to innovation is then $\eta \equiv \psi/\iota$, and Southern imitation relative to innovation is $v \equiv \mu/\iota$.

Northern imitation relative to innovation η and Southern imitation relative to innovation v describe the fraction of product cycles that occur through Northern imitation and Southern imitation, respectively. The ratio η describes the composition of Northern R&D between innovation and imitation. Also, the ratio of Northern imitation to Southern imitation ψ/μ describes the composition of imitation between the North and the South. These measures also allow us to fully characterize the composition of the product cycle between innovation, Northern imitation, and Southern imitation. For example, if both η and v rise (due to some parameter change), then fewer product cycles occur through innovation of products that have not yet been imitated. On the other hand, if v rises while η falls (or stays constant), then more product cycles occur though Southern imitation (relative to Northern imitation): the mix of imitation has shifted toward the South.

3. Substitution in R&D

Having specified the system that determines the equilibrium for a world where innovation occurs in the North and imitation occurs in the North and the South, we can examine how the equilibrium is affected by shifts in the resource supplies, resource requirements in R&D, or R&D subsidies. Since we have transformed the model to determine Northern imitation relative to innovation, the model demonstrates whether each force generates any shift in the composition of Northern R&D between innovation and imitation. We can also determine effects on the relative rates of imitation in the North compared to the South. We examine three types of changes: changes in labor supplies, changes in the difficulty of R&D, and changes in R&D subsidies.

3.1. Resource supplies

We begin with changes in the Northern and Southern labor supplies, remembering that the labor supplies are measured in efficiency units (units of labor needed to produce one unit of output). Thus, an increase in a labor supply can stem not only from population increases, but also from educational programs or any other type of training that enhances productivity. Also, the effects of migration can be examined by a decrease in the Southern labor supply together with an increase in the Northern labor supply.

Although innovation occurs only in the North, the rate of innovation is essentially driven by the world supply of resources. Increases in Southern resources permit an expansion of Southern imitation, both absolutely and relative to the rate of innovation. With more Southern imitation, more products are produced in the South, which frees up Northern resources for more innovation. Since Northern imitation does not locate production any differently than Northern innovation, it remains in a fixed proportion relative to the rate of innovation as resources are varied—the composition of Northern R&D remains unchanged. However, whereas increases in Southern resources shift the composition of imitation toward the South, increases in Northern resources shift the composition of imitation toward the North. Thus, migration of labor from the South to the North would lead to a shift in imitation that mirrors the movement of workers.

Proposition 1. *An increase in Northern or Southern resources increases the rates of innovation, Northern imitation, and Southern imitation. Aggregate expenditure rises. An increase in Southern resources expands Southern imitation relative to innovation, while an increase in Northern resources contracts it. Neither resource supply affects Northern imitation relative to innovation. Consequently, an increase in Southern resources expands Southern imitation relative to Northern imitation, while an increase in Northern resources contracts it. An increase in Northern resources increases the Northern-to-Southern relative wage, but an increase in Southern resources decreases it.*

That an increase in the Northern labor supply leads to an increase in the Northern-to-Southern relative wage and that an increase in the Southern labor supply decreases it may seem opposed to simple supply-and-demand logic. However, in this model, there is no direct effect of labor supplies on the Northern-to-Southern relative wage. The effects are generated indirectly through the ratio of Southern imitation relative to innovation $v \equiv \mu/\iota$. An increase in v causes the Northern-to-Southern wage to fall due to the reduction in the relative reward to innovation compared to Southern imitation. When there is more Southern imitation relative to innovation, the reward to innovation falls due to the shorter expected duration of profits. Therefore, the Northern-to-Southern relative wage must rise to increase the level of profits and thus counteract their shorter duration. Since an increase in the Northern labor supply reduces the ratio of Southern imitation relative to innovation $v \equiv \mu/\iota$ (by expanding innovation), it leads to an increase in the Northern-to-Southern relative wage w. The increase in the Northern-to-Southern relative wage and the increase in aggregate expenditure due to an increase in Northern resources raise the profits of Southern imitators so that the rate of Southern imitation also rises.

