Price discrimination and quality improvement

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Abstract. This paper models quality improvements when multiple quality levels can sell, owing to differences in consumers' valuations of quality improvements. Firms can collude to price discriminate, so that consumers with high valuations pay a price premium, while others receive a quality level below the highest available. Imposing minimum quality standards or price ceilings can ensure that only the highest quality level of each product is sold. Such intervention reduces the quality-adjusted price paid by consumers but also reduces the incentives for firms to innovate. When enough consumers have high valuations, such intervention must be welfare reducing, owing to reduced innovation. JEL Classification: O31, L16

Discrimination par les prix et amélioration de la qualité. Ce mémoire présente un modèle d'amélioration de la qualité quand on peut vendre des produits à divers niveaux de qualité à cause des différences dans les évaluations d'amélioration de qualité par les consommateurs. Les entreprises peuvent entrer en collusion pour faire de la discrimination par les prix de manière à ce que les consommateurs qui apprécient davantage la qualité paient une prime pendant que les autres consommateurs reçoivent une qualité au-dessous de ce qui est la meilleure qualité disponible. Si on impose des normes de qualité minimale ou des plafonds aux prix, on peut s'assurer que seuls les produits de la plus haute qualité seront vendus. De telles interventions réduisent le niveau de prix ajusté pour la qualité payé par les consommateurs, mais réduisent aussi les incitations des entreprises à innover. Quand un nombre suffisant de consommateurs apprécient beaucoup la qualité, de telles interventions peuvent réduire le niveau de bien-être à cause des innovations moins importantes.

1. Introduction

Improvements in the speed of computers and in the quality of other high-technology products have been amazing. However, the benefits of faster speed and higher qual-

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ity differ across consumers. Not all consumers buy the best, because the best generally carries a higher price and not everyone finds the higher quality justifies the higher price. For many high-tech products, some consumers purchase quality levels below the highest available.

The property that some consumers do not benefit from the latest technologies is a distortion that might warrant intervention. Holding all else equal, welfare could be improved by imposing a price ceiling sufficient to induce all consumers to purchase the state-of-the-art. But all else is not equal. The profits firms will earn entice them to develop new technologies. While regulation can generate static benefits for consumers, those gains would come at the expense of losses due to fewer innovations. Past analysis has not addressed this reduction in innovation. In this paper I determine whether the static gains or the dynamic losses dominate and under what conditions.

In my model, consumers differ in how much they value quality improvements. Firms can construct price-quality schedules such that consumers self-select. Quality enthusiasts pay a higher price for a higher quality level, while other consumers may purchase quality levels below the highest available. The fraction of consumers who are quality enthusiasts determines whether firms induce consumer types to separate, since separation means delayed sales to low-valuation consumers for the most recent innovator.

When too few consumers highly value quality improvements, only the highest quality level of each product sells in equilibrium. The rate of innovation does not depend on how many consumers are quality enthusiasts within this lower range. When enough consumers highly value quality improvements, however, both the highest and the second-highest quality level of each product sell in equilibrium, since producers set prices that induce consumer types to separate. The rate of innovation then increases with the fraction of consumers who are quality enthusiasts within this upper range, since larger profits provide a greater reward to innovation.

The incentives of firms to induce separation do not always match welfare objectives. When firms induce separation, lifetime utility for consumers can be above or below the level that would be enjoyed in the absence of price discrimination. High-valuation consumers pay more and low-valuation consumers receive a lower quality. However, the larger reward to innovation generates a faster rate of innovation. The benefits of faster innovation can more than offset the losses, owing to paying a higher quality-adjusted price when enough consumers are quality enthusiasts. Thus, government intervention to curtail price discrimination is not always warranted, since both types of consumers can be better off under price discrimination, owing to the faster rate of innovation. Examining only static effects risks exaggerating the need for regulation. This policy implication is the main conclusion of the paper.

This paper is unique in that a model of innovation is employed in which multiple quality levels sell to address issues of government policy intervention. The necessary ingredients are consumers that differ in their valuation of quality and a rate of innovation that responds to profit incentives. In contrast to representative agent models of quality improvement – such as Grossman and Helpman (1991), Segerstrom (1991), and Aghion and Howitt (1992) – here consumers differ in their valuation of quality, so that multiple quality levels may sell in equilibrium. In contrast to static models of quality differentiation, where multiple quality levels do sell in equilibrium – such as Jaskold Gabszewicz and Thisse (1979) and Shaked and Sutton (1982, 1983) – here, consumer heterogeneity influences the rate of quality improvement through profit incentives. In contrast to models of antitrust policy with a subsequent innovation – such as Chang (1995) and Green and Scotchmer (1995) – here, consumers differ in their valuation of quality, and quality can be repeatedly improved.

In section 2 I specify the maximization problem for heterogeneous consumers, where consumer types are based on the valuation of quality improvements. In section 3 I consider how firms choose prices and innovation intensities. In section 4 the solution is found for a steady-state pooling equilibrium and for a steady-state separating equilibrium, including the condition for whether firms induce consumer types to separate. In section 5 how the fraction of consumers who are quality enthusiasts affects the rate of innovation and aggregate expenditure is determined. Welfare analysis is performed and the potential for government intervention to improve welfare is explored. The conclusion is presented in section 7. Proofs appear in the appendix. A web appendix with additional material is located at http://economics.sbs.ohio-state.edu/pdf/glass/incsup.pdf.

2. Heterogeneous consumers

The specification of the consumer's problem follows Grossman and Helpman (1991) but allows the valuation of quality improvements to vary across consumer types. The economy is composed of two types of consumers, $\omega \in (L, H)$, labelled low and high based on the willingness to pay for quality improvements λ^ω. Consumers choose from a continuum of products $j \in [0,1]$. Consumers differ in their assessment of how much better each quality level m of product j is than the previous quality level $m-1$. Quality level m of product j provides quality $q_m(j) \equiv (\lambda^\omega)^m$ as perceived by consumers of type ω.[1] By the definition of quality improvement, all consumers agree that new generations are better than the old: $q_m(j) > q_{m-1}(j) \rightarrow (\lambda^\omega)^m > (\lambda^\omega)^{m-1} \rightarrow \lambda^\omega > 1$. High-type consumers (*quality enthusiasts*) value quality improvement more than low-type consumers: $\lambda^H > \lambda^L$. A fraction f^H of consumers are high type, while the remaining $f^L \equiv 1 - f^H$ are low type. For symmetry, the fraction of consumers who are quality enthusiasts is assumed to be the same for all products. Consumers are assumed to have equal incomes, for simplicity. If quality enthusiasts are wealthier, then statements about the fraction of consumers should be reinterpreted as applying to the fraction of spending by these consumers.

1 All products start at time $t = 0$ at quality level $m = 0$, so the base quality is $q_0(j) = (\lambda^\omega)^0 = 1$.

A consumer of type ω has additively separable intertemporal preferences given by lifetime utility

$$U^{\omega} = \int_0^{\infty} e^{-\rho t} \log u^{\omega}(t) dt, \tag{1}$$

where ρ is the common subjective discount factor. Instantaneous utility is

$$\log u^{\omega}(t) = \int_0^1 \log \sum_m (\lambda^{\omega})^m x_m^{\omega}(j,t) dj, \tag{2}$$

where $(\lambda^{\omega})^m$ is the assessment by type ω consumers of quality level m and $x_m^{\omega}(j,t)$ is consumption by type ω consumers of quality level m of product j at time t.

A consumer of type ω maximizes lifetime utility subject to an intertemporal budget constraint. Since preferences are homothetic, aggregate demand for each consumer type is found by maximizing lifetime utility subject to the aggregate intertemporal budget constraint

$$\int_0^{\infty} e^{-R(t)} E^{\omega}(t) dt \leq A^{\omega}(0) + \int_0^{\infty} e^{-R(t)} Y^{\omega}(t) dt, \tag{3}$$

where $R(t) = \int_0^t r(s) ds$ is the cumulative interest rate up to time t and $A^{\omega}(0) = f^{\omega} A(0)$ is the aggregate value of initial asset holdings by type ω consumers. Individuals hold assets in the form of ownership in firms, but with a diversified portfolio, any capital losses appear as capital gains elsewhere, so only initial asset holdings remain. Aggregate labour income of all type ω consumers is $Y^{\omega}(t) = f^{\omega} Lw(t)$, where $w(t)$ is the wage at time t and L is the labour supply. Thus $Lw(t)$ is total labour income at time t and $f^{\omega} Lw(t)$ is the share of total labour income belonging to type ω consumers. Normalize the wage to one: $w(t) = 1$. Aggregate expenditure of all type ω consumers is

$$E^{\omega}(t) = \int_0^1 \left[\sum_m p_m(j,t) x_m^{\omega}(j,t) \right] dj, \tag{4}$$

where $p_m(j,t)$ is the price of quality level m of product j at time t, $E^{\omega}(t) = f^{\omega} E(t)$, and $E(t)$ is aggregate expenditure.

A consumer's maximization problem can be broken into three stages: the allocation of lifetime wealth across time, the allocation of expenditure at each instant across products, and the allocation of expenditure at each instant for each product across available quality levels. In the final stage, consumers of each type allocate expenditure for each product at each instant to the quality level $\widetilde{m}^{\omega}(j,t)$ offering the lowest quality-adjusted price, $p_m(j,t)/(\lambda^{\omega})^m$. Settle indifference in favour of the higher quality level so that the quality level selected is unique. The steady-state follows Grossman and Helpman (1991), but owing to the different valuations of quality improvements, consumers can disagree on which quality level provides the lowest quality-adjusted price.

In the second stage, consumers of each type then evenly spread expenditure across the unit measure of all products, $E^{\omega}(j,t) = E^{\omega}(t)$, since the elasticity of substitution is constant at unity. Consumers of type ω demand

$$x^{\omega}_{\widetilde{m}}(j,t) = E^{\omega}(t)/p_{\widetilde{m}}(j,t) \tag{5}$$

units of quality level $\widetilde{m}^{\omega}(j,t)$ of product j and no units of other quality levels of that product. Thus, imposing the expression for consumer demand (5) and separating terms, instantaneous utility (2) becomes

$$\log u^{\omega}(t) = \log E^{\omega}(t) + \int_0^1 [\widetilde{m}^{\omega}(j,t)\log\lambda^{\omega} - \log p_{\widetilde{m}}(j,t)]\,dj, \tag{6}$$

and lifetime utility (1) becomes

$$U^{\omega} = \int_0^{\infty} e^{-\rho t}\left[\log E^{\omega}(t) + \int_0^1 [\widetilde{m}^{\omega}(j,t)\log\lambda^{\omega} - \log p_{\widetilde{m}}(j,t)]\,dj\right]dt \tag{7}$$

by substituting for instantaneous utility.

In the first stage, consumers of each type evenly spread lifetime expenditure across time, $E^{\omega}(t) = E^{\omega}$, since the utility function for each consumer type is time separable and the aggregate price levels (the aggregate high-quality price level and aggregate low-quality price level) will be shown not to vary across time $\log p_{\widetilde{m}}(j,t) = \log p_{\widetilde{m}}(j)$. Since aggregate expenditure is constant across time, the interest rate at each point in time reflects the discount rate $r(t) = \rho$, so $R(t) = \rho t$ in the intertemporal budget constraint. The model keeps these two stages simple to focus attention on the final stage, where quality choice is determined. Whether enough consumers are quality enthusiasts determines whether firms choose prices that cause consumer types to separate, as seen by characterizing firm behaviour.

3. Firm behaviour

To produce a quality level of a product, a firm must first design it. Firms are willing to endure the costs of developing higher quality levels of existing products because they earn profits in the product market if successful. The potential for quality improvement is unbounded.

3.1. Innovation

Modelling innovation success as a continuous Poisson process follows the assumptions made by Grossman and Helpman (1991). Innovation resembles a lottery: firms endure a cost for a chance at winning a payoff. For any firm, undertaking innovation intensity $\tilde{\imath}$ for a time interval dt requires $a\tilde{\imath}dt$ units of labour and leads to success with probability $\tilde{\imath}dt$. An innovation success means that the firm discovers how to produce the quality level m, where the previous quality level in the industry was $m-1$. A higher investment in innovation yields a higher probability of success, but no level of investment in innovation can guarantee success. Only the current level of innovation activity determines the chance of innovation success, since innovation is memory-less for simplicity. Assume innovation races occur simultaneously for all products, with all firms able to target the quality level above the current highest. Finally, assume free entry into innovation, with an endless pool of potential innovators.