3.2. R&D subsidies

Governments may attempt to directly affect R&D through the use of R&D subsidies. Grossman and Helpman (1991) derive differing results depending on whether followers (firms who did not make the previous innovation) are efficient (have innovation costs low enough that engage in innovation) or not; see also Segerstrom et al. (1990). They find that a subsidy to innovation increases the rate of innovation and decreases the rate of Southern imitation for the case of efficient followers, but increases the rate of Southern imitation for the case of inefficient followers. Likewise, they find that a subsidy to Southern imitation increases the rate of Southern imitation and decreases the rate of innovation for the case of efficient followers, but increases the rate of innovation for the case of inefficient followers.

R&D subsidies can be analyzed by adding terms $(1-\sigma_i)$ to the cost side of the R&D valuation conditions. An increase in the subsidy to Southern imitation σ_S unambiguously increases the rate of innovation, as well as the rates of Southern and Northern imitation. The positive effects on innovation and Southern imitation are similar to the results from the inefficient followers equilibrium in Grossman and Helpman (1991), even though followers are assumed to be efficient here. A subsidy to Southern imitation makes Southern imitation less costly, and thus the reward to Southern imitation must fall. From (10), the reward to Southern imitation is reduced by an increase in innovation (the intensity of innovation and the rate of innovation are the same here as innovation targets all markets).

The composition of imitation shifts away from the South, while the composition of Northern R&D is unaffected. So while subsidizing Southern imitation does succeed in encouraging Southern imitation, it indirectly encourages Northern imitation and innovation to a greater degree. A smaller fraction of products will exhibit cycles that include Southern production, so Southern R&D subsidies do not work to shift production to the South. The expanded imitation in the South crowds out production.

Proposition 2. *An increase in the Southern imitation subsidy increases the rates of innovation, Northern imitation, and Southern imitation. Aggregate expenditure also increases.*

The Northern-to-Southern relative wage and Southern imitation relative to innovation decrease. Northern imitation relative to innovation remains unchanged. Consequently, subsidies to Southern imitation decrease Southern imitation relative to Northern imitation—more of the world's imitation is done in the North.

On the Northern side, governments generally experience difficulty distinguishing innovative R&D from imitative R&D. Hence, like Davidson and Segerstrom (1998), we analyze a general R&D subsidy σ to all Northern R&D ($\mathrm{d}\sigma_\mathrm{N} = \mathrm{d}\sigma_\mathrm{M} = \mathrm{d}\sigma$). The composition of imitation shifts toward the South, while the composition of Northern R&D is unaffected. Thus, a general Northern R&D subsidy has effects like those of an innovation subsidy in inefficient followers equilibrium of Grossman and Helpman (1991), but we have shown that the rate of Northern imitation also rises.

Proposition 3. *An increase in the general Northern R&D subsidy increases the rates of innovation, Northern imitation, and Southern imitation. Aggregate expenditure falls. The Northern-to-Southern relative wage and Southern imitation relative to innovation rise. Northern imitation relative to innovation remains unchanged. Consequently, Southern imitation relative to Northern imitation rises—more of the world's imitation is done in the South.*

3.3. Resource requirements in R&D

Other changes in the composition of R&D across countries may be generated by changes in the inherent difficulty of conducting R&D. Southern imitation could become easier over time due to better communication, enhanced absorptive capacity, or the like. Or a change in the resource requirement for Southern imitation could be influenced by policy. A strengthening of intellectual property rights (IPR) protection in the South could make imitation more difficult, as argued by Glass and Saggi (2002). Perhaps the North pushing for stronger IPR protection in the South could be as a reaction to the reduction in the difficulty of Southern imitation that would otherwise have occurred due to other factors. An increase in the difficulty of Southern imitation should reduce the rate of Southern imitation, but its effects on Northern imitation and innovation are less obvious. Increased Northern imitation might take the place of some or all of the reduction in Southern imitation, so a rise in innovation is not assured. Probably innovation still rises because Northern imitation reduces an innovator's profits whereas Southern imitation cuts those profits to zero. Still, we need to look to see what the model suggests will happen.