Given the innovation intensity ι of other firms, a firm's strategy is to pick an innovation intensity $\tilde{\iota}$ and a price p to charge for its product once successful in innovation. Look for the steady-state equilibrium in stationary strategies, where firms' strategies are time invariant (but may depend on the current state of the economy). Firms already producing a quality level of a product can be shown to not engage in further innovation on that product; the proof mirrors that in Grossman and Helpman (1991). Producing firms lose value when subsequent innovation is successful, so other firms stand to gain more (net) and undertake innovation.[2] Therefore, a firm's problem can be broken into an innovation stage and a production stage.

In the innovation stage, a firm endures innovation costs of $a\tilde{\iota}dt$ and gains an expected reward of $v\tilde{\iota}dt$ (enjoys gains v with probability $\tilde{\iota}dt$). Each firm chooses its intensity of innovation $\tilde{\iota}$ to maximize its expected gain from innovation.

$$\max_{\tilde{\iota}\geq 0} \int_0^\infty e^{-(\rho+\iota)t}(v-a)\tilde{\iota}dt = \max_{\tilde{\iota}\geq 0}\left(\frac{v-a}{\rho+\iota}\right)\tilde{\iota} \leftrightarrow \max_{\tilde{\iota}\geq 0}(v-a)\tilde{\iota}. \tag{8}$$

The term $e^{-\iota t}$ captures the probability that no other firm will have succeeded in innovating in the same industry prior to time t. Each non-producing firm chooses its innovation intensity to maximize the difference between the expected reward and the costs of innovation. Firms engage in innovation with non-negative intensity whenever the expected gains are no less than their costs. To generate finite rates of innovation, the expected gains must not exceed their cost, with equality when innovation occurs with positive intensity.

$$v \leq a, \tilde{\iota} > 0 \Leftrightarrow v = a. \tag{9}$$

The product market outcome determines the expected gains from innovation. The value of a producing firm is the present discounted stream of its instantaneous profits, where its product is continually targeted for quality improvement by non-producing firms.

3.2. Production

In the production stage, each producing firm chooses the price of its product to maximize its value, given the prices and innovation intensities of other firms. At any moment, at most two different quality levels of any product sell, owing to the two types of consumers. The two quality levels are the two highest existing quality

2 Lerner (1997) finds empirical support for this feature in the hard disk drive industry.

levels, since all quality levels cost the same to produce but consumers value higher quality levels more. Label high and low the two quality levels of a product produced at any point in time, although the labelling of a given quality level changes over time (high when initially invented, low after subsequent innovation).

Depending on the fraction of consumers who are quality enthusiasts, separation or pooling of consumer types may occur (details are provided in the next subsection). If separating, the most recently successful innovator produces the high quality level and the previous innovator produces the low quality level. If pooling, the most recently successful innovator sells the high quality level, the only quality level of that product that sells.

Under pooling, a successful innovator becomes the top firm and the previous top firm is priced out of the market. The reward to successful innovation when pooling prevails is the present discounted value of profits from selling the high quality level to all consumers until rival innovation occurs and terminates the profit stream.

$$v^P = \int_0^\infty e^{-(\rho+\iota^P)t}\pi^P dt = \frac{\pi^P}{\rho+\iota^P}. \tag{10}$$

At each instant, the firm earns instantaneous profits π^P if rival innovation has not yet occurred, where ι^P is the innovation intensity under pooling.

Under separation, a successful innovator becomes the top firm, the previous top firm becomes the trailing firm, and the previous trailing firm is priced out of the market. The reward for successful innovation when separation prevails is the present discounted value of profits from selling the high quality level until rival innovation knocks the firm down to the trailing position.

$$v^1 = \int_0^\infty e^{-(\rho+\iota^S)t}(\pi^1 + \iota^S v^2)\, dt = \frac{\pi^1 + \iota^S v^2}{\rho+\iota^S}. \tag{11}$$

At each instant, the top firm earns instantaneous profits π^1 if rival innovation has not yet occurred, where ι^S is the innovation intensity under separation. Upon rival innovation, the firm maintains only the trailing firm value (until rival innovation occurs again and terminates the profit stream).

$$v^2 = \int_0^\infty e^{-(\rho+\iota^S)t}\pi^2 dt = \frac{\pi^2}{\rho+\iota^S}. \tag{12}$$

At each instant, the trailing firm earns instantaneous profits π^2 if rival innovation has not yet occurred. Combine the two valuation equations by substituting v^2 into v^1, leaving the value of a top firm under separation as its discounted stream of expected instantaneous profits from selling first high quality and then low quality.

$$v^S \equiv v^1 = \frac{\pi^1 + \iota^S \dfrac{\pi^2}{\rho+\iota^S}}{\rho+\iota^S}. \tag{13}$$

Here, the erosion of firm value due to technological depreciation occurs gradually, with repeated innovations being necessary to push a quality level of a product into obsolescence.[3]

Firms know the two different valuations of quality present in the population, λ^L and λ^H, as well as the fraction of consumers of each type, f^L and f^H, but cannot observe the willingness to pay for quality of any individual consumer. Thus, if firms want to sell different quality levels to different consumer types, they must offer prices such that consumers self-select (second-degree price discrimination).

If the high quality level is priced at a premium of $p^1/p^2 \leq \lambda^H$ relative to the low quality level, high-type consumers purchase the high quality level, while low-type consumers purchase the low quality level. In addition, if the low quality level is priced at a premium of $p^2/p^3 \leq \lambda^L$ relative to the quality level just out of the market, low-type consumers purchase the low quality level rather than the quality level below it. Finally, the lowest price for the quality level just out of the market that would achieve non-negative profits $p^3 = w = 1$, ties down the highest prices for the high and the low quality levels that induce consumer types to separate: $p^1 = \lambda^H \lambda^L$ and $p^2 = \lambda^L$. More generally, the incentive compatibility constraint for consumers of type ω, $p^k/p^{k+n} \leq (\lambda^\omega)^n$, indicates that consumers will buy quality level k rather than $k+n$ if the price ratio is no more than their willingness to pay for the n additional quality increments.

Normalize the labour requirement in production to one. Define $\delta^L \equiv 1/\lambda^L$ and $\delta^H \equiv 1/\lambda^H$. Let E^P and E^S refer to aggregate expenditure E under pooling or separation, respectively. Under pooling, the top firm charges price $p^P = \lambda^L$ and makes sales $x^P = E^P/p^P = E^P\delta^L$, yielding instantaneous profits.

$$\pi^P = E^P(1 - \delta^L). \tag{14}$$

The top firm engages in limit pricing against its inactive rival one quality level below. Both high- and low-type consumers buy the highest quality level available.

Under separation, the top firm charges price $p^1 = \lambda^H \lambda^L$ and makes sales $x^1 = f^H E^S/p^1 = f^H E^S \delta^H \delta^L$, yielding instantaneous profits.

$$\pi^1 = f^H E^S(1 - \delta^H \delta^L). \tag{15}$$

The trailing firm charges price $p^2 = \lambda^L$ and makes sales $x^2 = f^L E^S/p^2 = f^L E^S \delta^L$, yielding instantaneous profits.

$$\pi^2 = f^L E^S(1 - \delta^L). \tag{16}$$

The trailing firm engages in limit pricing against its inactive rival one quality level below, while the top firm engages in limit pricing against the trailing firm. High-type consumers buy the high quality level and low-type consumers buy the low quality level.

3 Lai (1998) introduces gradual product obsolescence into a product variety model of endogenous growth.

Inserting profits into producing firm valuation gives the value of a top firm under pooling,

$$v^P = \frac{E^P(1-\delta^L)}{\rho + \iota^P}, \tag{17}$$

and under separation,

$$v^S = \frac{f^H E^S(1-\delta^H\delta^L) + \dfrac{\iota^S}{\rho+\iota^S}(1-f^H)E^S(1-\delta^L)}{\rho+\iota^S}. \tag{18}$$

Thus, the innovation condition (9) when the rate of innovation is positive becomes

$$\max\{v^P, v^S\} = a, \tag{19}$$

owing to the trait that the top firm can essentially dictate whether separation or pooling occurs, as seen by considering the equilibrium of the repeated pricing game.

3.3. Implicit collusion

The pooling prices described above are the Nash equilibrium of the static pricing game. Given that the trailing firm is charging a price equal to one, the top firm's best response is $p^P = \lambda^L$. The top firm would not want to lower its price, since industry demand is unit elastic. Nor would the top firm want to raise its price and thus lose low-type consumers to the trailing firm (provided few enough consumers are quality enthusiasts $f^H < \widehat{f^H}$ as defined in the web appendix). Similarly, given that the top firm is charging the price $p^P = \lambda^L$, the trailing firm's best response is to charge a price of one, its marginal cost. If the trailing firm were to lower its price below marginal costs, it would generate negative profits, and raising its price would generate no sales.

The separating prices described above are not a Nash equilibrium of the static pricing game. Given that the trailing firm is charging the price $p^2 = \lambda^L$, the top firm's best response is $p^1 = \lambda^H\lambda^L$. The top firm would not want to raise its price and thus lose high-type consumers to the trailing firm, nor would it want to lower its price, since industry demand is unit elastic. But, given that the top firm is charging the price $p^1 = \lambda^H\lambda^L$, the trailing firm's best response is not $p^2 = \lambda^L$. The trailing firm would have an incentive to lower its price infinitesimally below λ^L and capture the entire market: high-type consumers would then buy the low quality level, since $\lambda^H\lambda^L/(\lambda^L - \varepsilon) > \lambda^H$.

A separating equilibrium can be supported as a subgame perfect equilibrium by the firms playing trigger strategies to support implicit collusion in the repeated game. In the repeated game, the top firm can deter the trailing firm from undercutting its price by threatening to punish any deviation. Restrict attention to *simple strategy profiles*, which specify the same punishment after any deviation and any history of actions. Suppose punishment can occur only after some lag $0 < l < \infty$.[4] Punishment can take the form of reverting to the one-shot Nash equilibrium of pooling. Under pooling, even when the trailing firm prices at its marginal cost of one, the top firm can price at $p^P = \lambda^L$ and capture the entire market, owing to its quality advantage. The ability of the trailing firm to shut down and thus ensure zero profits prevents any more severe punishment.

At time τ, the top firm chooses the price p^1 and the trailing firm picks the price p^2 if both firms have always done so in the history up to time $\tau - l$; otherwise, the top firm would pick p^P and the trailing firm would price at its cost of one during the permanent punishment phase. The punishment phase is subgame perfect, since the Nash equilibrium of the one-shot game (pooling) is merely repeated. Implicit collusion is subgame perfect if and only if both firms gain a higher value from cooperating than from deviating.

Given the top firm's strategy, the trailing firm never (for finite discount rate and sufficiently short lag in retaliation) deviates from p^2, because higher instantaneous profits would require sacrifice of future profits following retaliation. For the trailing firm to cooperate, its value v^2 from cooperation must exceed the value v^D stemming from a deviation. A trailing firm that defects earns instantaneous deviating profits until either the top firm retaliates or rival innovation occurs and terminates the profit stream.

$$v^D = \int_0^l e^{-(\rho+\iota^S)t}\pi^D dt = \frac{\pi^D}{\rho + \iota^S}[1 - e^{-(\rho+\iota^S)l}]. \tag{20}$$

At each instant, the defecting trailing firm earns instantaneous profits π^D if rival innovation has not yet occurred and the lag until retaliation has not yet expired. Instantaneous profits for a trailing firm that deviates are maximized by charging a deviation price $p^D = \lambda^L - \varepsilon$ (infinitesimally below $p^2 = \lambda^L$) and thus capturing the entire market. The instantaneous profits the trailing firm earns when deviating prior to when the top firm can retaliate are

$$\pi^D = E^S(1 - \delta^L). \tag{21}$$

Once the lag has passed, the top firm retaliates by charging the price p^P forever, so the trailing firm earns no further profits.