Increases in the resource requirement in Southern imitation reduce the rate of innovation. Also intuitively, Southern imitation relative to innovation contracts when Southern imitation becomes more difficult, mirroring the response to the availability of resources above. Northern imitation relative to innovation (the composition of R&D) is unaffected, so the composition of imitation shifts towards more Northern imitation relative to Southern imitation. When Southern imitation becomes more difficult, the reward to imitation needs to rise in order to offset the higher cost of imitation. Aggregate expenditure and the Northern-to-Southern relative wage both rise. Larger aggregate expenditure leads to more sales, and a higher relative wage leads to a bigger profit margin since imitators charge a price equal to the relative wage. Also, innovation falls, so the profits for a successful Southern imitator are not only larger, they last for longer as well. Thus, the model suggests that stronger Southern IPR protection may fail to increase innovation.

Proposition 4. *An increase in the resource requirement in Southern imitation decreases the rates of innovation, Northern imitation, and Southern imitation. Aggregate expenditure and the Northern-to-Southern relative wage rise. Southern imitation relative to innovation declines. Northern imitation relative to innovation remains unchanged. Consequently, Southern imitation relative to Northern imitation falls—less of the world's imitation is done in the South.*

Suppose both Northern imitation and innovation become more difficult ($\mathrm{d}a_{\mathrm{N}} = a_{\mathrm{N}}\,\mathrm{d}\kappa$ and $\mathrm{d}a_{\mathrm{M}} = a_{\mathrm{M}}\,\mathrm{d}\kappa$). Increases in the overall difficulty of Northern R&D reduce the rate of innovation. Also intuitively, Southern imitation relative to innovation expands. Northern imitation relative to innovation (the composition of R&D) is unaffected, so the composition of imitation shifts towards more Southern imitation relative to Northern imitation.

Proposition 5. *An increase in the resource requirements in Northern R&D decreases the rates of innovation, Northern imitation, and Southern imitation. Aggregate expenditure and the Northern-to-Southern relative wage fall. Southern imitation relative to innovation increases. Northern imitation relative to innovation remains unchanged. Consequently, Southern imitation relative to Northern imitation rises—more of the world's imitation is done in the South.*

3.4. Summary and adjustment

In all, we have considered the effects of labor supply changes, R&D subsidies, and R&D efficiency. The sign of the effects of these changes on the rates of innovation ι, Northern imitation ψ, Southern imitation μ, Southern imitation relative to innovation $v \equiv \mu/\iota$ and Southern imitation relative to Northern imitation μ/ψ, aggregate expenditure E, and the Northern-to-Southern relative wage w are captured in Table 1. None of these changes had

Table 1
Summary of results[a]

	ι	ψ	μ	$v \equiv \mu/\iota$	μ/ψ	E	w
L_{N}	+	+	+	−	−	+	+
L_{S}	+	+	+	+	+	+	−
σ_{S}	+	+	+	−	−	+	−
σ	+	+	+	+	+	−	+
a_{S}	−	−	−	−	−	+	+
κ	−	−	−	+	+	−	−

[a] These parameters have no effect on $\eta \equiv \psi/\iota$.

any effect on the ratio Northern imitation relative to innovation $\eta \equiv \psi/\iota$, so the composition of Northern R&D remains the same. Since each change affects Northern and Southern imitation in the same way, the effect on the overall rate of imitation is always clear.

In Segerstrom's (1991) linear R&D model, R&D subsidies have counter-intuitive effects. If innovation is subsidized, the intensity of imitation rises while the intensity of innovation falls. If (Northern) imitation is subsidized, the intensity of innovation rises while the intensity of imitation falls. Cheng and Tao (1999) and Davidson and Segerstrom (1998) have demonstrated that these counter-intuitive results stem from the instability of the equilibrium in terms of ad hoc dynamics. If innovators increase their efforts when the reward to innovation exceeds costs (and likewise for imitation), there does not appear to be any sensible adjustment process that will lead the economy to the new equilibrium. Davidson and Segerstrom derive a mutual R&D condition by solving the imitation valuation condition for aggregate expenditure and substituting that expression into the innovation valuation condition. They find that a subsidy to innovation shifts the mutual R&D condition to require a higher intensity of imitation for any given intensity of innovation. The peculiar results in Segerstrom's model arise for selective R&D subsidies—subsidies to innovation but not imitation or imitation but not innovation. As noted earlier, there is good reason to doubt that a government could ever implement such a policy as they could not distinguish innovative R&D from imitative R&D in practice.