When cooperating, the trailing firm sells only to low-type consumers, so instantaneous profits are lower than when it is defecting (prior to retaliation), but the expected duration of profits is shorter when it is defecting, owing to retaliation by the top firm after lag l. For cooperation to be supported, the value under separation must exceed the value under deviating $v^2 > v^D$, which, by using the expressions for

4 Segerstrom (1991) employs similar arguments to support collusion between two firms producing the same quality level following imitation (which does not occur here, owing to assumed perfect patent protection).

the relevant values and profits, simplifies to $f^L > 1 - e^{-(\rho+\iota^S)l}$. Substituting $f^L = 1 - f^H$ and taking logs of both sides show that cooperation requires that the lag must be sufficiently short.

$$l < \bar{l} \equiv -\frac{\log(f^H)}{\rho + \iota^S}. \tag{22}$$

As the fraction of consumers who are high type f^H becomes larger, the labour supply L larger, or the resource requirement in innovation a smaller (through increasing ι^S), the maximum lag shrinks. The temptation for the trailing firm to deviate grows with f^H, since the deviation value from selling to the entire market becomes large relative to the value from serving only low-type consumers. This temptation shrinks for faster ι^S, owing to the chance that its profit stream will have been terminated by innovation before the punishment kicks in.

In the limit as the lag becomes infinitesimally small, $l \to 0$, the trailing firm always cooperates, so the top firm can always induce separation. If separation leads to a greater firm value, the top firm chooses to induce separation. Separation occurs whenever the value of a top firm is higher under separation $v^S > v^P$, while pooling occurs otherwise $v^S \leq v^P$. If separation generates a higher value, the top firm will follow its strategy profile described above.

Technically, a continuum of separating equilibria could be supported. Implicit collusion can support any $p^2 = \psi^L$ and $p^1 = \psi^L \psi^H$ such that $1 \leq \psi^L \leq \lambda^L$ and $\lambda^L \leq \psi^H \leq \lambda^H$. The separating equilibrium refers to one with the highest price levels, which yields the highest profits for both firms and thus the fastest innovation. Since profits are highest, firms will select the greatest degree of collusion, unless government antitrust concerns intervene.

4. Solution

In equilibrium, high- and low-type consumers maximize their lifetime utility subject to the intertemporal budget constraint, firms maximize their value given prices and innovation intensities chosen by other firms, and labour markets clear. Provided the economy has enough resources to support a positive rate of innovation, in equilibrium, producing firms earn profits sufficient in expectation to compensate for their innovation expenses. Rival innovation eventually eliminates the profit stream from production, but not immediately if multiple quality levels sell in equilibrium.

In a steady-state equilibrium with positive innovation, a constant fraction of all products experiences quality improvement at each instant. In this section the rate of innovation and aggregate expenditure under pooling are found, where all consumers buy the same quality level at the same price, and under separation, where firms induce consumer types to separate through price discrimination. Then, the condition on the fraction of quality enthusiasts needed to generate separation rather than pooling is derived.

4.1. Pooling equilibrium

For the pooling equilibrium, insert the producing firm valuation equation (17) into the innovation equilibrium condition (9) to yield the valuation condition, which requires the expected reward from innovation to equal its cost.

$$E^P(1-\delta^L) = a(\rho + \iota^P). \tag{23}$$

Then, impose the resource constraint, which requires the labour demand for innovation and production to equal the labour supply.

$$a\iota^P + E^P\delta^L = L. \tag{24}$$

The pooling equilibrium is the aggregate expenditure,

$$E^P = L + \rho a, \tag{25}$$

and rate of innovation,

$$\iota^P = \frac{L}{a}(1-\delta^L) - \rho\delta^L, \tag{26}$$

that solve this pair of equations.

The pooling equilibrium resembles the solution for homogeneous consumers all having willingness to pay for quality improvements λ^L, as in Grossman and Helpman (1991), because prices are determined by the willingness to pay for quality of low-type consumers. Even though some consumers are willing to pay more for quality, in the pooling equilibrium their higher willingness to pay for quality is not revealed.

4.2. Separating equilibrium

For the separating equilibrium, insert the producing firm valuation equation (18) into the innovation equilibrium condition (9) to yield the valuation condition.

$$E^S\left[f^H(1-\delta^H\delta^L) + \frac{\iota^S}{\rho+\iota^S}(1-f^H)(1-\delta^L)\right] = a(\rho+\iota^S). \tag{27}$$

Then impose the resource constraint.

$$a\iota^S + E^S\delta^L[1-f^H(1-\delta^H)] = L. \tag{28}$$

The separating equilibrium is the aggregate expenditure and rate of innovation that solve this pair of equations. The fraction of consumers who are quality enthusiasts determines whether the pooling or the separating equilibrium emerges, as is explored next.

4.3. Condition for separation

What fraction of consumers must be quality enthusiasts to give firms sufficient motive to induce consumer types to separate? Examining firm optimization leads to the conditions for separation to prevail. If the fraction of consumers who are quality enthusiasts is large enough, then separation results, since separation yields higher value for the most recent innovator than pooling.

Setting the value of a top firm under separation (18) equal to that under pooling (17) for a given rate of innovation determines the boundary fraction of consumers who are quality enthusiasts needed for firms to induce consumer types to separate.

$$f^* = \frac{1}{1 + \left(\dfrac{L}{\rho a} + 1\right)(1 - \delta^H)}. \tag{29}$$

If too small a fraction of consumers are quality enthusiasts, then firms do not induce consumer types to separate; to gain the added premium from a small fraction of high-type consumers, a successful innovator would sacrifice profits from selling to a large fraction of low-type consumers.

PROPOSITION 1. *(Equilibrium): If enough consumers are quality enthusiasts,* $f^H > f^*$*, a separating equilibrium prevails: the real expenditure and the rate of innovation solve the resource constraint (28) and the valuation condition (27). Otherwise, a pooling equilibrium prevails, with real expenditure (25) and rate of innovation (26).*

5. Innovation

In this section how the equilibrium rate of innovation and aggregate expenditure respond to changes in the fraction of consumers who are quality enthusiasts is determined.

In a pooling equilibrium, the price charged for the high quality level is determined by the willingness to pay for quality of low-type consumers. While some consumers do value quality improvements more, they do not pay any more in a pooling equilibrium. Thus, having more quality enthusiasts does not increase profits and thus does not increase the reward to innovation or the rate of innovation.