Can innovators and imitators find their way to the new equilibrium in our model? Yes. As in the Davidson and Segerstrom model, any general Northern R&D subsidy cancels out of the mutual R&D condition. Solving the Northern imitation valuation condition (12) for aggregate expenditure E and substituting into the innovation valuation condition (13)—each with a $(1-\sigma)$ on the cost side—yields our version of the mutual R&D condition

$$2a_{\mathrm{M}}(\rho+\iota_{\mathrm{N}}+\iota_{\mathrm{S}}) = a_{\mathrm{N}}\left[\rho+\iota_{\mathrm{N}}+\iota_{\mathrm{S}}+\iota_{\mathrm{M}}\left(1-\frac{a_{\mathrm{M}}}{a_{\mathrm{N}}}\right)\right], \tag{21}$$

or

$$\iota_{\mathrm{M}} = \left(\frac{2a_{\mathrm{M}}-a_{\mathrm{N}}}{a_{\mathrm{N}}-a_{\mathrm{M}}}\right)(\rho+\iota_{\mathrm{N}}+\iota_{\mathrm{S}}), \tag{22}$$

where innovation is more difficult than imitation $2a_{\mathrm{M}} > a_{\mathrm{N}} > a_{\mathrm{M}}$ but not by too large a margin (for both innovation and Northern imitation to occur in equilibrium). As in the Davidson and Segerstrom model, the mutual R&D condition is unaffected by a general R&D subsidy because the costs of both types of R&D fall to the same degree. The intensity of Northern imitation ι_{M} is a fixed ratio relative to the intensity of innovation ι_{N}, so the two move together, both rising in a sensible fashion as in Davidson and Segerstrom's model. Similarly, when both Northern imitation and innovation become more difficult to the same degree, the mutual R&D condition remains unaffected. Other parameters such as labor supplies, subsidies to Southern imitation, and the difficulty of Southern imitation do not enter the mutual R&D condition (21) and thus must affect innovation and Northern imitation to the same degree.

When a general R&D subsidy lowers the costs of innovation and Northern imitation to the same degree, the reward exceeds the cost of both innovation and Northern imitation. So both intensities should rise according to the ad hoc adjustment process. In fact, the intensities should rise in concert along the mutual R&D condition so as to keep the excess returns in innovation equal to the excess returns in Northern imitation. Otherwise, if innovation yielded an even bigger excess return than in imitation, imitators might still switch to investing in innovation. So the economy can move along the mutual R&D condition until the new equilibrium is reached.

4. Conclusion

This paper studies the effect on aggregate expenditure, the Northern-to-Southern relative wage, the rates of innovation, Northern imitation and Southern imitation, the composition of Northern R&D between innovation and imitation, and the composition of imitation between the North and the South of changes in resource supplies, R&D efficiency, and R&D subsidies. The model determines whether results for two-country models where the North exclusively innovates such as Grossman and Helpman (1991) are sensitive to adding Northern imitation.

We find that this more general model allows us to address interesting issues that could not be addressed in the simpler models. For example, when Southern imitation increases due to Southern imitation becoming easier, Southern R&D subsidies, or an increase in the Southern labor supply (measured in efficiency units), what happens to Northern imitation? Such increases in Southern imitation appear to generally be reinforced (rather than offset) by increases in Northern imitation. Despite the increase in the overall rate of imitation (or perhaps because of it), the rate of innovation rises in response. The composition of imitation across countries has implications for the Northern-to-Southern relative wage and hence the distribution of income across countries. Increases in Northern imitation relative to Southern imitation, whatever their cause, generally seem to raise the Northern-to-Southern relative wage. The adjustment in the relative wage works to help restore balance to the increase in the return on Northern relative to Southern imitation since the relative wage is a cost in the North but contributes to revenue in the South by allowing Southern imitators to charge higher prices. The goal of this paper has been to help build a fuller understanding of the rich set of interactions between the different types of R&D in different countries. Surely, there are yet more discoveries to be made in this area.