In a separating equilibrium, however, high-type consumers pay a premium reflecting their greater willingness to pay for quality. Thus, having more quality enthusiasts increases profits from selling high quality levels and decreases profits from selling low quality levels, with a net positive effect on the reward to innovation when separation prevails, as established through differentiation in the proof to proposition 2 below (see the appendix for the proof). Consequently, shifting the distribution towards quality enthusiasts increases the rate of innovation when more than one quality level sells but has no effect when only one quality level sells.

What are the upper and lower bounds on the rate of innovation and aggregate expenditure under separation? How do these bounds compare with the rate of innovation and aggregate expenditure under pooling? Establishing these bounds will complete the description of how aggregate expenditure and innovation vary with f^H and thus help in establishing the welfare results to follow.

In the limit, as the distribution becomes degenerate on high-type consumers (the upper bound), the rate of innovation under separation exceeds the rate of innovation under pooling,

$$\lim_{f^H \to 1} \iota^S = \bar{\iota} \equiv \frac{L}{a}(1 - \delta^H \delta^L) - \rho \delta^H \delta^L > \iota^P, \tag{30}$$

and aggregate expenditure under separation approaches aggregate expenditure under pooling.

$$\lim_{f^H \to 1} E^S = L + \rho a = E^P. \tag{31}$$

In this upper limit, the rate of innovation under separation exceeds even that for a pooling equilibrium with all quality enthusiasts $(L/a)(1 - \delta^H) - \rho\delta^H$. This difference arises because the presence of low-type consumers allows the top firm to extract a higher price (through implicit collusion with the trailing firm) than when there are only high-type consumers. The separating equilibrium does not exist when all consumers are quality enthusiasts ($f^H = 1$) because there are no low-type consumers remaining.

In the limit, as the distribution approaches the lower boundary for separation f^*, the rate of innovation under separation approaches the rate of innovation under pooling,

$$\lim_{f^H \to f^*} \iota^S = \frac{L}{a}(1 - \delta^L) - \rho \delta^L = \iota^P, \tag{32}$$

and aggregate expenditure under separation exceeds aggregate expenditure under pooling.

$$\lim_{f^H \to f^*} E^S = \bar{E} \equiv \frac{L + \rho a}{1 - f^H(1 - \delta^H)} > E^P. \tag{33}$$

The rate of innovation in the separating equilibrium lies between the bounds of a minimum equal to the rate of innovation under pooling and a maximum above the rate of innovation under pooling: $\iota^P \leq \iota^S \leq \bar{\iota}$. Similarly, aggregate expenditure in the separating equilibrium lies between the bounds of a minimum equal to aggregate expenditure under pooling and a maximum above aggregate expenditure under pooling: $E^P \leq E^S \leq \bar{E}$. However, the extreme $f^H \to 1$ where the rate of innovation is the highest $\iota^S \to \bar{\iota}$ is where aggregate expenditure is the lowest $E^S \to E^P$, and the extreme $f^H \to f^*$ where aggregate expenditure is the highest $E^S \to \bar{E}$ is where the rate of innovation is the lowest $\iota^S \to \iota^P$ within a separating equilibrium.

Overall, how does the rate of innovation respond to increases in the fraction of consumers who are quality enthusiasts?

PROPOSITION 2. *(Comparative Statics): Starting from zero, as the fraction of consumers who are quality enthusiasts rises, the rate of innovation and real expenditure initially remain constant at the levels given by (26) and (25), and then eventually the rate of innovation rises and real expenditure falls once more than one quality level sells.*

In the next section the impact on welfare of permitting separation for parameters such that separation would occur is analysed.

6. Welfare

Given that the rate of innovation and aggregate expenditure when separation prevails is higher than the rate of innovation were pooling to prevail, is welfare higher under separation than under pooling? Would government policies, such as minimum quality standards or price ceilings, necessarily lower welfare by reducing the number of different quality levels of each product selling in equilibrium? Not necessarily: while the rate of innovation is faster when separation prevails, under separation high-type consumers pay a higher price for the highest quality level available, while low-type consumers settle for a lower quality level relative to the quality frontier.

6.1. Welfare expressions

By the law of large numbers, the expected number of quality improvements in period t is $\overline{m}(t) = \iota t$. In a pooling equilibrium, instantaneous utility (6) for type ω consumers is

$$\log u_P^\omega(t) = \log E_P^\omega + \iota^P t \log \lambda^\omega - \log \lambda^L, \tag{34}$$

and lifetime utility (7) for type ω consumers is

$$U_P^\omega = \frac{\log E_P^\omega + \dfrac{\iota^P}{\rho} \log \lambda^\omega - \log \lambda^L}{\rho}. \tag{35}$$

The second term is the positive contribution of expected quality improvements to lifetime utility; the last term is the negative impact of price. The middle term is larger for high-type consumers than for low-type consumers, owing to their high valuation of quality. Firm profits do not contribute to welfare because they are offset (in present discounted value terms) by costs of innovation because of free entry into innovation.

Under separation, instantaneous utility (6) for type ω consumers is

$$\log u_S^{\omega}(t) = \log E_S^{\omega} + \iota^S t \log \lambda^{\omega} - \log \lambda^L - \log \lambda^{\omega}, \tag{36}$$

and lifetime utility (7) for type ω consumers is

$$U_S^{\omega} = \frac{\log E_S^{\omega} + \dfrac{\iota^S}{\rho} \log \lambda^{\omega} - \log \lambda^L - \log \lambda^{\omega}}{\rho}. \tag{37}$$

Compared with lifetime utility under pooling, under separation all consumers benefit from a higher rate of innovation and larger aggregate expenditure, but high-type consumers suffer from a higher price and low-type consumers suffer from a lower quality level (the $-\log \lambda^{\omega}$ term).

6.2. Welfare comparison

Define $\Delta U^{\omega} \equiv U_S^{\omega} - U_P^{\omega}$ as the difference in lifetime utility between separation and pooling, for a given set of parameters. Lifetime utility for type ω consumers is higher under separation if and only if the difference,

$$\Delta U^{\omega} = \frac{\Delta \log E^{\omega} + \left(\dfrac{\Delta \iota}{\rho} - 1\right) \log \lambda^{\omega}}{\rho}, \tag{38}$$

is positive, where $\Delta \log E^{\omega} \equiv \log E_S^{\omega} - \log E_P^{\omega} = \log(E_S^{\omega}/E_P^{\omega})$ is the difference in aggregate expenditure between separation and pooling. The difference in utility is the sum of three effects: difference in expenditure, rate of innovation, and price (or quality received relative to the highest available). The negative effect of higher price (or lower quality level relative to the highest available) is constant across time. The benefits of a faster rate of innovation grow over time as a greater difference develops in the highest quality level expected to be available. For lifetime utility to be higher under separation than pooling, the positive level and growth effects from larger aggregate expenditure and faster innovation under separation must offset this negative level effect.

PROPOSITION 3. *(Welfare): When the fraction of consumers who are quality enthusiasts is near the lower bound for separation to occur $f^H \to f^*$, consumers of both types experience a fall in lifetime utility from separation. When enough consumers are quality enthusiasts $f^H \to 1$, consumers of both types experience a rise in lifetime utility from separation.*

In the limit, since only enough consumers are quality enthusiasts to support separation $f^H \to f^*$, the increase in aggregate expenditure fails to counter the

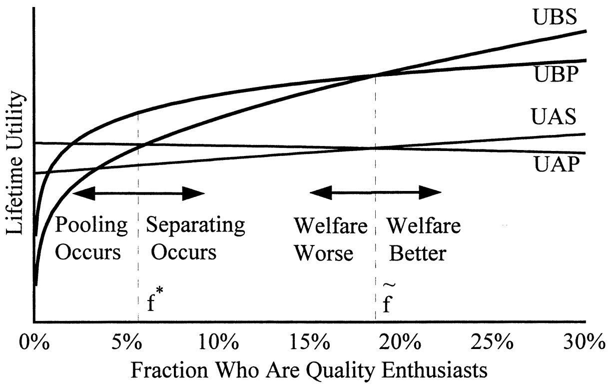

FIGURE 1 Welfare comparison

negative level effect of higher prices for high-type consumers and consuming below the highest quality level available for low-type consumers. In the limit, since all consumers are quality enthusiasts $f^H \to 1$, the faster rate of innovation is enough to counter the negative level effect of higher prices for high-type consumers and consuming below the highest quality level available for low-type consumers.

Figure 1 shows how lifetime utility initially is higher under pooling than under separation and eventually is higher under separation than under pooling, as the fraction of consumers who are quality enthusiasts increases. The proof of proposition 3 establishes these traits. Owing to the switch in welfare:

COROLLARY 4. *The fraction of consumers who are quality enthusiasts that is needed for consumers to benefit from separation is larger than that needed for separation to occur* $\tilde{f} > f^*$.[5]

Thus, firms induce separation for some range (f^H just above f^* but below $\tilde{f}$) where the lifetime utility of both types of consumer would be higher if separation did not occur. This property introduces a limited role for government intervention to curtail separation when separation reduces welfare.

6.3. Government intervention

Accordingly, minimum quality standards (or imposing a price ceiling at λ^L) can reduce the rate of innovation but still raise welfare: if the government bans sales of anything but the highest quality level available (and the ban binds), firms will slow down their attempts to improve the quality of products. Within the range of f^H just

5 The threshold for consumers to benefit from separation may depend on consumer type without negating any of the welfare results.

above f^*, where welfare is lower for both types of consumer under separation than under pooling, government intervention could raise the welfare of both types. If the fraction of consumers who are quality enthusiasts is not much more than is required for firms to induce separation, the slower rate of quality improvement is more than offset by a better pricing outcome for high-type consumers and no quality distortion for low-type consumers. Thus, minimum quality standards can be welfare enhancing even though innovation suffers.

There is no guarantee, however, that quality standards or price ceilings will be welfare improving; the outcome depends on the fraction of consumers who are quality enthusiasts. Within the range of f^H just below one, where welfare is higher for both types of consumer under separation than under pooling, the government should not intervene to induce pooling, because doing so would lower the welfare of both consumer types. The analysis performed here captures how separation provides greater incentives for innovation. Static analysis ignoring how the rate of innovation responds to profit incentives risks overstating the role for government intervention by failing to consider a positive aspect of separation.

7. Conclusion

Multiple quality levels of many high-tech products sell, since consumers' valuations of quality contain some idiosyncratic component. How does the rate of quality improvement of high-tech products respond to the fact that not all consumers buy the best version? How does the rate of quality improvement respond if the government intervenes to prevent all but the highest quality level of each product from selling? This paper provides some intriguing answers to these questions.

Quality enthusiasts buy higher quality levels and pay a higher price if a sufficient fraction of consumers are quality enthusiasts, so that firms induce consumer types to separate. If too few consumers are quality enthusiasts, then firms do not induce consumer types to separate: all consumers purchase the highest quality level available of each product. In a pooling equilibrium, increasing the fraction of consumers who are quality enthusiasts does not create any more incentive for innovation, because differences in willingness to pay for quality are not revealed.

Once enough consumers are quality enthusiasts, a separating equilibrium emerges. In a separating equilibrium, increasing the fraction of consumers who are quality enthusiasts does create more incentive for innovation, since the higher willingness to pay for quality of high-type consumers is revealed. Consequently, the fraction of consumers who are quality enthusiasts can alter the rate of innovation, but only if multiple quality levels are sold in equilibrium.

Regulations can have differing effects on welfare, depending on the distribution of consumer types (through effects on the rate of innovation). Within the range where multiple quality levels sell, for some range all consumers are worse off under separation than they would be under pooling, and the government could potentially increase welfare by imposing minimum quality standards or price ceilings. However, for some range all consumers are better off under separation than they would be under pooling, and the government should not intervene. Consumers can be better off under separation if the gain from faster innovation offsets the static loss from paying a higher price or consuming a quality level below the highest available. Understanding that the higher prices paid under separation help to support a faster rate of innovation is vital for avoiding interventions that would reduce welfare.