Appendix A

Assign $\imath_N = \imath$, $\imath_M = \eta\imath/n_N$, and $\imath_S = v\imath/(1-n_S)$. The expressions for the market measures become $n_N = (1-\eta)(1-v)$, $n_M = \eta(1-v)$, and $n_S = v$. Solving the Northern imitation valuation condition (12) for aggregate expenditure

$$E = \frac{2wa_M[\imath + \rho(1-v)]}{(1-w\delta)(1-v)}.$$

Assign this expression and then solve the Southern imitation valuation condition (11) for the Northern-to-Southern relative wage

$$w = 1 + \frac{a_S(\rho+\imath)(1-v)(1-\delta)}{2a_M[\imath+\rho(1-v)] + \delta a_S(\rho+\imath)(1-v)}.$$

Assign this expression and then the Northern resource constraint (16), Southern resource constraint (17) and innovation valuation condition (13) become

$$\begin{aligned}\mathrm{RN} \equiv\ & \imath(a_{\mathrm{N}}+\eta a_{\mathrm{M}})(1-\delta)-L_{\mathrm{N}}(1-\delta)+\delta(2a_{\mathrm{M}}[\imath+\rho(1-v)] \\ & +a_{\mathrm{S}}(\rho+\imath)(1-v))=0,\end{aligned}$$

$$\begin{aligned}\mathrm{RS} \equiv\ & v a_{\mathrm{S}}\imath(1-\delta)(1-v)-L_{\mathrm{S}}(1-\delta)(1-v)+v(2a_{\mathrm{M}}[\imath+\rho(1-v)] \\ & +\delta a_{\mathrm{S}}(\rho+\imath)(1-v))=0,\end{aligned}$$

$$\mathrm{VN} \equiv (2a_{\mathrm{M}}-a_{\mathrm{N}})[\imath+\rho(1-v)(1-\eta)]-\eta a_{\mathrm{M}}\imath=0.$$

Totally differentiate this reduced system in the endogenous variables $\{\imath, \eta, \mu\}$ with respect to the parameters $\{L_{\mathrm{N}}, L_{\mathrm{S}}, a_{\mathrm{N}}, a_{\mathrm{M}}, a_{\mathrm{S}}\}$; σ_{S}, and σ are suppressed for brevity

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & 0 & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}\begin{bmatrix} \mathrm{d}\imath \\ \mathrm{d}\eta \\ \mathrm{d}v \end{bmatrix}=\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix},$$

where

$$\begin{aligned}
b_{11} &= (a_{\mathrm{N}}+\eta a_{\mathrm{M}})(1-\delta)+\delta(2a_{\mathrm{M}}+a_{\mathrm{S}}(1-v)),\\
b_{12} &= a_{\mathrm{M}}\imath(1-\delta),\\
b_{13} &= -\delta[2\rho a_{\mathrm{M}}+a_{\mathrm{S}}(\rho+\imath)],\\
b_{21} &= v[2a_{\mathrm{M}}+a_{\mathrm{S}}(1-v)],\\
b_{23} &= -[v^2 a_{\mathrm{S}}\imath(1-\delta)+2a_{\mathrm{M}}[\imath+\rho(1-v)]-\delta v a_{\mathrm{S}}(\rho+\imath)]+L_{\mathrm{S}}(1-\delta),\\
b_{31} &= (2-\eta)a_{\mathrm{M}}-a_{\mathrm{N}},\\
b_{32} &= -\rho(2a_{\mathrm{M}}-a_{\mathrm{N}})(1-v)-\imath a_{\mathrm{M}},\\
b_{33} &= -\rho(2a_{\mathrm{M}}-a_{\mathrm{N}})(1-\eta),
\end{aligned}$$

and

$$\begin{aligned}
c_1 &= (1-\delta)\,\mathrm{d}L_{\mathrm{N}}-\delta(\rho+\imath)(1-v)\,\mathrm{d}a_{\mathrm{S}}-\imath(1-\delta)\,\mathrm{d}a_{\mathrm{N}}\\
&\quad-[\imath(2\delta+\eta(1-\delta))+2\delta\rho(1-v)]\,\mathrm{d}a_{\mathrm{M}},\\
c_2 &= (1-v)[(1-\delta)\,\mathrm{d}L_{\mathrm{S}}-v(\rho\delta+\imath)\,\mathrm{d}a_{\mathrm{S}}]-2v[\imath+\rho(1-v)]\,\mathrm{d}a_{\mathrm{M}},\\
c_3 &= -[\imath(2-\eta)+2\rho(1-\eta)(1-v)]\,\mathrm{d}a_{\mathrm{M}}+[\imath+\rho(1-\eta)(1-v)]\,\mathrm{d}a_{\mathrm{N}}.
\end{aligned}$$