Beyond the policy implications examined in this paper, this model proves useful for examining other issues that arise where multiple quality levels sell, especially international trade issues. Glass (1997, 1998), Glass and Saggi (1998), and Yang and Maskus (1999) use this model as the basis for models of international trade, foreign direct investment, or licensing between countries. Analysis with multiple quality levels permits the quality mix of production to vary across countries. Resource accumulation and government policies then alter the rate of quality improvement and the allocation of production of different quality levels across countries.

Appendix

Proof of proposition 1

Suppose that the economy is currently in a pooling equilibrium, so the rate of innovation under pooling ι^P is taken as given. For a firm to be indifferent between pooling and separation, the firm must receive the same value $v^S(\iota^P) = v^P(\iota^P) = a$ at the boundary f^*. To solve $v^S(\iota^P) = a$ for f^H, substitute $E^S = (L - a\iota^S)/[\delta^L(1 - f^H(1 - \delta^H))]$ from (28) and $\iota^S = \iota^P$ into the expression for the value of a top firm under separation (18) and set equal to a to achieve the condition.

$$\frac{f^H(1 - \delta^H\delta^L) + \dfrac{\iota^P}{\rho + \iota^P}(1 - f^H)(1 - \delta^L)}{\rho + \iota^P} = \frac{\delta^L[1 - f^H(1 - \delta^H)]}{\dfrac{L}{a} - \iota^P}. \tag{A1}$$

Substituting $\iota^P = (L/a)(1 - \delta^L) - \rho\delta^L$ from (26) and solving for f^H generates the boundary distribution f^*.

Proof of proposition 2

By proposition 1 and aggregate expenditure (25) and innovation (26), $\partial E^P/\partial f^H = \partial\iota^P/\partial f^H = 0$ for the pooling equilibrium $0 < f^H \leq f^*$. For the separating equilibrium $f^* < f^H < 1$, write the system of the resource constraint and the valuation condition as

$$\lambda^H\lambda^L a\iota^S + E^S[\lambda^H - f^H(\lambda^H - 1)] - \lambda^H\lambda^L L = 0 \tag{A2}$$

$$E^S[\rho(\lambda^H\lambda^L - 1) + \iota^S[f^H(\lambda^H - 1) + \lambda^H(\lambda^L - 1)]] - a\lambda^H\lambda^L(\rho + \iota^S)^2 = 0. \tag{A3}$$

The derivatives with respect to the fraction of consumers who are quality enthusiasts f^H are

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \partial E^S \\ \partial \iota^S \end{bmatrix} = \begin{bmatrix} -E^S(\lambda^H - 1)\partial f^H \\ E^S \iota^S f^H(\lambda^H - 1)\partial f^H \end{bmatrix}, \tag{A4}$$

where

$$b_{11} = \lambda^H - f^H(\lambda^H - 1) > 0$$

$$b_{12} = \lambda^H \lambda^L a > 0$$

$$b_{21} = \rho(\lambda^H \lambda^L - 1) + \iota^S[f^H(\lambda^H - 1) + \lambda^H(\lambda^L - 1)] > 0$$

$$b_{22} = E^S[f^H(\lambda^H - 1) + \lambda^H(\lambda^L - 1)] - 2a\lambda^H\lambda^L(\rho + \iota^S) < 0,$$

so $|B| = b_{11}b_{22} - b_{21}b_{12} < 0$. A greater fraction of consumers who are quality enthusiasts decreases aggregate expenditure and increases the rate of innovation in a separating equilibrium.

$$\frac{\partial E^S}{\partial f^H} = \frac{\rho\lambda^H(\lambda^L - 1)(E^S)^2[\rho[f^H\lambda^H(\lambda^L - 1) + \lambda^H - 1] + \iota^S(\lambda^H\lambda^L - 1)]}{(\rho + \iota^S)^2|B|} < 0 \tag{A5}$$

$$\frac{\partial \iota^S}{\partial f^H} = -\frac{\lambda^H E^S[\rho(\lambda^H\lambda^L - 1) + \iota^S\lambda^L(\lambda^H - 1)]}{|B|} > 0. \tag{A6}$$

Proof of proposition 3
In the limit as the fraction of consumers who are quality enthusiasts falls to the minimum share needed to support a separating equilibrium $f^H \to f^*$, the rate of innovation under separation falls to the rate of innovation under pooling $\Delta\iota = 0$. Thus, lifetime utility is lower,

$$\lim_{f^H \to f^*} \Delta U^\omega = \frac{\Delta \log E^\omega - \log \lambda^\omega}{\rho} < 0 \Leftrightarrow \frac{E_S^\omega}{E_P^\omega} < \lambda^\omega, \tag{A7}$$

if aggregate expenditure under separation relative to pooling is not sufficiently large to cover the higher price (or lower quality level consumed).

$$\lim_{f^H \to f^*} \frac{E_S^\omega}{E_P^\omega} = \frac{1}{1 - f^*(1 - \delta^H)}. \tag{A8}$$

Adding this expenditure effect to the negative price effect, relative aggregate expenditure is less than the quality increment $E_S^\omega / E_P^\omega < \lambda^\omega$, so lifetime utility falls $\Delta U^\omega > 0$, since $\delta^\omega + f^*(1 - \delta^H) < 1$ is true by the definition of quality $\delta^\omega < 1$ and $0 < f^H < 1$.

In the limit, since all consumers are quality enthusiasts $f^H \rightarrow 1$, aggregate expenditure under separation returns to the level of aggregate expenditure under pooling $\Delta E = 0$. Thus, lifetime utility is higher under separation,

$$\lim_{f^H \rightarrow 1} \Delta U^{\omega} = \frac{\left(\dfrac{\Delta\iota}{\rho} - 1\right) \log \lambda^{\omega}}{\rho} > 0 \Leftrightarrow \iota^S - \iota^P > \rho, \tag{A9}$$

when the rate of innovation under separation exceeds the rate of innovation under pooling by more than the discount rate.

$$\lim_{f^H \rightarrow 1} \frac{\iota^S - \iota^P}{\rho} = \left[\frac{L}{a\rho} + 1\right] \delta^L (1 - \delta^H). \tag{A10}$$

The rate of innovation rises by more than the discount rate $(\iota^S - \iota^P)/\rho > 1$, so lifetime utility rises $\Delta U^{\omega} > 0$.

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