To simplify expressions for the derivatives, apply the definitions of each resource requirement in imitation relative to innovation $a_{\mathrm{M}} \equiv \theta a_{\mathrm{N}}$ and $a_{\mathrm{S}} \equiv \gamma a_{\mathrm{N}}$. The determinant is positive

$$\beta \equiv |B| = \theta a_{\mathrm{N}}^3 \imath^2\left[\gamma(1-v)(2\theta+\delta\gamma)+2\theta\delta\gamma(1+v)+\frac{4\theta^2}{1-v}\right]>0.$$

A.1. Proof of Proposition 1

$$\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{N}}} = \frac{\theta a_{\mathrm{N}}^2 \imath^2 (1-\delta)[2\theta + \gamma(1-v)^2]}{(1-v)\beta} > 0, \qquad \frac{\mathrm{d}\eta}{\mathrm{d}L_{\mathrm{N}}} = 0,$$

$$\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{S}}} = \frac{\delta\gamma\theta a_{\mathrm{N}}^2 \imath^2 (1-\delta)(1-v)}{\beta} > 0, \qquad \frac{\mathrm{d}\eta}{\mathrm{d}L_{\mathrm{S}}} = 0,$$

$$\frac{\mathrm{d}v}{\mathrm{d}L_{\mathrm{N}}} = -\left(\frac{v(1-v)[2\theta + \gamma(1-v)]}{\imath[2\theta + \gamma(1-v)^2]}\right)\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{N}}} < 0,$$

$$\frac{\mathrm{d}v}{\mathrm{d}L_{\mathrm{S}}} = \left(\frac{2\theta + \delta\gamma(1-v)}{\imath\delta\gamma}\right)\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{S}}} > 0,$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}L_i} = \eta\frac{\mathrm{d}\imath}{\mathrm{d}L_i} + \imath\frac{\mathrm{d}\eta}{\mathrm{d}L_i} = \eta\frac{\mathrm{d}\imath}{\mathrm{d}L_i} > 0, \quad i = \mathrm{N}, \mathrm{S},$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}L_{\mathrm{N}}} = \frac{2\theta v^2}{2\theta + \gamma(1-v)^2}\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{N}}} > 0,$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}L_{\mathrm{S}}} = v\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{S}}} + \imath\frac{\mathrm{d}v}{\mathrm{d}L_{\mathrm{S}}} > 0,$$

$$\frac{\mathrm{d}E}{\mathrm{d}L_{\mathrm{N}}} = \left(\frac{a_{\mathrm{N}}[2\theta + \gamma(1-v)]^2}{(1-\delta)[2\theta + \gamma(1-v)^2]}\right)\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{N}}} > 0,$$

$$\frac{\mathrm{d}E}{\mathrm{d}L_{\mathrm{S}}} = \underbrace{\frac{\partial E}{\partial \imath}}_{+}\frac{\mathrm{d}\imath}{\mathrm{d}L_{\mathrm{S}}} + \underbrace{\frac{\partial E}{\partial v}}_{+}\frac{\mathrm{d}v}{\mathrm{d}L_{\mathrm{S}}} > 0,$$

$$\frac{\mathrm{d}w}{\mathrm{d}L_{\mathrm{N}}} = \underbrace{\frac{\partial w}{\partial v}}_{-}\frac{\mathrm{d}v}{\mathrm{d}L_{\mathrm{N}}} > 0, \frac{\mathrm{d}w}{\mathrm{d}L_{\mathrm{S}}} = \underbrace{\frac{\partial w}{\partial v}}_{-}\frac{\mathrm{d}v}{\mathrm{d}L_{\mathrm{S}}} < 0.$$

A.2. Proof of Proposition 2

$$\frac{\mathrm{d}\imath}{\mathrm{d}\sigma_{\mathrm{S}}} = \frac{\delta\gamma\theta a_{\mathrm{N}}^2 \imath^3 [2\theta + \gamma(1-v)(1-v+\delta v)]}{\beta} > 0, \qquad \frac{\mathrm{d}\eta}{\mathrm{d}\sigma_{\mathrm{S}}} = 0,$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\sigma_{\mathrm{S}}} = \eta\frac{\mathrm{d}\imath}{\mathrm{d}\sigma_{\mathrm{S}}} + \imath\frac{\mathrm{d}\eta}{\mathrm{d}\sigma_{\mathrm{S}}} = \eta\frac{\mathrm{d}\imath}{\mathrm{d}\sigma_{\mathrm{S}}} > 0,$$

$$\frac{\mathrm{d}v}{\mathrm{d}\sigma_{\mathrm{S}}} = -\left(\frac{\gamma v(1-\delta)(1-v)^2}{\imath[2\theta + \gamma(1-v)(1-v+\delta v)]}\right)\frac{\mathrm{d}\imath}{\mathrm{d}\sigma_{\mathrm{S}}} < 0,$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}\sigma_{\mathrm{S}}} = \left(\frac{v[2\theta + \gamma\delta(1-v)]}{2\theta + \gamma(1-v)(1-v+\delta v)}\right)\frac{\mathrm{d}\imath}{\mathrm{d}\sigma_{\mathrm{S}}} > 0,$$

$$\frac{\mathrm{d}E}{\mathrm{d}\sigma_{\mathrm{S}}} = \left(\frac{a_{\mathrm{N}}[\delta v\gamma^2(1-v)^2 + 2\theta(2\theta + \gamma(1-v)(1-v+2\delta v))]}{\delta(1-v)[2\theta + \gamma(1-v)(1-v+\delta v)]}\right)\frac{\mathrm{d}\imath}{\mathrm{d}\sigma_{\mathrm{S}}} > 0,$$

$$\frac{\mathrm{d}w}{\mathrm{d}\sigma_{\mathrm{S}}} = -\left(\frac{2\theta(1-\delta)}{\delta\iota[2\theta+\gamma\delta(1-v)]}\right)\frac{\mathrm{d}\iota}{\mathrm{d}\sigma_{\mathrm{S}}} < 0.$$

A.3. Proof of Proposition 3

$$\frac{\mathrm{d}\iota}{\mathrm{d}\sigma} = \frac{2\theta^2\delta a_{\mathrm{N}}^3\iota^3[2\theta+\gamma(1-v)]}{(1-v)\beta} > 0, \qquad \frac{\mathrm{d}\eta}{\mathrm{d}\sigma} = 0,$$
$$\frac{\mathrm{d}\psi}{\mathrm{d}\sigma} = \eta\frac{\mathrm{d}\iota}{\mathrm{d}\sigma} + \iota\frac{\mathrm{d}\eta}{\mathrm{d}\sigma} = \eta\frac{\mathrm{d}\iota}{\mathrm{d}\sigma} > 0,$$
$$\frac{\mathrm{d}v}{\mathrm{d}\sigma} = \left(\frac{2\theta v(1-v)(1-\delta)}{\delta\iota[2\theta+\gamma(1-v)]}\right)\frac{\mathrm{d}\iota}{\mathrm{d}\sigma} > 0,$$
$$\frac{\mathrm{d}\mu}{\mathrm{d}\sigma} = v\frac{\mathrm{d}\iota}{\mathrm{d}\sigma} + \iota\frac{\mathrm{d}v}{\mathrm{d}\sigma} > 0,$$
$$\frac{\mathrm{d}E}{\mathrm{d}\sigma} = -\left(\frac{2\theta a_{\mathrm{N}}}{\delta}\right)\frac{\mathrm{d}\iota}{\mathrm{d}\sigma} < 0,$$
$$\frac{\mathrm{d}w}{\mathrm{d}\sigma} = \left(\frac{\gamma(1-v)(1-\delta)[2\theta(1-v+\delta v)+\gamma\delta(1-v)]}{\delta\iota[2\theta+\gamma(1-v)]^2}\right)\frac{\mathrm{d}\iota}{\mathrm{d}\sigma} > 0.$$

A.4. Proof of Proposition 4

$$\frac{\mathrm{d}\iota}{\mathrm{d}a_{\mathrm{S}}} = -\frac{\delta\theta a_{\mathrm{N}}^2\iota^3[2\theta+\delta\gamma(1-v)]}{\beta} < 0, \qquad \frac{\mathrm{d}\eta}{\mathrm{d}a_{\mathrm{S}}} = 0,$$
$$\frac{\mathrm{d}\psi}{\mathrm{d}a_{\mathrm{S}}} = \eta\frac{\mathrm{d}\iota}{\mathrm{d}a_{\mathrm{S}}} + \iota\frac{\mathrm{d}\eta}{\mathrm{d}a_{\mathrm{S}}} = \eta\frac{\mathrm{d}\iota}{\mathrm{d}a_{\mathrm{S}}} < 0,$$
$$\frac{\mathrm{d}v}{\mathrm{d}a_{\mathrm{S}}} = \left(\frac{2\theta v(1-\delta)(1-v)}{\delta\iota[2\theta+\gamma(1-v)]}\right)\frac{\mathrm{d}\iota}{\mathrm{d}a_{\mathrm{S}}} < 0,$$
$$\frac{\mathrm{d}\mu}{\mathrm{d}a_{\mathrm{S}}} = v\frac{\mathrm{d}\iota}{\mathrm{d}a_{\mathrm{S}}} + \iota\frac{\mathrm{d}v}{\mathrm{d}a_{\mathrm{S}}} < 0,$$
$$\frac{\mathrm{d}E}{\mathrm{d}a_{\mathrm{S}}} = -\left(\frac{2\theta a_{\mathrm{N}}}{\delta}\right)\frac{\mathrm{d}\iota}{\mathrm{d}a_{\mathrm{S}}} > 0,$$
$$\frac{\mathrm{d}w}{\mathrm{d}a_{\mathrm{S}}} = \frac{\partial w}{\partial a_{\mathrm{S}}} + \frac{\partial w}{\partial v}\frac{\mathrm{d}v}{\mathrm{d}a_{\mathrm{S}}} > 0.$$

A.5. Proof of Proposition 5

$$\frac{\mathrm{d}\iota}{\mathrm{d}\kappa} = -\frac{2\theta^2 a_{\mathrm{N}}^3\iota^3[2\theta+\gamma(1-v)(1-v+\delta v)]}{(1-v)\beta} < 0, \qquad \frac{\mathrm{d}\eta}{\mathrm{d}\kappa} = 0,$$
$$\frac{\mathrm{d}\psi}{\mathrm{d}\kappa} = \eta\frac{\mathrm{d}\iota}{\mathrm{d}\kappa} + \iota\frac{\mathrm{d}\eta}{\mathrm{d}\kappa} = \eta\frac{\mathrm{d}\iota}{\mathrm{d}\kappa} < 0,$$

$$\frac{\mathrm{d}v}{\mathrm{d}\kappa} = \left(\frac{\gamma v(1-\delta)(1-v)^2}{2\theta + \gamma(1-v)(1-v+\delta v)}\right)\frac{\mathrm{d}\iota}{\mathrm{d}\kappa} > 0,$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}\kappa} = \left(\frac{v[2\theta + \gamma\delta(1-v)]}{2\theta + \gamma(1-v)(1-v+\delta v)}\right)\frac{\mathrm{d}\iota}{\mathrm{d}\kappa} < 0,$$

$$\frac{\mathrm{d}E}{\mathrm{d}\kappa} = \left(\frac{a_{\mathrm{N}}[4\theta^2(1-\delta)(1+v) + (1-v+\delta v+\delta)[2\theta - \gamma\delta(1-v)]^2 + \chi]}{(1-\delta)(1-v)[2\theta + \gamma(1-v)(1-v+\delta v)]}\right)\frac{\mathrm{d}\iota}{\mathrm{d}\kappa} < 0,$$

where

$$\chi \equiv 2\theta\gamma(1-v)[\iota v(1-\delta) + 4(1-v+\delta v) + 1 + 3\delta] > 0,$$

and

$$\frac{\mathrm{d}w}{\mathrm{d}\kappa} = \frac{\partial w}{\partial \kappa} + \frac{\partial w}{\partial v}\frac{\mathrm{d}v}{\mathrm{d}\kappa} < 0.$$

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